Class : 12

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COMMON QUARTERLY EXAMINATION-2024-25

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

Answer all the questions: 20x1=20

1. If A is a 3x3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T=
(a) A (b) B (c) I_3 (d) B^T

2. The radius of the circle passing through the point (6,2) two of whose diameters are x+y=6 and x+2y=4 is
(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

3. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then
(a) $c = \pm 3$ (b) $c = \pm\sqrt{3}$ (c) $c > 0$ (d) $0 < c < 1$

4. The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
(a) $1 + i$ (b) i (c) 1 (d) 0

5. If α, β and γ are the roots of x^3+px^2+qx+r, then $\sum \frac{1}{\alpha}$ is
(a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

6. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

7. If $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ ia
(a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1

8. The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is
(a) a parabola (b) a hyperpola (c) an ellipse (d) a circle

9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zerovectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to
(a) 81 (b) 9 (c) 27 (d) 18

10. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

11. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =
(a) 0 (b) $\sin\theta$ (c) $\cos\theta$ (d) 1

12. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta) y - (\cos\theta) z = 0$, $(\cos\theta) x - y + z$, $(\sin\theta) x + y - z = 0$ has a non-trivial solution then is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

13. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(a) 2 (b) 4 (c) 1 (d) ∞

14. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

15. $\sin^{-1}\left(\tan \frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$, then x is a root of the equatoin.

(a) $x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

16. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

17. If A is an invertible matrix of order 2. Then $\det[A^{-1}] =$

(a) det [A] (b) 1 / det [A] (c) 1 (d) 0

18. If $-i + 3$ is a roots of $x^2 - 6x + k = 0$, then the value of k is

(a) 5 (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 10

19. The sum of the focal distances from any point of the ellipse $9x^2 + 16y^2 = 144$ is

(a) 32 (b) 18 (c) 16 (d) 8

20. The value of $[\hat{i} - \hat{j}, \hat{j} - \hat{k}, \hat{k} - \hat{i}]$ is

(a) 0 (b) 1 (c) 2 (d) 3

PART - II

Answer any 7 Questions. Question Nunmber 30 is compulsory **7x2=14**

21. Find a polynomial equation of minimum degree with rational coefficients, havng $2 + \sqrt{3}i$ as a root.

22. Show that the equation $z^2 = \bar{z}$ has four solutions.

23. Find the equation of Ellipse: If focus $(\pm 3, 0)$ and $e = 1/2$.

24. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, then prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$.

25. If $\operatorname{adj} A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

26. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$ find c.

27. Find the value of : $2\cos^{-1}(1/2) + \sin^{-1}(1/2)$.

28. Find the angle between the planes $\vec{r} \cdot (\hat{i} + \hat{j} - 2\hat{k}) = 3$ and $x - 2y + z = 2$.

29. Find the value of $\sum_{n=1}^{10} i^{n+50}$

30. Find the rank of the matrix. $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

PART - III

Answer any 7 Questions. Question Number 40 is compulsory. 7x3=21

31. Solve the cubic equation: $2x^3 - 9x^2 + 10x = 3$.
32. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.
33. If $|z| = 2$, show that $3 \leq |z + 3 + 4i| \leq 7$.
34. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x+2y+3=0$.
35. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I_2$.
36. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)
37. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
38. Find the magnitude and the direction cosines of the torque about the point (2,0,-1) of a force $2\hat{i} + \hat{j} - \hat{k}$ whose line of action passes through the origin.
39. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$
40. Simplify : $\left(\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}\right)^{12}$

PART - IV

Answer all the questions 7x5=35

41. a) By vector method, prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

(OR)

b) The prices of three commodities A,B and C are Rs x, y and z per unit respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P,Q and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A,B and C. (Use matrix inversion method to solve the problem.)

42. a) On lighting a rocket cracker is get projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z - i}{z + 2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

43\. a) Find the value of $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

(OR)

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

44\. a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(OR)

b) Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

45\. a) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$ ii) $xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$

(OR)

b) If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$.

46\. a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinte number of solutions.

(OR)

b) Find the equation of the circle passing through the points (1,1), (2,-1) and (3,2).

47\. a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x+6y+6z=9$.

(OR)

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

MOUNT CARMEL MISSION MAT. HR. SEC. SCHOOL
KALLAKURICHI

STD: XII

SUB: MATHS

COMMON QUARTERLY EXAM – 2024-25

ANSWER KEYS

PART - I

1. (c) I_3
2. (b) $2\sqrt{5}$
3. (b) $c = \pm\sqrt{3}$
4. (a) $1+i$
5. (a) $-\frac{q}{r}$
6. (c) $\frac{2}{\sqrt{3}}$
7. (a) $|\vec{a}||\vec{b}||\vec{c}|$
8. (c) an ellipse
9. (a) 81
10. (b) $\frac{1}{\sqrt{5}}$
11. (d) 1
12. (d) $\frac{\pi}{4}$
13. (a) 2
14. (b) imaginary axis
15. (b) $x^2 - x - 12 = 0$
16. (b) -1
17. (b) $\frac{1}{\det[A]}$
18. (d) 10
19. (d) 8
20. (a) 0

