

Sketching of Curves

Steps	1) $y = f(x) = x^2 - x - 6$	2) $y = f(x) = x^3 - 6x - 9$	3) $f(x) = \frac{-1}{3}(x^3 - 3x + 2)$
Curve			
i) Domain Range	$(-\infty, \infty)$ $y \geq \frac{-25}{4}$	$(-\infty, \infty)$ $(-\infty, \infty)$	$(-\infty, \infty)$ $(-\infty, \infty)$
ii) Intercepts	x-intercept = -2, 3 y-intercept = -6	x-intercept = 3 y-intercept = -9	x-intercept = -2, 1 y-intercept = $\frac{-2}{3}$
iii) Critical Points	$f'(x) = 0$ $\Rightarrow x = \frac{1}{2}$	$f'(x) = 0 \Rightarrow 3(x^2 - 2) = 0$ $\Rightarrow x = \pm\sqrt{2}$	$f'(x) = 0 \Rightarrow \frac{-1}{3}(3x^2 - 3) = 0$ $\Rightarrow x = \pm 1$
iv) Local Extrema	At $x = \frac{1}{2} \Rightarrow f''(x) = 2 > 0$ Local Min = $f\left(\frac{1}{2}\right) = \frac{-25}{4}$	$f''(x) = 6x$ Local Max = $f(-\sqrt{2}) = 4\sqrt{2} - 9$ Local Min = $f(\sqrt{2}) = -4\sqrt{2} - 9$	$f''(x) = -2x$ Local Min = $f(-1) = \frac{-4}{3}$ Local Max = $f(1) = 0$
v) Intervals of Concavity	$f''(x) = 2 > 0 \forall x \in R$ Concave up on R	$f''(x) = 0 \Rightarrow x = 0$ $x < 0 \Rightarrow f''(x) = -ve \Rightarrow$ Concave down $x > 0 \Rightarrow f''(x) = +ve \Rightarrow$ Concave up	$f''(x) = -2x$ & $f''(x) = 0 \Rightarrow x = 0$ $x < 0 \Rightarrow f''(x) = +ve \Rightarrow$ Concave up $x > 0 \Rightarrow f''(x) = -ve \Rightarrow$ Concave down
vi) Points of inflections	No point of inflection	Point of inflection = $(0, -9)$	Point of inflection = $\left(0, \frac{-2}{3}\right)$
vii) Asymptotes of the curve	No Asymptote	No Asymptotes	No Asymptotes

Steps	4) $y = \frac{x^2 - 3x}{x - 1}$	5) $y = \frac{3x}{x^2 - 1}$	6) $y = x\sqrt{4 - x}$
Curve			
i) Domain Range	$R - \{1\}$ $(-\infty, \infty)$	$R - \{-1, 1\}$ $(-\infty, \infty)$	$x \leq 4$ $\left(-\infty, \frac{16}{3\sqrt{3}}\right)$
ii) Intercepts	x-intercept = 0, 3 y-intercept = 0	x-intercept = 0 y-intercept = 0	x-intercept = 0, 4 y-intercept = 0
iii) Critical Points	$f'(x) = \frac{x^2 - 2x + 3}{(x-1)^2}$ Critical no $\Rightarrow x = 1$	$f'(x) = \frac{-3(x^2 + 1)}{(x^2 - 1)^2}$ Critical Nos $x = \pm 1$	$f'(x) = \frac{8 - 3x}{2\sqrt{4 - x}}$ Critical nos $x = \frac{8}{3}, 4$
iv) Local Extrema	$x = 1 \notin R - \{1\}$ No local Extrema	Since $x = \pm 1 \notin R - \{-1, 1\}$ No local Extrema	$f'(x) = \frac{3x - 16}{4(4 - x)^{3/2}}$ Local Max = $f\left(\frac{8}{3}\right) = \frac{16}{3\sqrt{3}}$
v) Intervals of Concavity	$f''(x) = \frac{-4}{(x-1)^3}$ $x < 1; f''(x) = +ve \Rightarrow$ Concave up $x > 1; f''(x) = -ve \Rightarrow$ Concave down	$f''(x) = \frac{6x(x^2 + 3)}{(x^2 - 1)^3} \Rightarrow x = -1, 0, 1$ $x < -1 \Rightarrow f''(x) = -ve \Rightarrow$ Concave down $-1 < x < 0 \Rightarrow f''(x) = +ve \Rightarrow$ Concave up $0 < x < 1 \Rightarrow f''(x) = -ve \Rightarrow$ Concave down $x > 1 \Rightarrow f''(x) = +ve \Rightarrow$ Concave up	$x < 4 \Rightarrow f''(x) = -ve \Rightarrow$ Concave down
vi) Points of inflections	No point of inflection	Point of inflection = $(0, 0)$	No point of inflection
vii) Asymptotes of the curve	Vertical Asymptote at $x = 1$ Slant Asymptote $y = x - 2$	Vertical Asymptotes at $x = -1$ & $x = 1$ Horizontal Asymptote at $y = 0$	No Asymptotes