QUARTERLY EXAMINATION - 2024

MATHEMATICS

STD - XII
TIME : 3.00 Hrs **MARKS : 90**

I Answer all the following questions. **20 x 1 = 20**

1. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is
   (a) 17 (b) 14 (c) 19 (d) 21

2. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is (a) cis $\frac{2\pi}{3}$ (b) cis $\frac{4\pi}{3}$ (c) -cis $\frac{2\pi}{3}$ (d) -cis $\frac{4\pi}{3}$

3. If α, β and γ are the roots of x^3+px^2+qx+r, then is $\Sigma\frac{1}{\alpha}$ is
   (a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

4. The domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ is
   (a) [1, 2] (b) [-1, 1] (c) [0, 1] (d) [-1, 0]

5. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
   a) 2ab (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

6. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then
   (a) $c=\pm3$ (b) $c=\pm\sqrt{3}$ (c) $c>0$ (d) $0<c<1$

7. If $A=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}$, $B = adj\,A$ and $C = 3A$, then $\frac{|adj\,B|}{|C|}=$
   (a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

8. The principal argument of $\frac{3}{-1+i}$ is
   (a) $-\frac{5\pi}{6}$ (b) $-\frac{2\pi}{3}$ (c) $-\frac{3\pi}{4}$ (d) $-\frac{\pi}{2}$

9. The number of real numbers in $[0, 2\pi]$ satisfying $sin^4 x - 2sin^2 x + 1$ is
   (a) 2 (b) 4 (c) 1 (d) ∞

10. The equation $tan^{-1} x - cot^{-1} x = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has
    (a) no solution (b) unique solution (c) two solutions (d) infinite number of solutions

11. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
    (a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

12. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j},\hat{i}+2\hat{j},\hat{i}+\hat{j}+\pi\hat{k}$ is (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

13. If $A=\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value x is

(a) $\frac{-4}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

14. If z is a non-zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

15. If $cot^{-1} 2$ and $cot^{-1} 3$ are two angles of a triangle, then the third angle is

(a) $\frac{p}{4}$ (b) $\frac{3p}{4}$ (c) $\frac{p}{6}$ (d) $\frac{p}{3}$

16. Area of quadrilateral with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

17. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

18. If f(x) = 0 has n roots, then f'(x) = 0 has .... roots.

(a) n (b) n - 1 (c) n+1 (d) (n - r)

19. Area of the circle $(x-2)^2 + (y-k)^2 = 25$ is

(a) 25π (b) 5π c) 10π d) 25

20. If $\vec{p} \times \vec{q} = 2\hat{i}+\hat{j}$, $\vec{r} \times \vec{s} = 3\hat{j}+2\hat{k}$ then $\vec{p}.(\vec{q} \times (\vec{r} \times \vec{s})) =$

(a) 9 (b) 6 (c) 2 (d) 5

PART - II K. Muruganandham

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II. Answer any seven questions. Q.No.30 is compulsory. **7 x 2 = 14**

21. Find $adj(adj(A))$ if $adjA=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

22. Find the rank of the matrix $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

23. Obtain the Cartesian equation for the locus of $z = x + iy$ in $\bar{z} = z^{-1}$

24. Find the principal value of $sin^{-1}(2)$, if it exists.

25. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

26. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.

27. Obtain the equation of the circles with radius 5cm and touching x-axis at the origin in general form.

28. If $2\hat{i}-\hat{j}+3\hat{k}, 3\hat{i}+2\hat{j}+\hat{k}, \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

29. If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z in the rectangular form.

30. Find the equation of the parabola, The end points of latus rectum (4, -8) and (4, 8)

PART - III

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III. Answer any seven questions. Q.No.40 is compulsory. **7 x 3 = 21**

31. Find the sum of the squares of the roots of $ax^4+bx^3+cx^2+dx+e=0, a\neq 0$.

32. Find the value of $2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$

33. Find the vertices, foci for the hyperbola $9x^2-16y^2=144$.

34. Find the inverse of matrix by using Gauss-Jordan method $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

35. Prove that the point of interesection of the tangents at 't_1' and 't_2' on the parabola $y^2=4ax$ is $[\, at_1t_2, a(t_1+t_2)]$

36. Find the angle between the straight line $\vec{r}=(2\hat{i}+3\hat{j}+\hat{k})+t(\hat{i}-\hat{j}+\hat{k})$ and the plane $2x-y+z=5$

37. Show that $\left|\frac{z-3}{z+3}\right|=2$ represents a circle.

38. Prove that : $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$

39. Show that the equation $z^2=\bar{z}$ has four solutions.

40. The vector equation in parametric form of a line is $\vec{r}=(3\hat{i}-2\hat{j}+6\hat{k})+t(2\hat{i}-\hat{j}+3\hat{k})$. Find (i) the direction consines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equations of the line.

PART - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Solve the following systems of linear equations by Gaussian elimination method:
$2x+4y+6z=22, 3x+8y+5z=27, -x+y+2z=2$

(OR)

b) Prove by vector method that $sin(\alpha + \beta) = sin\, \alpha\, cos\, \beta + cos\, \alpha\, sin\, \alpha\beta$

42. a) Find the value of $cot^{-1}(1) + sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - sec^{-1}(-\sqrt{2})$

(OR)

b) Test for consistency of the following system of linear equations and if possible solve : $x - y + z = -9,\ 2x - 2y + 2z = -18,\ 3x - 3y + 3z + 27 = 0$

43. a) Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ (OR)

b) Find the equation of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to $10x - 3y + 9 = 0.$

44. a) If $z = x + iy$ and $arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

(OR)

b) Identify the type of conic and find centre, foci, vertices, and directrices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

45. a) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}, \vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$ verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

(OR)

b) Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

46. a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$

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(OR)

b) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

47. a) Find the value of $tan\left(sin^{-1}\frac{3}{5} + cot^{-1}\frac{3}{2}\right)$ (OR)

b) Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point (0,1,-5) and parallel to the straight lines

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + s(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$.