T

COMMON QUARTERLY EXAMINATION - 2024

Standard - XII

Reg.No.

Time: 3.00 hrs. **MATHEMATICS** *Marks:90*

I. Choose the best answer: **20×1=20**

1. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj\ (AB)| =$

   a) -40 b) -80 c) -60 d) -20

2. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$

   a) $\left(\cos^2\frac{\theta}{2}\right)A$ b) $\left(\cos^2\frac{\theta}{2}\right)A^T$ c) $\left(\cos^2\theta\right)I$ d) $\left(\sin^2\frac{\theta}{2}\right)A$

M.SATHEESH LINGAM M.A.,M.Sc.,M.Ed.,M.Phil.,DUC.,
P.G. Asst.
APARNA MAT. HR. SEC. SCHOOL
MURUGESAPURAM
PARAMANKURICHI - 628 213

3. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively

   a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

   c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

4. The conjugate of a complex number is $\frac{1}{i-2}$. Then the complex number is

   a) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

5. If z is a complex number such that $z \in C\backslash R$ and $z + \frac{1}{z} \in R$, then $|z|$ is

   a) 0 b) 1 c) 2 d) 3

6. The principal argument of $\frac{3}{-1+i}$ is

   a) $\frac{-5\pi}{6}$ b) $\frac{-2\pi}{3}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{2}$

7. If f and g are polynomials of degrees m and n respectively, and if h(x) = (fog)(x), then the degree of h is

   a) mn b) m + n c) m^n d) n^m

8. A polynomial equation in x of degree n always has

   a) n distinct roots b) n real roots c) n complex roots d) at most one root

9. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

   a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

10. If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then

    a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $|\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| > \frac{1}{\sqrt{2}}$

M.SATHEESH LINGAM M.A.,M.SC.,M.Ed.,M.Phil.,DUC.,
P.G. Asst.
APARNA MAT. HR. SEC. SCHOOL
MURUGESAPURAM
PARAMANKURICHI - 628 213

11. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

a) $-\pi \le x \le 0$ b) $0 \le x \le \pi$ c) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ d) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

12. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017}+y^{2018}+z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

a) 0 b) 1 c) 2 d) 3

13. If the function $f(x)=\sin^{-1}(x^2-3)$ then x blelongs to

a) $[-1, 1]$ b) $(\sqrt{2}, 2)$ c) $(-2, -\sqrt{2}) \cup [\sqrt{2}, 2]$ d) $[-2, -\sqrt{2}]$

14. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

a) $\frac{4}{3}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{3}{2}$

15. Identify the conic $3x^2 + 3y^2 - 4x + 3y + 10 = 0$ is

a) Circle b) Ellipse c) It is not a conic d) Hyperbola

16. If P(x, y) be any point on $16x^2 + 25y^2 = 400$ with foci F_1 (3, 0) and F_2 (-3, 0) then $PF_1 + PF_2$ is

a) 8 b) 6 c) 10 d) 12

17. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ d) $(3\sqrt{3}, -2\sqrt{2})$

18. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) π d) $\frac{\pi}{4}$

19. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is

a) (-5, 5) b) (-6, 7) c) (5, -5) d) (6, -7)

20. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

a) 0 b) 1 c) 2 d) 3

Answer any 7 questions. (Question number 30 is compulsory) **7×2=14**

M.SATHEESH LINGAM M.A.,M.SC.,M.Ed.,M.Phil.,DUC.,
P.G. Asst.
APARNA MAT. HR. SEC. SCHOOL
MURUGESAPURAM
PARAMANKURICHI - 628 213

21. If adj (A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ = , find A^{-1}. Ex 1.1 (9)

22. Solve the following system of linear equations by matrix inversion method.
$2x - y = 8$, $3x + 2y = -2$ Ex 1.3 (1(ii))

23. Find the square root of $6 - 8i$. Eg 2.17

24. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}\,i$ as a root. Ex 3.2 (2)

25. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$ Eg 4.27

26. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$ Ex 4.2 (5(ii))

27. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c. Eg 5.12

28. Find the length of Latus rectum of the parabola $y^2 = 4ax$. Eg 5.14

29. A particle acted upon by the forces $(3\hat{i} - 2\hat{j} + 2\hat{k})$ and $(2\hat{i} + \hat{j} - \hat{k})$ is displaced from the point (1, 3, -1) to the point (4, -1, λ). If the work done by the forces is 16 units, find the value of λ. Eg 6.10

30. Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin perpendicular to $6\hat{i} + 2\hat{j} - 3\hat{k}$. Eg 6.38

Answer any 7 questions. (Question number 40 is compulsory) **7×3=21**

31. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$ Ex 1.1 (7)

32. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form. Eg 1.18

33. If $|z| = 2$ show that $3 \le |z + 3 + 4i| \le 7$ Eg 2.13

34. If $\omega \neq 1$ is a cube root of unity, then the show that
$\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = -1$ Eg 2.8 (1)

35. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1. Ex 3.1 (4)

36. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$ Ex 4.1 (5)

37. Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$ (Hint : use parametric form) Ex 5.4 (6)

38. The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex. Eg 5.34

39. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$ Eg 6.18

40. Find the equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})+1=0$ and $\vec{r}.(2\hat{i}-3\hat{j}+5\hat{k})=2$ and the point (-1, 2, 1). Eg 6.53

Answer all the questions: **7x5=35**

41. a) If $A=\frac{1}{7}\begin{bmatrix}6 & -3 & a\\ b & -2 & 6\\ 2 & c & 3\end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. Eg 1.12 **(OR)**

b) Solve the following equation : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ Eg 3.28

42. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$ then show that $x^2 + y^2 + 3x - 3y + 2 = 0$ Exc 2.7(6)

(OR)

b) If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2=\left(t_1+\frac{2}{t_1}\right)$ Ex 5.4(8)

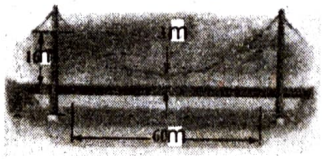
43. a) Solve the equation $z^3 + 8i = 0$, where $z \in C$. Eg 2.34 **(OR)**

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection. Ex 5.5(9)

44. a) Solve the equation $(x - 2)(x - 7)(x - 3)(x + 2) + 19 = 0$ Eg 3.23 **(OR)**

b) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ Ex 4.10

45. a) Parabolic cable of a 60m portion of the road bed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. Ex 5.5(5)



(OR)

b) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ Ex 6.1(10)

46. a) Simplify : $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$ Ex 4.5(8) **(OR)**

b) Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are skew and hence find the shortest distance between them. Ex 6.5(5)

47. a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$ Eg 6.44 **(OR)**

b) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$ have Ex 1.6(3)
i) no solution ii) a unique solution iii) an infinite number of solutions.

M.SATHEESH LINGAM M.A.,M.SC.,M.Ed.,M.Phil.,DUC.,
P.G. Asst.
APARNA MAT. HR. SEC. SCHOOL
MURUGESAPURAM
PARAMANKURICHI - 628 213