COMMON QUARTERLY EXAMINATION - 2024

Standard XII Reg.No. [| | | | |]

MATHEMATICS

Time : 3.00 hrs Part - I Marks : 90

I. Choose the correct answer: 20 x 1 = 20

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

2. If A is non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

3. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

4. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a) 17 b) 14 c) 19 d) 21

5. The area of the triangle formed by the complex numbers z, iz and z+iz in the Argand's diagram is

a) $\frac{1}{2}|z|^2$ b) $|z|^2$ c) $\frac{3}{2}|z|^2$ d) $2|z|^2$

6. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

a) z b) $\bar{z}$ c) $\frac{1}{z}$ d) 1

7. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

a) -110° b) -70° c) 70° d) 110°

8. Choose the wrong statement.

a) $|z|^2 = 1 \Rightarrow \frac{1}{z} = \bar{z}$ b) $\mathrm{Re}(z) \le |z|$

c) $||z_1| - |z_2|| \ge |z_1 + z_2|$ d) $|z^n| = |z|^n$

9. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

10. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r(-1)^r x^r$

a) 0 b) n c) < n d) r

11. The period of $y = \cos 6x + \sin 4x$ is

a) π b) $\frac{5\pi}{6}$ c) 3π d) 4π

12. The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is

a) $\pi - x$ b) $x - \frac{\pi}{2}$ c) $\frac{\pi}{2} - x$ d) $x - \pi$

13. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

14. if $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is

a) 4 b) 5 c) 2 d) 3

15. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

16. An ellipse has OB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{\sqrt{3}}$

17. The locus of a point whose distance from (–2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is

a) a parabola b) a hyperbola c) an ellipse d) a circle

18. If a vector $\bar{\alpha}$ lies in the plane of $\bar{\beta}$ and $\bar{\gamma}$, then

a) $[\bar{\alpha}, \bar{\beta}, \bar{\gamma}] = 1$ b) $[\bar{\alpha}, \bar{\beta}, \bar{\gamma}] = -1$ c) $[\bar{\alpha}, \bar{\beta}, \bar{\gamma}] = 0$ d) $[\bar{\alpha}, \bar{\beta}, \bar{\gamma}] = 2$

19. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{b} \cdot \vec{c} \neq 0$ and $\vec{a} \cdot \vec{b} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

a) perpendicular b) parallel

c) inclined at an angle $\frac{\pi}{3}$ d) inclined at an angle $\frac{\pi}{6}$

20. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

a) 0 b) 1 c) 2 d) 3

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) $7 \times 2 = 14$

21. Find the rank by minor method : $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$.

22. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

23. Write in rectangular form : $\overline{(5+9i)+(2-4i)}$

24. Show that $|z-2-i| = 3$ represent a circle and find its centre and radius.

25. If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$, construct a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$

26. Find a polynomial equation of minimum degree with rational coefficients having 2i+3 as a root.

27. Find all the values of x such that $-10\pi \le x \le 10\pi$ and $\sin x = 0$

28. Find the general equation of the circle whose diameter is the line segment joining the points (–4,–2) and (1,1)

29. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$, find $\vec{a} \cdot (\vec{b} \times \vec{c})$

30. Prove that $\sum_{n=1}^{204} \left(i^{n+1} + i^{n+2}\right) = 0$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) $7 \times 3 = 21$

31. Solve by matrix inversion method : $2x - y = 8$, $3x + 2y = -2$

32. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

33. If $|z| = 3$, show that $7 \le |z + 6 - 8i| \le 13$

34. If $\omega \ne 1$ is a cube root of unity, show that $(1 + \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$

35. Solve : $x^3 - 3x^2 - 33x + 35 = 0$

36. Find the exact number of real and imaginary zeros of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$

37. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

38. Prove that $\cos^{-1}(\cos 10) = 4\pi - 10$

39. A particle acted on by constant forces $8\hat{i} + 2\hat{j} - 6\hat{k}$ and $6\hat{i} + 2\hat{j} - 2\hat{k}$ is displaced from the point (1,2,3) to the point (5,4,1), prove that work done by forces is 80 units.

40. Find the equations of the tangent and normal to the circle $x^2 + y^2 = 25$ at P(–3,4)

Part - IV

IV. **Answer all the questions.** 7 x 5 = 35

41. a) If $F(\alpha)=\begin{vmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{vmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$ *(OR)*

b) If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$. Find all roots.

42. a) If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$. Show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$ *(OR)*

b) Find the domain of f(x) =

$$f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$$

43. a) If $z=x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$. Show that the locus of z is $2x^2+2y^2+x-2y=0$

(OR)

b) If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$

44. a) Find the vertex, focus, directrix and length of latus rectum of the parabola $x^2-4x-5y-1=0$ *(OR)*

b) Prove by vector method : $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

45. a) Investigate the values of λ and μ the system of linear equation $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. *(OR)*

b) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight line

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1} \text{ and } \frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$$

46. a) Evaluate : $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$ *(OR)*

b) Solve by Cramer's rule : $3x+3y-z=11$, $2x-y+2z=9$, $4x+3y+2z=25$

47. a) A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre on either sides. *(OR)*

b) If $2\cos\alpha=x+\frac{1}{x}$ and $2\cos\beta=y+\frac{1}{y}$, show that,

i) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$ ii) $x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$

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