17. Solution

$AA^{-1} = I$

Taking det on both sides

$|AA^{-1}| = |I|$

$|A|\,|A^{-1}| = |I|$ $(\because |AB| = |A||B|)$

$|A^{-1}| = \frac{1}{|A|}$ $(\because I = 1.)$

Since, $|A| \neq 0.$

Hence $|A^{-1}| = \frac{1}{|A|}.$

18. Solution

Quadratic equation of the form,

$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0.$

Given,

$-i+3$ is a root of

$x^2 - 6x + k = 0.$

Sum of roots

$(-i+3) + (-i+3) = -6$

$-2i + 6 = -6$

$-2i = -12$

$\boxed{i = 6}$

Product of roots

$(-i+3)(-i+3) = k$

$-i^2+9 = k.$

$1+9 = k$

$\boxed{k=10}$

(OR)

Let $\alpha = -i+3$

The other root is $\beta = 3+i$

product of roots $= \alpha\beta$

$\alpha\beta = (-i+3)(3+i) = k.$

$\Rightarrow -i^2+3^2 = k$

$1+9 = k$

$\therefore \boxed{k=10}$

19. $9x^2+16y^2 = 144$

$\div 144 \Rightarrow$

$$\frac{9x^2}{144} + \frac{16y^2}{144} = \frac{144}{144}$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Here, $a^2 = 16$, $b^2 = 9$

$a = 4$, $b = 3$

Length of Major axis $= 2a$

$2(4) = 8$

Since the sum of the focal distance from any point on the ellipse is equal to its major axis

$\therefore$ Required sum $= 8.$

20.

$[\hat{i}-\hat{j}, \hat{j}-\hat{k}, \hat{k}-\hat{i}]$

$= [\hat{i}-\hat{j}].[(\hat{j}-\hat{k})\times(\hat{k}-\hat{i})]$

$= (\hat{i}-\hat{j}).[\hat{j}\times\hat{k} - \hat{j}\times\hat{i} - \hat{k}\times\hat{k} + \hat{k} + \hat{i}]$

$= \hat{i}.\hat{i} + \hat{i}.\hat{k} + \hat{i}.\hat{j} - \hat{j}.\hat{i} - \hat{j}.\hat{k} - \hat{j}.\hat{j}$

$= \hat{i}^2 + 0 + 0 - 0 - 0 - \hat{j}^2$

$= 1-1 = 0.$

PART - II

21) Given that, $2+\sqrt{3}i$ is a root of the polynomial.

Therefore $2-\sqrt{3}i$ is also a root of the polynomial.

Let $\alpha = 2+\sqrt{3}i$ and $\beta = 2-\sqrt{3}i$

$\alpha+\beta = 2+\sqrt{3}i + 2-\sqrt{3}i = 4$

$\boxed{\alpha+\beta = 4}$

$\alpha\beta = (2+\sqrt{3}i)(2-\sqrt{3}i)$

$= 2^2-(\sqrt{3}i)^2 = 4+3 = 7$

The equation of the quadratic polynomial whose roots are

$x^2-(\alpha+\beta)x+\alpha\beta = 0.$

$\therefore x^2-4x+7 = 0.$

which is the required minimum degree polynomial with rational coefficients.

22) We have, $z^2 = \bar{z}$

$\Rightarrow |z|^2 = |z|$

$\Rightarrow |z|^2 - |z| = 0$

$\Rightarrow |z|(|z|-1) = 0$

$\Rightarrow |z| = 0 \; ; \; |z| - 1 = 0$

$|z| = 1.$

$|z| = 0 \Rightarrow z = 0$ is a solution

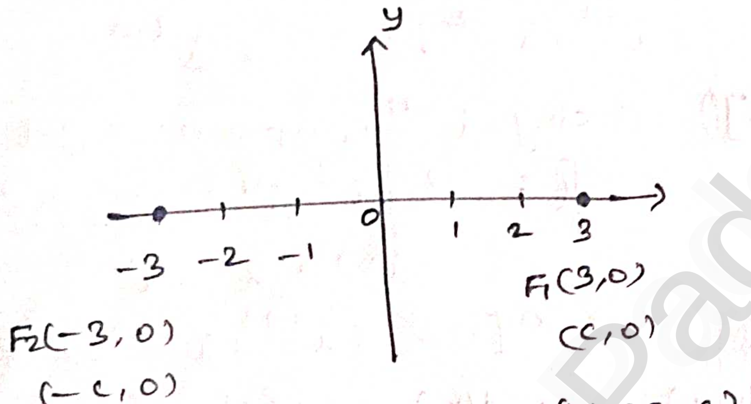
$|z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$

Given $z^2 = \bar{z} \Rightarrow z^2 = \frac{1}{z}$

$z^3 = 1.$

It has 3 non-zero solutions
Hence including zero solution, There are four solutions.

23)



$(\because ae = c)$

The major axis along x-axis.

