QL

QUARTERLY EXAMINATION - 2024

MARK : 90

STD : XII

MATHEMATICS

TIME : 3 hrs

PART – A

CHOOSE THE CORRET ANSWER :- 20 X 1 = 20

1. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|} =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

3. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$.

The system has infinitely many solutions if

(a) $\lambda = 7, \mu \neq -5$ (b) $\lambda = -7, \mu = 5$

(c) $\lambda \neq 7, \mu \neq -5$ (d) $\lambda = 7, \mu = -5$

4. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

5. If $(1+i)(1+2i)(1+3i)\cdots(1+ni) = x+iy$, then $2 \cdot 5 \cdot 10 \cdots (1+n^2)$ is

(a) 1 (b) i (c) x^2+y^2 (d) $1+n^2$

6. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

7. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is

(a) mn (b) $m+n$ (c) m^n (d) n^m

8. The number of positive roots of the polynomial $\sum_{r=0}^{n} {}^{n}C_r (-1)^r x^r$ is

(a) 0 (b) n (c) $< n$ (d) r

9. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(a) 0 (b) 1 (c) 2 (d) 3

10. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

11. $\sin(\tan^{-1} x), |x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

12. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(a) $15 < m < 65$ (b) $35 < m < 85$ (c) $-85 < m < -35$ (d) $-35 < m < 15$

13. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

(a) 3 (b) -1 (c) 1 (d) 9

14. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3) is

(a) $x^2 + y^2 - 6y - 7 = 0$ (b) $x^2 + y^2 - 6y + 7 = 0$
(c) $x^2 + y^2 - 6y - 5 = 0$ (d) $x^2 + y^2 - 6y + 5 = 0$

15. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

16. The coordinates of the point where the line $\vec{r} = (6\hat{\imath} - \hat{\jmath} - 3\hat{k}) + t(-\hat{\imath} + 4\hat{k})$ meets the plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 3$ are

(a) (2,1,0) (b) (7, −1, −7) (c) (1, 2, −6) (d) (5, −1, 1)

17. If $x + iy = \frac{1}{1+\cos\theta + i\sin\theta}$ then $sin^{-1}x$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{5\pi}{12}$

18. The value of $\sin\left[\frac{\pi}{2} - sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is

(a) ½ (b) 1 (c) 0 (d) $\frac{\sqrt{3}}{2}$

19. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$ then the value of c is

(a) $3\sqrt{17}$ (b) $-3\sqrt{17}$ (c) $\pm 3\sqrt{17}$ (d) 5

20. If $P(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vetor then the value of P is

(a) 1 (b) $\pm\frac{1}{2}$ (c) $\pm\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

PART – B

ANSWER ANY SEVEN QUESTIONS (Q.NO : 30 IS COMPULSORY) :- 7 X 2 = 14

21. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

22. Find z^{-1}, if $z = (2+3i)(1-i)$.

23. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.

24. Find the value of $tan^{-1}\left(tan\left(-\frac{\pi}{6}\right)\right)$

25. If $cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $cos\,\theta$.

26. Find the equation of the hyperbola with Foci $(\pm 2,\ 0)$, $e = \frac{3}{2}$.

27. Find the equations of tangent to the ellipse $x^2 + 4y^2 = 32$ at $(4,2)$.

28. Determine whether the three vectors $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \hat{\jmath} + 3\hat{k}$ are coplanar.

29. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$.

30. If $\omega \neq 1$ is a cube root of unity, show that $\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega} = 0$

PART – C

ANSWER ANY SEVEN QUESTIONS (Q.NO : 40 IS COMPULSORY) : 7 X 3 = 21

31. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

32. Test the consistency of the system of linear equations

$x - y + z = -9,\ \ 2x - y + z = 4,\ \ 3x - y + z = 6,\ \ 4x - y + 2z = 7.$

33. Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.

34. Find the principal argument $Arg z$, when $z = \frac{-2}{1+i\sqrt{3}}$.

35. Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

36. Find the domain of $f(x) = sin^{-1}\left(\frac{|x|-2}{3}\right) + cos^{-1}\left(\frac{1-|x|}{4}\right)$.

37. A concrete bridge is designed as a parabolic arch. The road over bridge is $40m$ long and the maximum height of the arch is $15m$. Write an equation of the parabolic arch.

38. Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.

39. Find the magnitude and direction cosines of the torque (moment) of a force represented by $3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ about the point with position vector $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ acting through a point whose position vector is $4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.

40. Prove that $cot^{-1}(7) + cot^{-1}(8) + cot^{-1}(18) = cot^{-1}(3)$.

PART – D

ANSWER ALL THE QUESTIONS :- 7 X 5 = 35

41a. Solve the following system of equations, using matrix inversion method:

$$2x_1 + 3x_2 + 3x_3 = 5,\ x_1 - 2x_2 + x_3 = -4,\ 3x_1 - x_2 - 2x_3 = 3.$$ (OR)

b. Identify the type of conic and find centre, foci, vertices, and directrices of

$$18x^2 + 12y^2 - 144x + 48y + 120 = 0$$

42a. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

(OR)

b. Find the value of $cos\left(sin^{-1}\left(\frac{4}{5}\right) - tan^{-1}\left(\frac{3}{4}\right)\right)$

43a. Solve the equations $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ *(OR)*

b. By vector method, prove that $cos(\alpha + \beta) = cos\,\alpha\, cos\,\beta - sin\,\alpha\, sin\,\beta$.

44a. Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution

(OR)

b. Obtain the Cartesian equation form of the locus of $z = x + iy$ in $Im\left(\frac{2z+1}{iz+1}\right) = 0$.

45a. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1, 2, 0), (2, 2, -1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

(OR)

b. Solve the equation $(z-1)^3 + 8 = 0$

46a. Solve : $cos\left(sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = sin\left\{cot^{-1}\left(\frac{3}{4}\right)\right\}$.

(OR)

b. Find the equation of the circle passing through the points $(1, 1)$, $(2, -1)$, and $(3, 2)$.

47a. Solve : $tan^{-1}2x + tan^{-1}3x = \frac{\pi}{4}$, if $6x^2 < 1$. *(OR)*

b. Show that the line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ and its point of contact.