VILLUPURAM DIST.

**Class : 12**

Register Number

# COMMON QUARTERLY EXAMINATION - 2024 - 25

## BUSINESS MATHEMATICS AND STATISTICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

### PART - A

I. Choose the correct answer. Answer all the questions. 20x1=20

I. Choose the correct answer 20×1= 20

1. If $T = \begin{matrix} A \\ B \end{matrix} \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$ is a transition probability matrix, then at equilibrium A is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

2. The rank of m×n matrix whose elements are unity is (a) 0 (b) 1 (c) m (d) n

3. If $\rho(A) = \rho(A, B) =$ the number of unknowns, then the system is

(a) Consistent and has infinitely many solutions (b) Consistent and has a unique solution
(c) Inconsistent (d) Consistent

4. 4 In a transition probability matrix, all the entries are greater than or equal to

(a) 2 (b) 1 (c) 0 (d) 3

5. $\int \frac{dx}{\sqrt{x^2-36}}$ is (a) $\sqrt{x^2-36}+c$ (b) $\log|x+\sqrt{x^2-36}|+c$
(c) $\log|x-\sqrt{x^2-36}|+c$ (d) $\log|x^2+\sqrt{x^2-36}|+c$

6. $\int 2^x\,dx$ is (a) $2^x \log 2 + c$ (b) $2^x + c$ (c) $\frac{2^x}{\log 2}+c$ (d) $\frac{\log 2}{2^x}+c$

7. $\int \frac{dx}{\sqrt{x^2-36}}$ is (a) $\sqrt{x^2-36}+c$ (b) $\log|x+\sqrt{x^2-36}|+c$
(c) $\log|x-\sqrt{x^2-36}|+c$ (d) $\log|x^2+\sqrt{x^2-36}|+c$

8. If MR and MC denotes the marginal revenue and marginal cost functions, then the profit function is

(a) $P = \int (MR - MC)\,dx + k$ (b) $P = \int (MR + MC)\,dx + k$
(c) $P = \int (MR)(MC)\,dx + k$ (d) $P = \int (R - C)\,dx + k$

9. The demand function for the marginal function $MR = 100 - 9x^2$ is

(a) $100 - 3x^2$ (b) $100x - 3x^2$ (c) $100x - 9x^2$ (d) $100 + 9x^2$

10. Area bounded by $y = |x|$ between the limits 0 and 2 is

(a) 1 sq. unit (b) 3 sq. units (c) 2 sq. units (d) 4 sq. units

11. The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{\frac{1}{2}} = x$ is

(a) of order 2 and degree 1 (b) of order 1 and degree 3
(c) of order 1 and degree 6 (d) of order 1 and degree 2

12. The integrating factor of the differential equation $\frac{dx}{dy} + Px = Q$ is

(a) $e^{\int P dx}$ (b) $\int P dx$ (c) $\int P dy$ (d) $e^{\int P dy}$

13. A homogeneous differential equation of the form $\frac{dx}{dy} = f\left(\frac{x}{y}\right)$ can be solved by making substitution,

(a) $x = vy$ (b) $y = vx$ (c) $y = v$ (d) $x = v$

14. $\Delta^2 y_0 =$ (a) $y_2 - 2y_1 + y_0$ (b) $y_2 + 2y_1 - y_0$ (c) $y_2 + 2y_1 + y_0$ (d) $y_2 + y_1 + 2y_0$

15. $E \equiv$ (a) $1 + \Delta$ (b) $1 - \Delta$ (c) $1 + \nabla$ (d) $1 - \nabla$

16. If $f(x) = x^2 + 2x + 2$ and the interval of differencing is unity then $\Delta f(x) =$
(a) $2x - 3$ (b) $2x + 3$ (c) $x + 3$ (d) $x - 3$

17. Given E(X) = 5 and E(Y) = –2, then E(X – Y) is
(a) 3 (b) 5 (c) 7 (d) –2

18. A formula or equation used to represent the probability distribution of a continuous random variable is called
(a) probability distribution (b) distribution function
(c) probability density function (d) mathematical expectation

19. If c is a constant, then E(c) is (a) 0 (b) 1 (c) $c f(c)$ (d) c

20. A discrete probability function $p(x)$ is always non-negative and always lies between
(a) 0 and ∞ (b) 0 and 1 (c) –1 and +1 (d) $-\infty$ and $+\infty$

## II. Answer Any SEVEN of the following (Qtn No. 30, Compulsory) $7 \times 2 = 14$

21. Solve the equations $2x + 3y = 7$, $3x + 5y = 9$ by Cramer's rule.

22. Find the rank of the matrix $\begin{bmatrix} 1 & 5 \\ 3 & 9 \end{bmatrix}$.

23. Integrate $e^{xloga} + e^{aloga} - e^{nlogx}$.

24. Solve: $\frac{dy}{dx} = y \sin 2x$

25. Evaluate $\Delta^2 \left(\frac{1}{x}\right)$ by taking '1' as the interval of differencing.

26. Prove $\nabla = \frac{E-1}{E}$.

27. Find $\Delta e^{ax}$.

28. Find the order and degree $\frac{d^3y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 + 2\frac{dy}{dx} = 0$.

29. Solve $9y'' - 12y' + 4y = 0$

30. Using graphic method, fi nd the value of $y$ when $x = 48$ from the following data:

| X | 40 | 50 | 60 | 70 |
|---|---|---|---|---|
| Y | 6.2 | 7.2 | 9.1 | 12 |

## III. Answer Any SEVEN of the following (Qtn No. 40, Compulsory) $7 \times 3 = 21$

31. A commodity was produced by using 3 units of labour and 2 units of capital, the total cost is ₹.62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is ₹ 56. What is the cost per unit of labour and capital? (Use determinant method).