$C(h,k)$ = Midpoint of F_1F_2

$= \left(\frac{3-3}{2}, \frac{0+0}{2}\right) = (0,0)$

$F_1F_2 = 2ae = 6$

$\Rightarrow 2c = 6 \Rightarrow \boxed{c = 3}$

$c = ae \Rightarrow ae = 3$

$a\left(\frac{1}{2}\right) = 3$

$\boxed{a = 6}$

$c^2 = a^2 - b^2$

$9 = 36 - b^2$

$b^2 = 36 - 9 \Rightarrow 27$

Equation of ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-0)^2}{36} + \frac{(y-0)^2}{27} = 0$$

$$\therefore \frac{x^2}{36} + \frac{y^2}{27} = 0.$$

$\therefore$ The required equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{27} = 0.$

24)

$[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}]$

$= [\vec{a}+0\vec{b}+\vec{c}, \vec{a}+\vec{b}+0\vec{c}, \vec{a}+0\vec{b}+\vec{c}]$

$$= \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$$

$= \{1(1-0)+1(1-1)\}[\vec{a}, \vec{b}, \vec{c}]$

$= (1+0)[\vec{a}, \vec{b}, \vec{c}]$

$= [\vec{a}, \vec{b}, \vec{c}]$

Hence proved.

25)

We compute,

$$|\text{adj}A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 9$$

So, we get, $A^{-1} = \pm \frac{1}{\sqrt{|\text{adj}A|}} \text{adj}(A)$

$$= \pm \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

4

26)

If $y = 4x + c$ is a tangent to the circle $x^2+y^2=9$

The condition for the line $y = mx + c$ to be a tangent to the circle $x^2+y^2=a^2$ is

$c^2 = a^2(1+m^2)$

Then,

$c = \pm\sqrt{9(1+16)}$

$c = \pm 3\sqrt{17}$.

27)

Let $\cos^{-1}\frac{1}{2} = x$ and $\sin^{-1}\left(\frac{1}{2}\right) = y$

$\cos x = \frac{1}{2}$

$\cos x = \cos\frac{\pi}{3}$ $\left(\because \frac{\pi}{3} \in [0,\pi]\right)$

$x = \frac{\pi}{3}$

$\sin y = \frac{1}{2} \Rightarrow \sin y = \sin\frac{\pi}{6}$

$x = \frac{\pi}{3}$ and $y = \frac{\pi}{6}$ $\left(\because \frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right)$

$2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

$= 2\left(\frac{\pi}{3}\right) + \frac{\pi}{6} = \frac{2\pi}{3} + \frac{\pi}{6}$

$= \frac{4\pi+\pi}{6} = \frac{5\pi}{6}$.

$\therefore 2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) = \frac{5\pi}{6}$.

28)

Given planes are,

$\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k}) = 3$ and

$2x-2y+z = 2 \Rightarrow \vec{r}\cdot(2\hat{i}-2\hat{j}+\hat{k}) = 2$

$\vec{n_1} = \hat{i}+\hat{j}-2\hat{k}$, and

$\vec{n_2} = 2\hat{i}-2\hat{j}+\hat{k}$

Angle between the plane is

$\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{|1(2)+1(-2)-2(1)|}{\sqrt{1^2+1^2+(-2)^2}\sqrt{2^2+(-2)^2+1^2}}$

$= \frac{|-2|}{\sqrt{6}\cdot\sqrt{9}} = \frac{2}{\sqrt{6}(3)} = \frac{2}{3\sqrt{6}}$

$\cos\theta = \frac{2}{3\sqrt{6}} \Rightarrow \theta = \cos^{-1}\frac{2}{3\sqrt{6}}$

29)

$i^{1+50} + i^{2+50} + \dots + i^{10+50}$

$= i^{51} + i^{52} + \dots + i^{60}$

Taking i^{50} common we get,

$i^{50}[(i+i^2+i^3+i^4) + (i^5+i^6+i^7+i^8) + i^9 + i^{10}]$

$= i^{50}[0 + (i^{4+1}+i^{4+2}+i^{4+3}+i^{4+4}) + (i^{8+1}+i^{8+2})]$

$= i^{50}[0+0+i+i^2]$

$(\because i+i^2+i^3+i^4=0)$

$= i^{50}[i-1] = i^{48+2}(i-1)$

$= i^2(i-1)$

$= -1(i-1) = -i+1 = 1-i$.

30)

Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$.

Then A is a matrix of order 3×3.

So, $\rho(A) \le \min\{3,3\} = 3$.

The highest order of minors of A is 3. There is only one Third order Minor of A.

It is $\begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{vmatrix}$

$= 3(6-6) - 2(6-6) + 5(3-3) = 0$

So, $\rho(A) < 3$

Next consider the second-order minors of A.

we find that the second order minor $\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3-2 = 1 \neq 0$

So, $\rho(A) = 2$.

PART – III

31)

Since the sum of the co-efficients is

$2 - 9 + 10 - 3 = 12 - 12 = 0$

$x = 1$ is a root of $f(x)$

$\therefore (x-1)$ is a factor of $f(x)$

To find the other factor, let us divide $f(x)$ by $x-1$

$$\begin{array}{c|cccc} 1 & 2 & -9 & 10 & -3 \\ & \downarrow & 2 & -7 & 3 \\ \hline 3 & 2 & -7 & 3 & \boxed{0} \\ & \downarrow & 6 & -3 & \\ \hline & 2 & -1 & \boxed{0} & \end{array}$$

$f(x) = (x-1)(x-3)(2x-1) = 0$

$x - 1 = 0$, $x = 1$ | $x - 3 = 0$, $x = 3$ | $2x - 1 = 0$, $x = \frac{1}{2}$

Hence the roots are $1, 3, \frac{1}{2}$

32)

