12 - [illegible] 24

1	d	$\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
2	d	1
3	d	$2A^{-1}$
4	d	$\sqrt{5}+2$
5	a	0
6	c	x^2+y^2
7	d	-4
8	a	$\frac{-q}{r}$
9	b	$\frac{\pi}{3}$
10	a	$\frac{\pi}{2}$
11	b	$1/\sqrt{5}$
12	c	$\sqrt{10}$
13	d	9
14	b	$\frac{1}{3}$
15	d	0
16	a	81
17	d	$\frac{\pi}{2}$
18	c	$\lvert A\rvert^{n-1}$
19	d	2
20	d	$x+iy$

21) $A=\begin{bmatrix}1&2&3\\2&1&4\\3&0&5\end{bmatrix} \sim \begin{bmatrix}1&2&3\\0&-3&-2\\0&-6&-4\end{bmatrix} \sim \begin{bmatrix}1&2&3\\0&-3&-2\\0&0&0\end{bmatrix}$

$\therefore \rho(A)=2.$

22) $\frac{z_1}{z_2}=\frac{3-2i}{6+4i}\times\frac{6-4i}{6-4i} = \frac{10}{52}-\frac{24}{52}i = \frac{5}{26}-\frac{6}{13}i$

23) $1+i \to 10-8i$

$=|9-9i|=\sqrt{162}$

$1+i \to 11+6i$

$=|10+5i|=\sqrt{125}$

$\therefore$ [illegible] : $11+6i$

24) $x^2=y \Rightarrow y^2-14y+45=0$

$\Rightarrow y=9,5.$

$y=9 \Rightarrow x=\pm 3$

$y=5 \Rightarrow x=\pm\sqrt{5}.$

$\therefore$ [illegible] : $3,-3,\sqrt{5},-\sqrt{5}.$

25) $LHS=\cos^{-1}\left(\frac{-3}{2\sqrt{3}}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}.$

26) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow (x-3)(x-2)+(y-4)(y+7)=0$

$\Rightarrow x^2+y^2-5x+3y-22=0$

27) $a=2$

$yt=x+at^2$

$\Rightarrow x-2y+8=0$

28) $[\vec{a}\ \vec{b}\ \vec{c}]=90$

$\begin{vmatrix}7&\lambda&-3\\1&2&-1\\-3&7&5\end{vmatrix}=90$

$\Rightarrow 2\lambda=-10 \Rightarrow \lambda=-5$

29) $\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{8-4+4}{\sqrt{12}\sqrt{24}}=\frac{\sqrt{2}}{3}$

$\theta=\cos^{-1}(\sqrt{2}/3)$

30) $AA^T=A^TA=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I_2$

$A \to$ [illegible].

31) $\underset{A}{\begin{bmatrix}2&-1\\3&2\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\y\end{bmatrix}}=\underset{B}{\begin{bmatrix}8\\-2\end{bmatrix}}$

$|A|=7,\ A^{-1}=\frac{1}{7}\begin{bmatrix}2&1\\-3&2\end{bmatrix}$

$X=A^{-1}B=\frac{1}{7}\begin{bmatrix}14\\-28\end{bmatrix}=\begin{bmatrix}2\\-4\end{bmatrix}$

$\therefore (x,y)=(2,-4).$

32) $[A|B]=\begin{bmatrix}2&2&1&5\\1&-1&1&1\\3&1&2&4\end{bmatrix} \sim \begin{bmatrix}1&-1&1&1\\2&2&1&5\\3&1&2&4\end{bmatrix} \sim \begin{bmatrix}1&-1&1&1\\0&4&-1&3\\0&4&-1&1\end{bmatrix}$

$\sim \begin{bmatrix}1&-1&1&1\\0&4&-1&3\\0&0&0&-2\end{bmatrix}$ $\rho(A)=2$, $\rho(A|B)=3.$

$\rho(A)\neq\rho(A|B)$ [illegible].

