Vnr12M

Virudhunagar District
Common Quarterly Examination - 2024

Standard 12

MATHEMATICS

Time Allowed: 3.00 Hours Maximum Marks: 90

PART - I

Choose the correct answer: **20×1=20**

1) If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|adj(AB)|$ =

a) −40 b) −80 c) −60 d) −20

2) The augmented matrix of a system of linear equations is $\begin{bmatrix}1 & 2 & 7 & 3\\ 0 & 1 & 4 & 6\\ 0 & 0 & \lambda-7 & \mu+5\end{bmatrix}$

The system has infinitely many solutions, if

a) $\lambda = 7, \mu \neq -5$ b) $\lambda = -7, \mu = 5$

c) $\lambda \neq 7, \mu \neq -5$ d) $\lambda = 7, \mu = -5$

3) The rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 6 & 8\\ -1 & -2 & -3 & -4\end{bmatrix}$ is

a) 1 b) 2 c) 4 d) 3

4) If $Z=\dfrac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

a) 0 b) 1 c) 2 d) 3

5) The principal argument of $\dfrac{3}{-1+i}$ is

a) $\dfrac{-5\pi}{6}$ b) $\dfrac{-2\pi}{3}$ c) $\dfrac{-3\pi}{4}$ d) $\dfrac{-\pi}{2}$

6) The value of $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

a) $\text{cis}\dfrac{2\pi}{3}$ b) $\text{cis}\dfrac{4\pi}{3}$ c) $-\text{cis}\dfrac{2\pi}{3}$ d) $-\text{cis}\dfrac{4\pi}{3}$

7) A polynomial equation in x of degree n always has

a) n distinct roots b) n real roots

c) n complex roots d) at most one root

8) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

9) $\sin^{-1}(\cos x) = \dfrac{\pi}{2} - x$ is valid for

a) $-\pi \leq x \leq 0$ b) $0 \leq x \leq \pi$ c) $\dfrac{-\pi}{2} \leq x \leq \dfrac{\pi}{2}$ d) $\dfrac{-\pi}{4} \leq x \leq \dfrac{3\pi}{4}$

10) $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to

a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

11) The range of $\cos^{-1}x$ is

a) $(0, \pi)$ b) $[-\pi, 0]$ c) $[0, \pi]$ d) $[0, \frac{\pi}{2}]$

12) If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is

a) $-\sqrt{\frac{24}{25}}$ b) $\sqrt{\frac{24}{25}}$ c) $\sqrt{\frac{1}{5}}$ d) $\frac{-1}{5}$

13) The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

a) $\frac{4}{3}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{3}{2}$

14) $2x^2 - y^2 = 7$ denotes

a) parabola b) circle c) hyperbola d) ellipse

15) The eccentricity of the parabola is

a) $e < 1$ b) $e = 1$ c) $e > 1$ d) None of these

16) The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ passing through the point

a) $(-5, 2)$ b) $(2, -5)$ c) $(5, -2)$ d) $(-2, 5)$

17) If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

a) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ b) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ c) 1 d) -1

18) The angle between the vector $3\vec{i} + 4\vec{j} + 5\vec{k}$ and the z-axis is

a) 30° b) 60° c) 45° d) 90°

19) Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

a) 0 b) 1 c) 2 d) 3

20) If the planes $\vec{r}.(2\vec{i} - \lambda\vec{j} + \vec{k}) = 3$ and $\vec{r}.(4\vec{i} + \vec{j} - \mu\vec{k}) = 5$ are parallel, then the value of λ and μ are

a) $\frac{1}{2}, -2$ b) $\frac{-1}{2}, 2$ c) $\frac{-1}{2}, -2$ d) $\frac{1}{2}, 2$

PART - II

Answer any 7 questions: (30 is compulsory) **7×2=14**

21) Find the inverse of $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$.

22) Simplify: $i \cdot i^2 \cdot i^3 \ldots\ldots i^{2000}$

23) Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.

24) Find the principal value of $\sin^{-1}(2)$, if it exists.

25) Obtain the equation of the circle for which (3, 4) and (2, −7) are the ends of a diameter.

26) Find the vertices, foci for the hyperbola $9x^2-16y^2 = 144$.

27) Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are parallel.

28) Find the volume of the parallelo piped whose coterminus edges are represented by the vectors $-6\vec{i}+14\vec{j}+10\vec{k}$, $14\vec{i}-10\vec{j}-6\vec{k}$ and $2\vec{i}+4\vec{j}-2\vec{k}$.

29) Show that $\left(2+i\sqrt{3}\right)^{10} + \left(2-i\sqrt{3}\right)^{10}$ is real.

30) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

PART - III

Answer any 7 of the following questions: (40 is compulsory) **7×3=21**

31) If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A = |A|I.

32) Examine the consistency of the equation $2x-3y+7z = 5$, $3x+y-3z = 13$, $2x+19y-47z = 32$.

33) If $z = \cos\theta+i\sin\theta$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$.

34) If α, β and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d = 0$, find the value of $\Sigma\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

35) Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$.

36) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$.

37) Find the equation of the parabola whose vertex is (5, −2) and focus (2, −2).

38) If the equation $3x^2+(3-p)xy+qy^2-2px-8pq = 0$ represents a circle, find p and q. Also determine the centre and radius of the circle.

39) Find the angle between the planes $\vec{r}.(\vec{i}+\vec{j}-2\vec{k}) = 3$ and $2x-2y+z = 2$.

40) Find the paint of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

PART - IV

Answer all the questions: **7×5=35**

41) a) If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

(OR)

b) Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532 = 0$.

42) a) If $z = x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$.

(OR)

b) If $\vec{a} = 2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b} = 3\vec{i}+5\vec{j}+2\vec{k}$, $c = -\vec{i}-2\vec{j}+3\vec{k}$, verify that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

43) a) Solve: $(x-5)(x-7)(x+6)(x+4) = 504$.

(OR)

b) Solve: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

44) a) Draw $y = \sin x$ in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $y = \sin^{-1}x$ in $[-1, 1]$

(OR)

b) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

45) a) Solve the following system of linear equation by Gaussian elimination method.

$2x-2y+3z = 2$, $x+2y-z = 3$, $3x-y+2z = 1$

(OR)

b) Find the equation of the circle passing through the points (1, 0), (–1, 0) and (0, 1).

46) a) Find the fourth roots of unity.

(OR)

b) Prove by vector method that $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

47) a) Find the vector (parametric and non-parametric) and Cartesian form of the equations of the plane passing through the three non-collinear points (3, 6, –2), (–1, –2, 6) and (6, 4, –2).

(OR)

b) From the equation whose roots are the squares of the roots of the cubic equation $x^3+ax^2+bx+c = 0$.

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