Given equation of the circle is

$3x^2 + (3-p)xy + qy^2 - 2px = 8pq$

For the circle,

co-efficient of $xy = 0$

$3 - p = 0 \Rightarrow p = 3$

Also, co-efficient of x^2 = co-efficient of y^2

$3 = q$

$\therefore$ Equation of the circle is

$3x^2 + 3y^2 - 6x = 8(3)(3)$

$3x^2 + 3y^2 - 6x - 72 = 0$

Dividing by 3, we get

$x^2 + y^2 - 2x - 24 = 0$

Here $2g = -2$

$\boxed{g = -1}$

$f = 0$ and $c = -24$

centre is $(-g, -f) = (1, 0)$

and $r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-1)^2 + 0 + 24}$

$= \sqrt{25}$

$r = 5$ units

33) $3 \le |z + 3 + 4i| \le 7$

$\big||z_1| - |z_2|\big| \le |z_1 + z_2| \le |z_1| + |z_2|$

Here, $z_1 = z$

$z_2 = 3 + 4i$

$$||z| - |3+4i|| \le |z+3+4i| \le |z| + |3+4i|$$

$$|2 - \sqrt{3^2+4^2}| \le |z+3+4i| \le 2 + \sqrt{3^2+4^2}$$

$$|2 - \sqrt{9+16}| \le |z+3+4i| \le 2 + \sqrt{9+16}$$

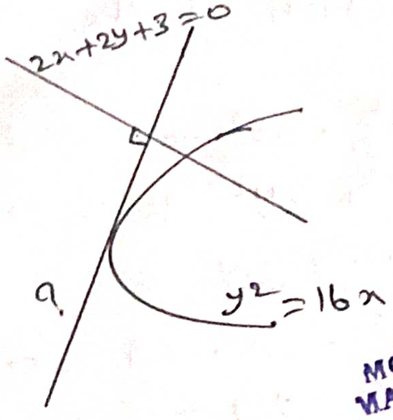
$$|2-5| \le |z+3+4i| \le 2+5$$

$$|-3| \le |z+3+4i| \le 7$$

$$3 \le |z+3+4i| \le 7.$$

Hence proved.

34)



$2x+2y+3=0$

Slope $= -\frac{a}{b} = \frac{-2}{2} = -1$

Slope of its $\perp^r = 1$

$y^2 = 16x \Rightarrow y^2 = 4ax$

$4a = 16 \Rightarrow a = \frac{16}{4} \Rightarrow \boxed{a=4}$

Equation of tangent is

$y = mx + a/m$

$y = 1x + \frac{4}{1}$

$x - y + 4 = 0.$

35) Given $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

$\text{adj} A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$|A| = 24 - 20$

$|A| = 4$

$A(\text{adj}A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$= \begin{bmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ — ①

$(\text{adj}A)A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \longrightarrow$ ②

$|A| I_2 = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ — ③

From ①, ② and ③.

It is prove that

$A(\text{adj}A) = (\text{adj}A)A = |A| I_2$

is verified.

36) Let x represent the number of question with correct answer and y represent the number of questions with wrong answers.

By the given data,

$x+y=100$ and —(1)

$1.x - \frac{1}{4}y = 80$

$4x-y=320$ —(2)

From (1) and (2)

$$\Delta = \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = -1-4 = -5$$

$$\Delta_1 = \begin{vmatrix} 100 & 1 \\ 320 & -1 \end{vmatrix} = -100-320 = -420$$

$$\Delta_2 = \begin{vmatrix} 1 & 100 \\ 4 & 320 \end{vmatrix} = 320-400 = -80$$

$$\therefore x = \frac{\Delta_1}{\Delta} = -\frac{420}{-5} = 84$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-80}{-5} = 16.$$

Hence, the number of questions with correct answer is 84.

37)

The discriminant

$$\Delta = (4p)^2 - 4(4)(p+2)$$
$$= 16(p^2-p-2)$$
$$= 16(p+1)(p-2)$$

$\Delta < 0$ if $-1 < p < 2$

$\Delta = 0$ if $p=-1$ or $p=2$

$\Delta > 0$ if $-\infty < p < -1$ or $2 < p < \infty$

Thus the given polynomial has imaginary roots if $-1<p<2$

equal real roots if $p=-1$, or $p=2$

distinct real roots if $-\infty<p<-1$ (or) $2<p<\infty$

38)

Let A be the point $(2,0,-1)$

Then the position vector of A is $\vec{OA} = 2\hat{i}-\hat{k}$

and therefore,

$$\vec{r} = \vec{AO} = -2\hat{i}+\hat{k}$$

$$\vec{F} = 2\hat{i}+\hat{j}-\hat{k}$$

The torque is

$$\vec{t} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= -\hat{i} - 2\hat{k}$$

The magnitude of the torque $= |-\hat{i}-2\hat{k}| = \sqrt{5}$

and the direction cosines of the torque are

$$\frac{-1}{\sqrt5}, 0, \frac{-2}{\sqrt5}.$$

39)

$$\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right]$$

$$\left(\because \frac{5\pi}{4} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$$

$$\sin^{-1}\left[\sin\left(\pi+\frac{\pi}{4}\right)\right]$$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{4}\right)\right]$$

$$= -\frac{\pi}{4} \in \left[-\pi/2, \pi/2\right]$$

40)