33) $(3x-2y+5)+i(-x+y-2) = (2x-y+3)+i(2y+2)$

$\Rightarrow x-y=-2$; $-x-y=0.$

$\therefore x=-1,\ y=1$

34) $3a=4 \Rightarrow a=4/3.$

$\therefore 9x^2-24x+12=0.$

$\Rightarrow x=2, 2/3.$

$\therefore$ [illegible] : $\frac{2}{3}, 4/3, 2.$

36) [illegible] : $c^2=a^2(1+m^2)$

$\Rightarrow c=\pm 3\sqrt{17}.$

37) $a^2=25,\ b^2=9,\ e=\frac{4}{5}$

$ae=5\times\frac{4}{5}=4.$

$C:(0,0)$

$F:(\pm ae,0)=(\pm 4,0)$

$V:(\pm a,0)=(\pm 5,0)$

$D: x=\pm a/e=\pm\frac{25}{4}.$

35) $10-3\pi \in [-\pi/2, \pi/2]$

$\sin^{-1}(\sin x)=\sin^{-1}(\sin(10-3\pi)) = 10-3\pi.$

38) $\vec{F}_1=5\sqrt{2}\hat{F}_1=3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{F}_2=10\sqrt{2}\hat{F}_2=10\hat{i}+6\hat{j}-8\hat{k}$

$\vec{F}=13\hat{i}+10\hat{j}-3\hat{k}$

$\vec{d}=\vec{AB}=2\hat{i}+4\hat{j}-\hat{k}$

$W=\vec{F}\cdot\vec{d}=26+40+3 = 69.$

39) $LHS=(\vec{a}\times\vec{b})\cdot[(b\times c)\times(c\times a)]$

$=(\vec{a}\times\vec{b})\cdot([\vec{a}\ \vec{b}\ \vec{c}]\vec{c})$

$=(\vec{a}\times\vec{b})\cdot\vec{c}\,[\vec{a}\ \vec{b}\ \vec{c}]$

$=[\vec{a}\ \vec{b}\ \vec{c}]^2$

40) $||z_1|-|z_2|| \le |z_1+z_2| \le |z_1|+|z_2|$

$|12+6+8i| \le |12|+|6+8i|$

≤ 12 — (1)

$|12+6+8i| \ge ||12|-|6+8i||$

$\ge |12-10| \ge 8$ — (2)

$\therefore 8 \le |12+6+8i| \le 12.$

41(a) $\Delta=-22$; $\Delta_x=-44$

$\Delta_y=-66$; $\Delta_z=-88$

$x=\frac{\Delta_x}{\Delta}=2$; $y=3, z=4.$

$\therefore (x,y,z)=(2,3,4).$

(b) $[A|B]=\begin{bmatrix}2&3&5&9\\7&3&-5&8\\2&3&\lambda&\mu\end{bmatrix} = \begin{bmatrix}2&3&5&9\\0&-15&-45&-47\\0&0&\lambda-5&\mu-9\end{bmatrix}$

(i) $\lambda=5 \Rightarrow \rho(A)\neq\rho(A|B)$

[illegible].

(ii) $\lambda\neq 5,\ \mu\in R.$

$\rho(A)=\rho(A|B)=3.$

[illegible].

(iii) $\lambda=5, \lambda=9 \Rightarrow \rho(A)=\rho(A|B)=2.$

[illegible].

42(a) $\frac{2z+1}{iz+1}=\frac{(2x+1)+i2y}{(1-y)+ix}$

$\mathrm{Im}\left(\frac{2z+1}{iz+1}\right)=0$

$\Rightarrow \frac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2}=0$

$\Rightarrow 2x^2+2y^2+x-2y=0$

(b) $\frac{19+9i}{5-3i}=2+3i$; $\frac{8+i}{1+2i}=2-3i$

$z=(2+3i)^{15}-(2-3i)^{15}$

$\bar{z}=-((2+3i)^{15}-(2-3i)^{15})$

$=-z. \Rightarrow z$ [illegible].

43(a) $1+2i$; $1-2i$

$\sqrt{3}$; $-\sqrt{3}.$

α,β [illegible].

