KK12M

Kanyakumari District

Common Quarterly Examination - September 2024

Time: 3.00 hrs

Standard 12

MATHEMATICS

Marks: 90

PART – I ($20 \times 1 = 20$)

(i) Answer all the questions.

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

(1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

2. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

(1) $(1, 0)$ (2) $(-1, 1)$ (3) $(0, 1)$ (4) $(1, 1)$

3. If $sin^{-1}x + sin^{-1}y = \frac{2\pi}{3}$ then $cos^{-1}x + cos^{-1}y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

4. If A = $\begin{bmatrix} 1 & tan\frac{\theta}{2} \\ -tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I_2, then B =

(1) $(cos^2\frac{\theta}{2})A$ (2) $(cos^2\frac{\theta}{2})A^T$ (3) $(cos^2\theta)I$ (4) $(sin^2\frac{\theta}{2})A$

5. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

6. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to

(1) 81 (2) 9 (3) 27 (4) 18

7. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

8. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

(1) $4(a^2+b^2)$ (2) $2(a^2+b^2)$ (3) a^2+b^2 (4) $\frac{1}{2}(a^2+b^2)$

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9. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}$, $\hat{i}+2\hat{j}$, $\hat{i}+\hat{j}+\pi\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

10. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

11. If f and g are polynomials of degree m and n respectively, and if $h(x)=(f o g)(x)$, then the degree of h is

(1) mn (2) $m+n$ (3) m^n (4) n^m

12. The value of $\sum_{n=1}^{13}(i^n+i^{n-1})$ is

(1) $1+i$ (2) i (3) 1 (4) 0

13. If $A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$ then $det(A^{-1})$ is equal to

(1) 0 (2) $\frac{1}{5}$ (3) $\frac{-1}{5}$ (4) does not exist

14. If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) -1 (3) 1 (4) 9

15. The number of rows in a truth table is 4. Then its corresponding compound statement is made up of ----- number of simple statements.

(1) 2 (2) 2^2 (3) 2^3 (4) 2^4

16. If $|z|=2$ and $arg(z)=\frac{\pi}{4}$ then $z=$

(1) $2(1+i)$ (2) $2(1-i)$ (3) $\sqrt{2}(1+i)$ (4) $\sqrt{2}(1-i)$

17. If α,β are the roots of $x^2+x+1=0$ and if $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are the roots of $x^2+px+q=0$ then p is equal to

(1) -1 (2) 1 (3) -2 (4) 2

18. If $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\hat{k}$, then $\vec{a}\times(\vec{a}.\vec{b})$ is equal to

(1) $3\vec{a}$ (2) $3\sqrt{14}$ (3) 0 (4) not defined

19. The number of tangents that can be drawn from $(1,2)$ to $x^2+y^2=5$ is

(1) 1 (2) 2 (3) 3 (4) 0

20. The value of $sin^{-1}(1)+cos^{-1}(1)$ is

(1) 1 (2) 0 (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

PART – II ($7 \times 2 = 14$)

Answer any seven questions. Question No.30 is compulsory.

21. Find the value of $sin^{-1}[sin\left(\frac{5\pi}{4}\right)]$

22. Find the modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$

23. If α, β are the roots of $17x^2 + 43x - 73 = 0$, find equation whose roots are $-\alpha$ and $-\beta$

24. Examine the position of the point $(2, 3)$ with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$

25. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$

26. Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.

27. Find z^{-1}, if $z = (2 + 3i)(1 - i)$

28. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I_2$

29. Verify the associative property under the binary operation $*$ defined by $a * b = a^b, \forall\, a, b \in N$

30. What is the eccentricity of the parabola $x^2 - 4x - 5y - 1 = 0$?

PART – III ($7 \times 3 = 21$)

Answer any seven questions. Question No.40 is compulsory.

31. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

32. Which one of the points $10 - 8i, 11 + 6i$ is closest to $1 + i$

33. Find the roots of the equation $x^4 - 3x^2 - 4 = 0$

34. The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

35. find a polynomial equation with integer coefficient with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

36. Prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$

37. If $cot^{-1}\left(\frac{1}{7}\right) = \theta$, then find the value of $cos\theta$

38. Find the equation of the ellipse whose Foci $(\pm 3, 0), e = \frac{1}{2}$

39. Solve by matrix inversion method: $5x + 2y = 4, 7x + 3y = 5$

40. Find $|adj(AB)|$ where $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

PART – IV (7 × 5 = 35)

Answer all the questions.

41. (a) Find the locus of a complex number $z = x + iy$ if $\left|\frac{z-4i}{z+4i}\right| = 1$

(OR)

(b) Find the equation of the circle passing through the points $(1,1), (2,-1)$ and $(3,2)$

42. (a) Find the eccentricity, length of latus rectum, length of transverse axis and the length of conjugate axis of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

(OR)

(b) Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + s(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + t(3\hat{i} + 2\hat{j} + 6\hat{k})$ intersecting. Also find the point of intersection.

43. (a) Find the condition on a, b, c so that the system of linear equations $x + y + z = a,\ x + 2y + 3z = b,\ 3x + 5y + 7z = c$ has one parameter family of solutions.

(OR)

(b) A rod of length $1.2m$ moves with its ends always touching the coordinate axes. Prove that the locus of a point P on the rod, which is $0.3m$ from the end in contact with x – axis is an ellipse. Find the eccentricity.

44. (a) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(OR)

(b) Construct truth table for $(\neg p \to r) \wedge (p \leftrightarrow q)$.

45. (a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

(OR)

(b) Solve $(x-5)(x-7)(x+6)(x+4) = 504$

46. (a) Find all the cube roots of $\sqrt{3} + i$

(OR)

(b) Prove by vector method that $\cos(A+B) = \cos A \cos B - \sin A \sin B$

47. (a) Solve by Cramer's Rule: $3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$

(OR)

(b) Find the parametric vector and cartesian form of the equations of the plane passing through the non-collinear points $(3, 6, -2), (-1, -2, 6), (6, -4, -2)$.

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