De Moivere's theorem

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$(\sin\theta + i\cos\theta)^n = \left[\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right]^n$

$= \cos n\left(\frac{\pi}{2} - \theta\right) + i\sin n\left(\frac{\pi}{2} - \theta\right)$

Given:

$\left(\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}\right)^{12} \Rightarrow n = 12$

$\left[\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right]^{12}$

$\Rightarrow \left[\cos\left(\frac{4\pi - 2\pi}{8}\right) + i\sin\left(\frac{4\pi - 2\pi}{8}\right)\right]^{12}$

$\Rightarrow \left[\cos\left(\frac{2\pi}{8}\right) + i\sin\left(\frac{2\pi}{8}\right)\right]^{12}$

$\Rightarrow \cos 12\left(\frac{\pi}{4}\right) + i\sin 12\left(\frac{\pi}{4}\right)$

$\Rightarrow \cos 3\pi + i\sin 3\pi$

$\Rightarrow -1 + i(0)$

$(\because \sin 3\pi = 0$

$\cos 3\pi =$

$\therefore \left(\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}\right)^{12} = -1$

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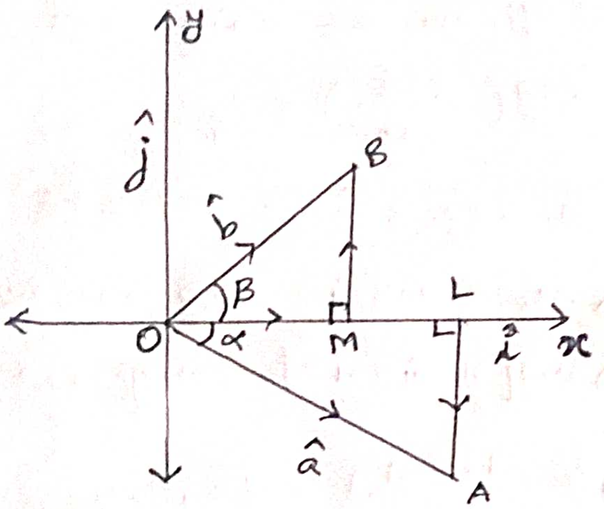
PART - IV

41) (a)

Let $\hat{a} = \vec{OA}$ and $\hat{b} = \vec{OB}$ be the unit vectors and which make angles α and β respectively, with positive x- axis,

where A and B are as in the Figure.



Draw AL and BM perpendicular to the x - axis.

Then $|\vec{OL}| = |\vec{OA}| \cos\alpha = \cos\alpha$,

$|\vec{LA}| = |\vec{OA}| \sin\alpha = \sin\alpha$

$\hat{a} = \vec{OA} = \vec{OL} + \vec{LA} = \cos\alpha \hat{i} - \sin\alpha \hat{j}$ —— (1)

$\hat{b} = \cos\beta \hat{i} + \sin\beta \hat{j}$ —— (2)

The angle between $\hat{a}$ and $\hat{b}$ is $\alpha + \beta$.

$\hat{a} \cdot \hat{b} = |\hat{a}| |\hat{b}| \cos(\alpha+\beta) = \cos(\alpha+\beta)$ → (3)

on the other hand, from (1) & (2)

$\hat{a} \cdot \hat{b} = (\cos\alpha \hat{i} - \sin\alpha \hat{j}) \cdot (\cos\beta \hat{i} + \sin\beta \hat{j})$

$= \cos\alpha \cos\beta - \sin\alpha \sin\beta$

From (3) and (4) we get

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(b)

Let the prices per unit for the commodities A, B and C be ₹x, ₹y, and ₹z.

By given data,

$2x - 4y + 5z = 15000$

$3x + y - 2z = 1000$

$-x + 3y + z = 4000$

The Matrix form of the system of equations is

$$\begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

$AX = B$ where $A = \begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}$,

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$

$\boxed{X = A^{-1} B}$

$$|A| = \begin{vmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{vmatrix} = 68$$

$$\text{adj}A = \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$A^{-1} = \frac{1}{68} \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$$

$X = A^{-1} B$

$$= \frac{1}{68} \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix} \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$$

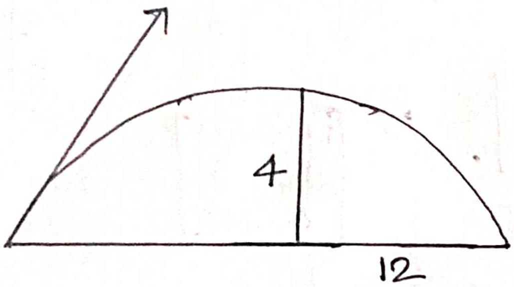
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{68}\begin{bmatrix} 136000 \\ 68000 \\ 204000 \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