[illegible] : $\alpha+\beta=1.$

[illegible] : $\alpha\beta=-9.$

$x^2-x-9=0.$

$\Rightarrow x=\frac{1\pm\sqrt{37}}{2}.$

(b). [illegible]

[illegible] : $1/3$; [illegible] $=3$

1/3	6	-5	-38	-5	6	
	0	2	-1	-13	-6	
3	6	-3	-39	-18	0	
	0	18	45	18		
-2	6	15	6	0		
	0	-12	-6			
-1/2	6	3				
	0	-3				
	6	0				

$\therefore$ [illegible] : $1/3, 3, -2, -1/2.$

44(a) $\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right]=\frac{\pi}{4}$

$\Rightarrow \frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}=1$

$\Rightarrow 2x^2=1 \Rightarrow x=\pm 1/\sqrt{2}.$

(b) $\sin^{-1}x, \cos^{-1}x$ [illegible] : $[-1,1].$

$-1\le\frac{|x|-2}{3}\le 1$; $-1\le\frac{1-|x|}{4}\le 1$

$\Rightarrow -1\le|x|\le 5$; $-5\le|x|\le 3$

$\Rightarrow |x|\le 5$

$\Rightarrow -5\le x\le 5.$

ie, $x\in[-5,5].$

45(a)

$x^2+y^2+2gx+2fy+c=0$

$2g+2f+c=-2$

$4g-2f+c=-5$

$6g+4f+c=-13$

$f=-\frac{1}{2}$; $g=\frac{-5}{2}$; $c=4.$

$x^2+y^2-5x-y+4=0.$

b) $(x-2)^2=5(y+1)$

$(x-h)^2=4a(y-k)$

$\Rightarrow a=5/4$

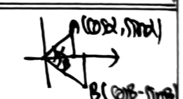
[illegible] : $(h,k)=(2,-1)$

[illegible] : $(h,k+a)=(2,1/4)$

[illegible] : $y=k-a=4y+9=0.$

[illegible] : $4a=5.$

(46) (a)



$\hat{a}=\cos\alpha\hat{i}+\sin\alpha\hat{j}$

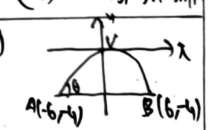
$\hat{b}=\cos\beta\hat{i}-\sin\beta\hat{j}$

$\hat{a}\cdot\hat{b}=\cos(\alpha+\beta)$ — (1)

$\hat{a}\cdot\hat{b}=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\therefore \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta.$

(b)



$x^2=-4ay$

$(6,-4)$ [illegible] $\Rightarrow 4a=9$

$\therefore x^2=-9y$ — (1)

$(-6,-4)$ [illegible] :

$2x=-9\frac{dy}{dx}.$

$\frac{dy}{dx}=\tan\theta=\frac{4}{3}.$

$\theta=\tan^{-1}(4/3).$

47 (a)

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&-1\\3&5&2\end{vmatrix} = 11\hat{i}-7\hat{j}+\hat{k}$

$(\vec{a}\times\vec{b})\times\vec{c}=-19\hat{i}-34\hat{j}-29\hat{k}$

$\vec{a}\cdot\vec{c}=-11$

$\vec{b}\cdot\vec{c}=-7.$

$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a} = -19\hat{i}-34\hat{j}-29\hat{k}$

$\therefore$ Result.

(b) $\vec{a}=3\hat{i}+6\hat{j}-2\hat{k}$

$\vec{b}=-\hat{i}-2\hat{j}+6\hat{k}$

$\vec{c}=6\hat{i}+4\hat{j}-2\hat{k}$

[illegible]:

$\vec{r}=(1-s-t)\vec{a}+s\vec{b}+t\vec{c}$

$\vec{r}=(1-s-t)(3\hat{i}+6\hat{j}-2\hat{k})+s(-\hat{i}-2\hat{j}+6\hat{k})+t(6\hat{i}+4\hat{j}-2\hat{k})$

[illegible]: $\begin{vmatrix}x-x_1, & y-y_1, & z-z_1\\x_2-x_1 & y_2-y_1, & z_2-z_1\\x_3-x_1, & y_3-y_1, & z_3-z_1\end{vmatrix}=0.$

$\begin{vmatrix}x-3&y-6&z+2\\-4&-8&8\\3&-2&0\end{vmatrix}=0$

$\Rightarrow 2x+3y+4z-16=0$

$(\vec{r}-\vec{a})\cdot((\vec{b}-\vec{a})\times(\vec{c}-\vec{a}))=0$

$\Rightarrow \vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})-16=0$