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COIMBATORE

Quarterly Examination - 2024

MATHEMATICS

Time : 3.00 hrs. Reg. No. Max. Marks : 90

PART - A

Answer all the questions. Choose the correct answer **20 x 1 = 20**

1. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ then $(A^T)^{-1}$ is

   a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj}A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ then k = a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1

3. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I_2 - A =$ a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

4. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is a) $\sqrt{3} - 2$ b) $\sqrt{3} + 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

5. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is a) 0 b) 1 c) –1 d) i

6. If $(1 + i)(1 + 2i)(1 + 3i) \ldots (1 + ni) = x + iy$ then $2.5.10 \ldots (i + n^2)$ is a) 1 b) i c) $x^2 + y^2$ d) $1 + n^2$

7. A zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) –4

8. If α, β, γ are the zeros of $x^3 + px^2 + qx + r$ then $\sum \frac{1}{\alpha}$ is a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

9. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π

10. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$ a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

11. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ then x is equal to a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

12. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

13. If $x + y = k$ is a normal to the parabola $y^2 = 12x$ then the value of k is a) 3 b) –1 c) 1 d) 9

14. The eccentricity of the ellipse $(x - 3)^2 + (y - 4)^2 = \frac{y^2}{9}$ is a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

15. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to a) 2 b) –1 c) 1 d) 0

16. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$ then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to a) 81 b) 9 c) 27 d) 18

17. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{2}$, $z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

18. If A is square matrix of order n then $|\text{adj}A|$ is a) $|A|^2$ b) $|A|^n$ c) $|A|^{n-1}$ d) $|A|$

19. The value of $[\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}]$ is equal to a) 0 b) 1 c) 2 d) 4

20. If $x^2 + y^2 = 1$, then the value of $\frac{1 + x + iy}{1 + x - iy}$ is a) $x - iy$ b) $2x$ c) $-2iy$ d) $x + iy$

PART - B

Answer any seven questions. Question number 30 is compulsory. **7 x 2 = 14**

21. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

22. If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, find $\frac{z_1}{z_2}$ in the rectangular form.

23. Which one of the points $10 - 8i$, $11 + 6i$ is closest to $1 + i$.

24. Solve the equation $x^4 - 14x^2 + 45 = 0$

25. Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

26. Obtain the equation of the circle for which (3, 4) and (2, –7) are the ends of a diameter.

27. Find the equation of the tangent at t = 2 to the parabola $y^2 = 8x$.

28. The volume of the parallelopiped whose conterminus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$, $-3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of λ.

29. Find the acute angle between the planes $\vec{r}.(2\hat{i}+2\hat{j}+2\hat{k}) = 11$ and $4x - 2y + 2z = 15$.

30. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

PART - C

Answer any seven questions. Question number 40 is compulsory. **7 x 3 = 21**

31. Solve by matrix inversion method $2x - y = 8$; $3x + 2y = -2$.
32. Test for consistency. $2x + 2y + z = 5$, $x - y + z = 1$, $3x + y + 2z = 4$
33. Find the values of the real numbers x and y of the complex nummbers $(3 - i)x - (2 - i)y + 2i + 5$ and $2x + (-1 + 2i)y + 3 + 2i$ are equal.
34. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$, if the roots form a arithmetic progression.
35. Simplify : $\sin^{-1}(\sin 10)$
36. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.
37. Find centre, foci, vertices and directrices of $\frac{x^2}{25} + \frac{y^2}{9} = 1$
38. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$ respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.
39. Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$
40. If $|z| = 2$, show that $8 \le |z + 6 + 8i| \le 12$

PART - D

Answer all the questions. **7 x 5 = 35**

41. a) Solve by Cramer's Rule. $3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$ **(OR)**
    b) Investigate the value of λ and μ the system of linear equation $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$ $2x + 3y + \lambda z = \mu$ have i) No solution ii) Unique solution iii) an infinite number of solutions.
42. a) If $z = x + iy$ is a complex number such that $\text{Im}\left[\frac{2z+1}{iz+1}\right] = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$ **(OR)**
    b) Show that $\left[\frac{19+9i}{5-3i}\right]^{15} - \left[\frac{8+i}{1+2i}\right]^{15}$ is purely imaginary.
43. a) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros. **(OR)** b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.
44. a) Solve : $\tan^{-1}\left[\frac{x-1}{x-2}\right] + \tan^{-1}\left[\frac{x+1}{x+2}\right] = \frac{\pi}{4}$ **(OR)** b) Find the domain of $\sin^{-1}\left[\frac{|x|-2}{3}\right] + \cos^{-1}\left[\frac{1-|x|}{4}\right]$
45. a) Find the equation of the circle passing through the points (1, 1), (2, –1) and (3, 2) **(OR)**
    b) Find the vertex, focus, directrix and length of latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$
46. a) By vector method prove that $\cos(\alpha + \beta) = \cos\alpha\ \cos\beta - \sin\alpha\ \sin\beta$ **(OR)**
    b) On lighting a rocket craker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.
47. a) If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = 3\hat{i}+5\hat{j}+2\hat{k}$ $\vec{c} = -\hat{i}-2\hat{j}+3\hat{k}$, verify that $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$
    (OR)
    b) Find the parametric vector, non-parametric vector and carteasian form of the equations of the plane passing through the three non-collinear points (3, 6, –2), (–1, –2, 6) and (6, 4, –2)