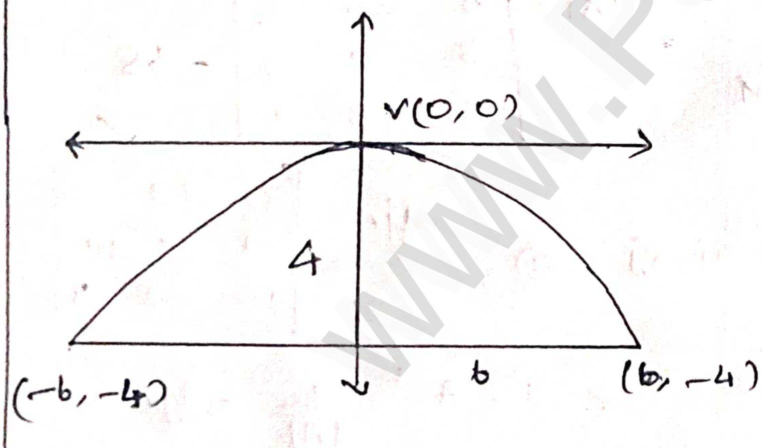
$\therefore x = 2000,\ y = 1000,\ z = 3000$

Hence the prices per unit of the commodities A, B and C are ₹ 2000, ₹ 1000 and ₹ 3000 respectively.

42) (a)

By taking the vertex at the origin, The parabola is open down ward.





Its equation is $x^2 = -4ay$ —(1)

It passes through $(6,-4)$

$\therefore 36 = -4a(-4) \Rightarrow 4a = \frac{36}{4} = 9$

(1) $\Rightarrow x^2 = -9y$

To find the slope at $(-6,-4)$

Differentiating with respect to 'x'

$2x = -9\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{2x}{9}$

At $(-6,-4) \Rightarrow \frac{dy}{dx} = -2\left(\frac{-6}{9}\right)$

$\frac{dy}{dx} = \frac{4}{3}$

$\therefore \tan\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore$ The angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$.

(b) Given $z = x+iy$ and

$\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$

$\Rightarrow \arg(z-i) - \arg(z+2) = \frac{\pi}{4}$

$\Rightarrow \arg(x+iy-i) - \arg(x+iy+2) = \frac{\pi}{4}$

$\Rightarrow \arg[x+i(y-1)] - \arg[(x+2)+iy] = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

$\left[\because \arg(a+ib) = \tan^{-1}\left(\frac{b}{a}\right)\right]$

$\Rightarrow \tan^{-1}\left[\frac{\frac{y-1}{x} - \frac{y}{x+2}}{1+\left(\frac{y-1}{x}\right)\left(\frac{y}{x+2}\right)}\right] = \frac{\pi}{4}$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$\Rightarrow \left[\frac{\frac{(x+2)(y-1) - xy}{\cancel{x(x+2)}}}{\frac{x(x+2)+y(y-1)}{\cancel{x(x+2)}}}\right] = \tan\frac{\pi}{4}$

$\left(\because \tan\frac{\pi}{4} = 1\right)$

$x^2+y^2+3x-3y+2=0$

Hence Proved

43) (a)

Let $\cot^{-1}(1) = x$

$\cot x = 1$

$\tan x = 1$

$\tan x = \tan\frac{\pi}{4}$

$x = \frac{\pi}{4} \quad (\because \frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2}))$

Let $\sin^{-1}(-\frac{\sqrt{3}}{2}) = y$

$\sin y = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3}$

$y = -\frac{\pi}{3}$

$(\because \sin(-\theta) = -\sin\theta)$

Let $\sec^{-1}(-\sqrt{2}) = z$

$\sec z = -\sqrt{2}$

$\cos z = -\frac{1}{\sqrt{2}}$

$\cos z = -\cos\frac{\pi}{4}$

$\cos z = \cos(\pi - \frac{\pi}{4})$

$\cos z = \cos\frac{3\pi}{4}$

$z = \frac{3\pi}{4}$

$\therefore \cot^{-1}(1) + \sin^{-1}(-\frac{\sqrt{3}}{2}) - \sec^{-1}(-\sqrt{2}) = \frac{\pi}{4} - \frac{\pi}{3} - \frac{3\pi}{4}$

$= -\frac{5\pi}{6}$

$\therefore \cot^{-1}(1) + \sin^{-1}(-\frac{\sqrt{3}}{2}) - \sec^{-1}(-\sqrt{2}) = -\frac{5\pi}{6}$

(b) $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$

$a = 1, \; b = -3, \; g = 135$

Zeros: $1+2i, \; 1-2i, \; \sqrt{3}, -\sqrt{3}, \; \alpha, \beta$

$\Sigma_1 = 1+\cancel{2i} + 1 - \cancel{2i} + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} + \alpha + \beta = -\frac{b}{a}$

$\Rightarrow 2 + \alpha + \beta = \frac{3}{1}$

$\alpha + \beta = 3 - 2$

$\boxed{\alpha + \beta = 1} \Rightarrow \beta = 1 - \alpha \quad \text{--- (1)}$

$\Sigma_6 = (1+2i)(1-2i)(\sqrt{3})(-\sqrt{3})(\alpha)(\beta) = \frac{g}{a}$

$\Rightarrow (1^2 - (2i)^2)(-\sqrt{3})^2 \alpha\beta = \frac{135}{1}$

$\Rightarrow (1+4)(-3)\alpha\beta = 135$

$\alpha\beta = -9 \longrightarrow (2)$

Sub eqn (1) in eqn (2)