32. If $f'(x) = 8x^3 - 2x$ and $f(2) = 8$, then find $f(x)$.

33. Integrate $\sqrt{1 - \sin 2x}$.

34. Evaluate $\Delta(\log ax)$.

35. Find $f(2.8)$ from the following table:

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| f(x) | 1 | 2 | 11 | 34 |

36. Solve $\frac{dy}{dx} + y\cos x = \sin x \cos x$.

37. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$..

38. Find $\Delta^2 e^x$.

39. If $p(x) = \begin{cases} \frac{x}{20}, x = 0,1,2,3,4,5 \\ 0, \quad otherwise \end{cases}$ Find (i) $P(X < 3)$ and (ii) $P(2 < X \leq 4)$

40. A fair die is thrown. Find out the expected value of its outcomes.

## IV. Answer ALL the questions. $7 \times 5 = 35$

41. a) Show that the equations $2x + y + z = 5, x + y + z = 4, x - y + 2z = 1$ are consistent and hence solve them. **(OR)**

b) Two types of soaps *A* and *B* are in the market. Their present market shares are 15% for *A* and 85% for *B*. Of those who bought *A* the previous year, 65% continue to buy it again while 35% switch over to *B*. Of those who bought *B* the previous year, 55% buy it again and 45% switch over to *A*. Find their market shares aft er one year and when is the equilibrium reached?

42. a) In a market survey three commodities A, B and C were considered. In finding out the index number some fi xed weights were assigned to the three varieties in each of the commodities. The table below provides the information regarding the consumption of three commodities according to the three varieties and also the total weight received by the commodity.

| Commodity veriet | Variety | | | Total weight |
|---|---|---|---|---|
| | I | II | III | |
| A | 1 | 2 | 3 | 11 |
| B | 2 | 4 | 5 | 21 |
| C | 3 | 5 | 6 | 27 |

Find the weights assigned to the three varieties by using Cramer's Rule. **(OR)**

b) Evaluate $\int \frac{7x-1}{x^2-5x+6} dx$

43. a) Find the diff erential equation of the family of curves $y = e^x (a\cos x + b\sin x)$ where a and *b* are arbitrary constants. **(OR)**

b) Solve $(D^2 + D - 6)y = e^{3x} + e^{-3x}$.

44. a) Suppose that $Q_d = 30 - 5p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2}$ *and* $Q_s = 6 + 3p$. Find the equilibrium price for market clearance **(OR)**

b) Evaluate $\Delta\left[\frac{1}{(x+1)(x+2)}\right]$ by taking '1' as the interval of differencing.

45. a) Following are the population of a district

| Year (x) | 1881 | 1891 | 1901 | 1911 | 1921 | 1931 |
|---|---|---|---|---|---|---|
| Population(y) thousands | 363 | 391 | 421 | - | 467 | 501 |

Find the population of the year 1911. **(OR)**

b) The population of a certain town is as follows:

| Year : x | 1941 | 1951 | 1961 | 1971 | 1981 | 1991 |
|---|---|---|---|---|---|---|
| Population in Lakhs y | 20 | 24 | 29 | 36 | 46 | 51 |

Using appropriate interpolation formula, estimate the population during the period 1946.

46. a) Using interpolation estimate the business done in 1985 from the following data

| Year | 1980 | 1983 | 1984 | 1986 |
|---|---|---|---|---|
| Business done (in Lakhs) | 150 | 235 | 365 | 525 |

(OR) b) The amount of bread (in hundreds of pounds) $x$ that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function $f(x)$is given by

$$f(x) = \begin{cases} Ax, & for\ 0 \le x < 10 \\ A(20-x), & for\ 10 \le x < 20 \\ 0, & otherwise \end{cases}$$

(a) Find the value of A.

(b) What is the probability that the number of pounds of bread that will be sold tomorrow is

(i) More than 10 pounds, (ii) Less than 10 pounds, and

(iii) Between 5 and 15 pounds?

47. a) The probability density function of a random variable $X$ is $f(x) = ke^{-|x|}, -\infty < x < \infty$. Find the value of k and also fi nd mean and variance for the random variable. **(OR)**

b) A continuous random variable $X$ has the following probability function

| Value of X =x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(x) | 0 | K | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2 + k$ |

(i) Find k (ii) Evaluate $p(x < 6)$, $p(x \le 6)$ and $p(0 < x < 5)$

(iii) If $P(X \le x) > \frac{1}{2}$ then find the minimum value of x.