QUARTERLY EXAMINATION - 2024

Class : 12 Reg.No

MATHEMATICS

Time : 3.00 Hours Marks : 90

PART – I

Answer all the questions . $20 \times 1 = 20$

1. If $\operatorname{adj}A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $\operatorname{adj}B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj(AB) is
   1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ 2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ 3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ 4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

2. If A is a 3 x 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$
   1) A 2) B 3) I_3 4) B^T

3. If $\rho(A) = \rho([A|B])$, then the system AX = B of linear equations is
   1) consistent and has a unique solution 2) consistent
   3) consistent and has infinitely many solution 4) inconsistent

4. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is
   1) 1 2) 2 3) 3 4) 5

5. If $\omega = \text{cis}\frac{2\pi}{3}$, then the number of distinct root of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$
   1) 1 2) 2 3) 3 4) 4

6. The area of the triangle formed by the complex numbers z, iz and $z + iz$ in the Argand's diagram is
   1) $\frac{1}{2}|z|^2$ 2) $|z|^2$ 3) $\frac{3}{2}|z|^2$ 4) $2|z|^2$

7. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r(-1)^r x^r$ is
   1) 0 2) n 3) $< n$ 4) r

8. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to
   1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ 2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ 3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ 4) $\tan^{-1}\left(\frac{1}{2}\right)$

9. If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then
   1) $|\alpha| \le \frac{1}{\sqrt{2}}$ 2) $|\alpha| \ge \frac{1}{\sqrt{2}}$ 3) $|\alpha| < \frac{1}{\sqrt{2}}$ 4) $|\alpha| > \frac{1}{\sqrt{2}}$

10. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is
    1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

11. The locus of a point whose distance from (–2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is
    (1) a parabola (2) a hyperbola (3) an ellipse (4) a circle

12. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3).
    (1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

13. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ having centr at (0, 3) is

(1) $x^2+y^2-6y-7=0$ (2) $x^2+y^2-6y+7=0$ (3) $x^2+y^2-6y-5=0$ (4) $x^2+y^2-6y+5=0$

14. The coordinates of the point where the line $\vec{r}=(6\hat{i}-\hat{j}-3\hat{k})+t(-\hat{i}+4\hat{k})$ meets the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=3$ are

(1) (2,1,0) (2) (7,–1,–7) (3) (1,2,–6) (4) (5,–1,1)

15. If the planes $\vec{r}.(2\hat{i}-\lambda\hat{j}+\hat{k})=3$ and $\vec{r}.(4\hat{i}+\hat{j}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2},-2$ (2) $\frac{-1}{2},2$ (3) $-\frac{1}{2},-2$ (4) $\frac{1}{2},2$

16. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(1) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=1$ (2) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=-1$ (3) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=0$ (4) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=2$

17. If $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=6$, then the value of $|z_1+z_2+z_3|$ is

1) 1 2) 2 3) 3 4) 4

18. A zero of x^2+4 is

1) 0 2) $2i$ 3) 2 4) –2

19. $\tan(\sin^{-1}x)$, $|x|<1$ is equal to

1) $\frac{x}{\sqrt{1-x^2}}$ 2) $\frac{1}{\sqrt{1-x^2}}$ 3) $\frac{1}{\sqrt{1+x^2}}$ 4) $\frac{x}{\sqrt{1+x^2}}$

20. If A= $\begin{bmatrix} tan\theta & sec\,\theta \\ sec\,\theta & tan\,\theta \end{bmatrix}$ and A(adjA) = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =

1) 0 2) 1 3) 2 4) - 1

PART – II

Answer any seven questions. Question No.30 is compulsory. **7 x 2 = 14**

21. If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive.

22. Find the rank $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

23. Write the following in the rectangular form: $\overline{3i}+\frac{1}{2-i}$.

24. Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular from.

25. If α, β, and γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of the coefficients.

26. Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

27. Determine whether $x+y-1=0$ is the equation of a diameter of the circle $x^2+y^2-6x+4y+c=0$ for all possible values of c.

28. Prove that $[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}]=0$

29. Find the acute angle between the planes $\vec{r}.(2\hat{\imath}+2\hat{\jmath}+2\hat{k}) = 11$ and $4x-2y+2z=15$.

30. Find the period and amplitude of $y=-4\sin(2x+3)$.

PART – III

Answer any Seven questions. Question No.40 is compulsory. **7 x 3 = 21**

31. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

32. Solve the following systems of linear equations by Cramer's rule:
$5x-2y+16=0, x+3y-7=0$.

33. If $|z|=3$, show that $7 \le |z+6-8i| \le 13$.

34. If $\omega \ne 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$.

35. Solve the equation $x^4-9x^2+20=0$.

36. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$.

37. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1+\frac{2}{t_1}\right)$

38. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$

39. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{\imath}+\hat{\jmath}-\hat{k}$ whose line of action passes through the origin.

40. Show that, if p,q,r are rational and $p+q+r=0$, then the roots of the equation $4px^2+3qx+2r=0$ are rational.

PART – IV

Answer all the questions. **7 x 5 = 35**

41. (a) Investigate for what values of λ and μ the system of linear equations
$x+2y+z=7, x+y+\lambda z=\mu, x+3y-5z=5$
has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

(b) The prices of three commodities A,B and C are ₹ x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P,Q ,and R earn ₹ 15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A,B ,and C. (Use matrix inversion method to solve the problem.)

42. (a) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$ show that $x^2+y^2=1$.

(OR)

(b) Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$

43. (a) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$ if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

(OR)

(b) Find the domain of $f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$.

44. (a) Find the equation of the circle passing through the points (1,1),(2,-1) and (3,2).

(OR)

(b) A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?

45. (a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

(OR)

(b) Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through thepoints (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.

46. (a) Find the vertex, focus, equation of directrix and length of the latus rectum of the following parabola $x^2 - 4y + 2 = 0$

(OR)

(b) Find the shortest distance between the lines $\vec{r} = (1-\lambda)\vec{i} + (\lambda - 2)\vec{j} + (3 - 2\lambda)\vec{k}$ and $\vec{r} = (\mu + 1)\vec{i} + (2\mu - 1)\vec{j} - (2\mu + 1)\vec{k}$.

47. (a) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

(OR)

(b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$.