$\alpha(1-\alpha) = -9$

$\alpha - \alpha^2 + 9 = 0$

$\alpha^2 - \alpha - 9 = 0$

$a = 1, \; b = -1, \; c = -9$

$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{1 \pm \sqrt{1 + 36}}{2}$

$\alpha = \frac{1+\sqrt{37}}{2}$; $\alpha = \frac{1-\sqrt{37}}{2}$

(1) $\Rightarrow \beta = \frac{1-\sqrt{37}}{2}$; (1) $\Rightarrow \beta = \frac{1+\sqrt{37}}{2}$

Zeros are

$1+2i, \; 1-2i, \; \sqrt{3}, \; -\sqrt{3}, \; \frac{1+\sqrt{37}}{2}, \; \frac{1-\sqrt{37}}{2}$

(44) (a)

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

$\div x^2$

$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 10\left(x + \frac{1}{x}\right) + 26 = 0$ —— ①

Let $y = x + \frac{1}{x}$ —— ②

On squaring

$y^2 = \left(x + \frac{1}{x}\right)^2$

$y^2 = x^2 + \frac{1}{x^2} + 2x\left(\frac{1}{x}\right)$

$(\because (a+b)^2 = a^2 + 2ab + b^2)$

$y^2 - 2 = x^2 + \frac{1}{x^2}$ → ③

Sub. ②, ③ in ①

$y^2 - 2 - 10y + 26 = 0$

$y^2 - 10y + 24 = 0$

$(y-4)(y-6) = 0$

24: -4, -6; -10

$y - 4 = 0$	$y - 6 = 0$
$y = 4$	$y = 6$
$x + \frac{1}{x} = 4$	$x + \frac{1}{x} = 6$
$\frac{x^2+1}{x} = 4$	$\frac{x^2+1}{x} = 6$
$x^2 + 1 = 4x$	$x^2 + 1 = 6x$
$x^2 - 4x + 1 = 0$	$x^2 - 6x + 1 = 0$
$a = 1,\ b = -4,\ c = 1$	$a = 1,\ b = -6,\ c = 1$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = \frac{6 \pm \sqrt{36 - 4}}{2}$
$= \frac{4 \pm \sqrt{16 - 4}}{2}$	$= \frac{6 \pm \sqrt{32}}{2}$
$\frac{4 \pm 2\sqrt{3}}{2}$	$= \frac{6 \pm 4\sqrt{2}}{2}$
$x = 2 \pm \sqrt{3}$	$x = 3 \pm 2\sqrt{2}$

Roots are $2+\sqrt{3},\ 2-\sqrt{3},\ 3+2\sqrt{2},\ 3-2\sqrt{2}$.

(b) By definition, The domain of $y = \cos^{-1}x$ is $-1 \le x \le 1$ (or) $|x| \le 1$.

$-1 \le \frac{2 + \sin x}{3} \le 1$

$-3 \le 2 + \sin x \le 3$

$-3 - 2 \le \sin x \le 3 - 2$

$-5 \le \sin x \le 1$

Reduces to,

$-1 \le \sin x \le 1$

$-\sin^{-1}(1) \le x \le \sin^{-1}(1)$

$-\frac{\pi}{2} \le x \le \pi/2$ $(\because \sin^{-1}(1) = \frac{\pi}{2})$

Thus, the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(45) (a)

Given $2\cos\alpha = x + \frac{1}{x}$

$\Rightarrow 2\cos\alpha = \frac{x^2+1}{x}$

$x^2 + 1 = 2x\cos\alpha$

$x^2 - 2x\cos\alpha + 1 = 0$

$$\Rightarrow x = \frac{2\cos\alpha \pm \sqrt{(-2\cos\alpha)^2 - 4(1)(1)}}{2}$$

$$= \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - 4}}{2}$$

$$\left(\because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{2\cos\alpha \pm 2\sqrt{-\sin^2\alpha}}{2}$$

$$(\because \sin^2\alpha + \cos^2\alpha = 1$$
$$\cos^2\alpha - 1 = -\sin^2\alpha)$$

$$= \frac{2\cos\alpha \pm i2\sin\alpha}{2}$$

$$\therefore x = \cos\alpha \pm i\sin\alpha.$$

Also,

$$2\cos\beta = y + \frac{1}{y}$$

lly

$$y = \cos\beta \pm i\sin\beta$$

(i) $\frac{x}{y} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta}$

$$\frac{x}{y} = \cos(\alpha-\beta) + i\sin(\alpha-\beta) \quad \text{——} (1)$$

and $\frac{y}{x} = \frac{1}{\frac{x}{y}} = \cos(\alpha-\beta) - i\sin(\alpha-\beta) \quad \text{——} (2)$

$(1) + (2) \Rightarrow 2\cos(\alpha-\beta)$

(ii) $xy = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$

$$= \cos(\alpha+\beta) + i\sin(\alpha+\beta)$$

$$\frac{1}{xy} = \cos(\alpha+\beta) - i\sin(\alpha+\beta)$$

$$xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$$

(b) By definition.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & -1 & -4 \end{vmatrix}$$

$$= 4\hat{i} + 4\hat{j}.$$

$$\vec{c}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= 8\hat{i} - 2\hat{j} - 6\hat{k}$$

Then,

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 8 & -2 & -6 \end{vmatrix}$$

$$= -24\hat{i} + 24\hat{j} - 40\hat{k} \quad \text{——} (1)$$

On the other hand, we have

$$[\vec{a},\vec{b},\vec{d}]\vec{c} - [\vec{a},\vec{b},\vec{c}]\vec{d}$$

$$= 28(3\hat{j} - \hat{k}) - 12(2\hat{i} + 5\hat{j} + \hat{k})$$

$$= -24\hat{i} + 24\hat{j} - 40\hat{k} \quad \text{——} (2)$$

From (1) & (2)

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{b},\vec{d}]\vec{c} - [\vec{a},\vec{b},\vec{c}]\vec{d}$$

Hence verified

(46) (a)

Here the number of unknowns is 3.

