Class : 12 Register No.

COMMON QUARTERLY EXAMINATION-2024-25

Time Allowed : 3.00 Hours] **MATHEMATICS** **[Max. Marks : 90**

PART - I

Answer all the questions: **20x1=20**

1. If A is a 3x3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T=
   (a) A (b) B (c) I_3 (d) B^T
2. The radius of the circle passing through the point (6,2) two of whose diameters are x+y=6 and x+2y=4 is
   (a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4
3. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then
   (a) $c = \pm 3$ (b) $c = \pm\sqrt{3}$ (c) $c > 0$ (d) $0 < c < 1$
4. The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
   (a) 1 + i (b) i (c) 1 (d) 0
5. If α, β and γ are the roots of x^3+px^2+qx+r, then $\sum \frac{1}{\alpha}$ is
   (a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$
6. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
   (a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$
7. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ ia
   (a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1
8. The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is
   (a) a parabola (b) a hyperpola (c) an ellipse (d) a circle
9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zerovectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to
   (a) 81 (b) 9 (c) 27 (d) 18
10. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to
    (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$
11. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =
    (a) 0 (b) sinθ (c) cosθ (d) 1

12. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta) y - (\cos\theta) z = 0$, $(\cos\theta) x - y + z$, $(\sin\theta) x + y - z = 0$ has a non-trivial solution then is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

13. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(a) 2 (b) 4 (c) 1 (d) ∞

14. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

15. $\text{Sin}^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$, then x is a root of the equatoin.

(a) $x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

16. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

17. If A is an invertible matrix of order 2. Then $\det[A^{-1}] =$

(a) det [A] (b) 1 / det [A] (c) 1 (d) 0

18. If $-i + 3$ is a roots of $x^2 - 6x + k = 0$, then the value of k is

(a) 5 (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 10

19. The sum of the focal distances from any point of the ellipse $9x^2 + 16y^2 = 144$ is

(a) 32 (b) 18 (c) 16 (d) 8

20. The value of $[\hat{i} - \hat{j}, \hat{j} - \hat{k}, \hat{k} - \hat{i}]$ is

(a) 0 (b) 1 (c) 2 (d) 3

PART - II

Answer any 7 Questions. Question Nunmber 30 is compulsory **7x2=14**

21. Find a polynomial equation of minimum degree with rational coefficients, havng $2 + \sqrt{3}i$ as a root.

22. Show that the equation $z^2 = \bar{z}$ has four solutions.

23. Find the equation of Ellipse: If focus $(\pm 3, 0)$ and $e = 1/2$.

24. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, then prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$.

25. If $\text{adj}\, A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

26. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$ find c.

27. Find the value of : $2\cos^{-1}(1/2) + \sin^{-1}(1/2)$.

28. Find the angle between the planes $\vec{r}.(\hat{i} + \hat{j} - 2\hat{k}) = 3$ and $x - 2y + z = 2$.

29. Find the value of $\sum_{n=1}^{10} i^{n+50}$

30. Find the rank of the matrix. $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

PART - III

Answer any 7 Questions. Question Number 40 is compulsory. **7x3=21**

31. Solve the cubic equation: $2x^3 - 9x^2 + 10x = 3$.
32. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.
33. If $|z| = 2$, show that $3 \le |z + 3 + 4i| \le 7$.
34. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x+2y+3=0$.
35. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_2$.
36. In a competitive examination, one mark is awarded for every correct answer while $1/4$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)
37. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
38. Find the magnitude and the direction cosines of the torque about the point (2,0,-1) of a force $2\hat{i} + \hat{j} - \hat{k}$ whose line of action passes through the origin.
39. Find the value of $\sin^{-1}\left(\sin\left(5\pi/4\right)\right)$
40. Simplify : $\left(\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}\right)^{12}$

PART - IV

Answer all the questions **7x5=35**

41. a) By vector method, prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

(OR)

b) The prices of three commodities A,B and C are Rs x, y and z per unit respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, PQ and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A,B and C. (Use matrix inversion method to solve the problem.)

42. a) On lighting a rocket cracker is get projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z - i}{z + 2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

43. a) Find the value of $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

(OR)

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

44. a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(OR)

b) Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

45. a) If $2\cos\alpha = x + {}^1/_x$ and $2\cos\beta = y + {}^1/_y$, show that

i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$ ii) $xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$

(OR)

b) If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$.

46. a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinte number of solutions.

(OR)

b) Find the equation of the circle passing through the points (1,1), (2,-1) and (3,2).

47. a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x+6y+6z=9$.

(OR)

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.