The Matrix form is

$AX = B$.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & \lambda \\ 1 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 7 \\ \mu \\ 5 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 1 & \lambda & \mu \\ 1 & 3 & -5 & 5 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 5 \\ 1 & 1 & \lambda & \mu \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & -1 & \lambda-1 & \mu-7 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & \lambda-7 & \mu-9 \end{array}\right]$$

(i) If $\lambda = 7$ and $\mu \neq 9$.
then $\rho(A) = 2$ and $\rho[A|B] = 3$
So, $\rho(A) \neq \rho[A|B]$
The system is inconsistent and has no solution.

(ii) If $\lambda \neq 7$, $\mu \neq 9$
$\rho(A) = 3$ and $\rho[A|B] = 3$
$\rho(A) = \rho[A|B] =$ Number of unknowns.
The system is consistent and has a unique solution

(iii) If $\lambda = 7$ and $\mu = 9$,
Then $\rho(A) = 2$ and $\rho[A|B] = 2$
$\rho(A) = \rho[A|B] = 2 <$ Number of unknowns
The system is consistent and has infinite many solutions.

(b)

Let the general equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ — ①

It passes through points $(1,1)$, $(2,-1)$ and $(3,2)$

$2g + 2f + c = -2$ — ②

$4g - 2f + c = -5$ — ③

$6g + 4f + c = -13$ — ④

② − ③ ⇒ $-2g + 4f = 3$ — ⑤

④ − ③ ⇒ $2g + 6f = -8$ — ⑥

⑤ + ⑥ ⇒ $f = -\frac{1}{2}$

Sub $f = -\frac{1}{2}$ in ⑥

$g = -\frac{5}{2}$

Sub $f = -\frac{1}{2}$, $g = -\frac{5}{2}$ in (2)

$c = 4$

∴ $x^2 + y^2 + 2\left(-\frac{5}{2}\right)x + 2\left(-\frac{1}{2}\right)y + 4 = 0$

$\Rightarrow x^2 + y^2 - 5x - y + 4 = 0$

(47) (a)

Given plane is passing through the points

$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ and

$\vec{b} = 9\hat{i} + 3\hat{j} + 6\hat{k}$

Equation of the given plane is $2x + by + 6z = 9$.

Parametric form of vector equation of the plane passing through two points and parallel to a vector is

$\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t\vec{c}, \ s, t \in R$

$\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k} + s(7\hat{i} + \hat{j} + 5\hat{k}) + t(2\hat{i} + 6\hat{j} + 6\hat{k}), \ s, t \in R$

cartesian equation of the plane is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0.$$

$-24x - 32y + 40z + 72 = 0$

$\div 8 \Rightarrow$

$3x + 4y - 5z - 9 = 0.$

The parametric form of vector equation is

$\vec{r} \cdot (3\hat{i} + 4\hat{j} - 5\hat{k}) = 9.$

(b)

$4x^2 + y^2 + 24x - 2y + 21 = 0,$

$\cancel{4(x^2 + 6x + 9 - 9) + (y^2 - 2y + 1 - 1) + 21 = 0}$

$\underbrace{4x^2 + 24x} + y^2 - 2y + 21 = 0$

$4(x^2 + 6x) + 1(y^2 - 2y) + 21 = 0$

$4[(x+3)^2 - 3^2] + 1[(y-1)^2 - 1^2] + 21 = 0$

$4(x+3)^2 - 36 + (y-1)^2 - 1 + 21 = 0$

$4(x+3)^2 + (y-1)^2 = 16$

$\div 16 \Rightarrow$

$$\frac{4(x+3)^2}{16} + \frac{(y-1)^2}{16} = \frac{16}{16}$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1.$$

$$\frac{X^2}{4} + \frac{Y^2}{16} = 1.$$

where $X = x + 3$

$Y = y - 1$

Major axis is along y-axis

$a^2 = 16$; $b^2 = 4$

$a = 4$; $b = 2$

$c^2 = a^2 - b^2 \Rightarrow 16 - 4 = 12$

$c^2 = 12 \Rightarrow c = 2\sqrt{3}$

	X, Y	x, y
Centre:	$C(0, 0)$	$x = X - 3$, $y = Y + 1$; $C(-3, 1)$
vertices:	$A(0, 4)$; $A'(0, -4)$	$A(-3, 5)$; $A'(-3, -3)$
Foci:	$F_1(0, 2\sqrt{3})$; $F_2(0, -2\sqrt{3})$	$F_1(-3, 2\sqrt{3}+1)$; $F_2(-3, -2\sqrt{3}+1)$

Length of latus rectum

$= \frac{2b^2}{a} = \frac{2(4)}{4} = 2.$

Hence proved.

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