Class : 12 Register

COMMON QUARTERLY EXAMINATION-2024-25

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART - I

Answer all the questions: **20x1=20**

1. If A is a 3x3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T=

 (a) A (b) B (c) I_3 (d) B^T

2. The radius of the circle passing through the point (6,2) two of whose diameters are x+y=6 and x+2y=4 is

 (a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

3. If the direction cosines of a line are $^1/_c$, $^1/_c$, $^1/_c$ then

 (a) $c = \pm 3$ (b) $c = \pm\sqrt{3}$ (c) $c > 0$ (d) $0 < c < 1$

4. The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is

 (a) 1 + i (b) i (c) 1 (d) 0

5. If α, β and γ are the roots of x^3+px^2+qx+r, then $\sum {^1/_\alpha}$ is

 (a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

6. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

 (a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

7. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ ia

 (a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $^1/_3|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1

8. The locus of a point whose distance from (-2,0) is $^2/_3$ times its distance from the line $x = {^{-9}/_2}$ is

 (a) a parabola (b) a hyperpola (c) an ellipse (d) a circle

9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zerovectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to

 (a) 81 (b) 9. (c) 27 (d) 18

10. If $\sin^{-1}x + \cot^{-1}(^1/_2) = {^\pi/_2}$, then x is equal to

 (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

11. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =

 (a) 0 (b) sinθ (c) cosθ (d) 1

12. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)\, y - (\cos\theta)\, z = 0$, $(\cos\theta)\, x - y + z$, $(\sin\theta)\, x + y - z = 0$ has a non-trivial solution then is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

13. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(a) 2 (b) 4 (c) 1 (d) ∞

14. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

15. $\operatorname{Sin}^{-1}\left(\tan \frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$, then x is a root of the equatoin.

(a) $x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

16. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(a) -2 (b) -1 (c) 1 (d) 2

17. If A is an invertible matrix of order 2. Then $\det [A^{-1}] =$

(a) $\det [A]$ (b) $1 / \det [A]$ (c) 1 (d) 0

18. If $-i + 3$ is a roots of $x^2 - 6x + k = 0$, then the value of k is

(a) 5 (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 10

19. The sum of the focal distances from any point of the ellipse $9x^2 + 16y^2 = 144$ is

(a) 32 (b) 18 (c) 16 (d) 8

20. The value of $[\hat{i} - \hat{j}, \hat{j} - \hat{k}, \hat{k} - \hat{i}]$ is

(a) 0 (b) 1 (c) 2 (d) 3

PART - II

Answer any 7 Questions. Question Nunmber 30 is compulsory **7x2=14**

21. Find a polynomial equation of minimum degree with rational coefficients, havng $2 + \sqrt{3}i$ as a root.

22. Show that the equation $z^2 = \bar{z}$ has four solutions.

23. Find the equation of Ellipse: If focus $(\pm 3, 0)$ and $e = 1/2$.

24. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, then prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$.

25. If $\operatorname{adj} A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

26. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$ find c.

27. Find the value of : $2 \cos^{-1} (1/2) + \sin^{-1} (1/2)$.

28. Find the angle between the planes $\vec{r} \cdot (\hat{i} + \hat{j} - 2\hat{k}) = 3$ and $x - 2y + z = 2$.

29. Find the value of $\sum_{n=1}^{10} i^{n+50}$

30. Find the rank of the matrix. $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

PART - III

Answer any 7 Questions. Question Number 40 is compulsory. **7x3=21**

31. Solve the cubic equation: $2x^3 - 9x^2 + 10x = 3$.

32. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

33. If $|z| = 2$, show that $3 \leq |z + 3 + 4i| \leq 7$.

34. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x+2y+3=0$.

35. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_2$.

36. In a competitive examination, one mark is awarded for every correct answer while $^1/_4$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)

37. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

38. Find the magnitude and the direction cosines of the torque about the point (2,0,-1) of a force $2\hat{i} + \hat{j} - \hat{k}$ whose line of action passes through the origin.

39. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

40. Simplify : $\left(\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}\right)^{12}$

PART - IV

Answer all the questions **7x5=35**

41. a) By vector method, prove that $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

(OR)

b) The prices of three commodities A,B and C are Rs x, y and z per unit respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, PQ and R earn ₹ 15,000, ₹ 1,000 and ₹ 4,000 respectively. Find the prices per unit of A,B and C. (Use matrix inversion method to solve the problem.)

42. a) On lighting a rocket cracker is get projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z - i}{z + 2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

43. a) Find the value of $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

(OR)

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

44. a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(OR)

b) Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

45. a) If $2\cos\alpha = x + {}^1/_x$ and $2\cos\beta = y + {}^1/_y$, show that

i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$ ii) $xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$

(OR)

b) If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

46. a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinte number of solutions.

(OR)

b) Find the equation of the circle passing through the points (1,1), (2,-1) and (3,2).

47. a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x+6y+6z=9$.

(OR)

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

12th Std

COMMON QUARTERLY EXAMINATION -2024-25
MATHEMATICS

Part – I

1. c) I_3
2. b) $2\sqrt{5}$
3. b) $c = \pm\sqrt{3}$.
4. a) $1+i$
5. a) $-2/π$
6. c) $2/\sqrt{3}$
7. a) $|\vec{a}||\vec{b}||\vec{c}|$
8. c) an ellipse
9. a) 81.
10. b) $1/\sqrt{5}$
11. d) 1
12. d) $\pi/4$.
13. a) 2
14. b) imaginary axis.
15. b) $x^2 - x - 12 = 0$.
16. b) -1.
17. b) $1/\det[A]$.
18. d) 10.
19. d) 8
20. a) 0.

Part – II

21. Solution:-

Since $2+\sqrt{3}i$ is a root,
$2-\sqrt{3}i$ is also a root,

Sum = 4 / Product = 7.

$x^2 - (\text{Sum})x + \text{Product} = 0$.

Hence,

$x^2 - 4x + 7 = 0$ is required polynomial equation. Ans.

22. Solution:-

we have, $z^2 = \bar{z}$

$\Rightarrow |z|^2 = |z|$

$\Rightarrow |z|(|z|-1) = 0$.

$\Rightarrow |z| = 0$, (or) $|z| = 1$.

$|z| = 0 \Rightarrow z = 0$ is a solution,

$|z| = 1$

$z\bar{z} = 1$

$\Rightarrow \bar{z} = \frac{1}{z}$.

Given $z^2 = \bar{z}$

$\Rightarrow z^2 = \frac{1}{z}$.

$\Rightarrow z^3 = 1$.

Hence, It has 3 non-zero solutions including zero soln. there are four solutions. Ans.

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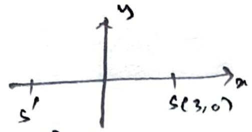
23. Solution:-



From the given data,

the major axis is along x-axis,
& the centre is (0,0).

The equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

$SS' = 2ae = 6$.

$ae = 3$. $e = \frac{1}{2}$.

i.e) $\boxed{a=6}$

$a^2 = 36$. / $b^2 = a^2 - a^2e^2$

The equation of the ellipse $= 36 - 9 = 27$.

is $\frac{x^2}{36}+\frac{y^2}{27}=1$ Ans.

24. Solution:-

we get

$[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$

$= [\vec{a}\ \vec{b}\ \vec{c}]$ Ans.

25. Solution:-

$|adj A| = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 9$

so, we get.

$A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj(A)$.

$= \pm \frac{1}{\sqrt{9}} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$= \pm \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Ans.

26. Solution:-

The condition for the line $y = mx + c$ to be a tangent to the circle $x^2+y^2=a^2$ is $c^2 = a^2(1+m^2)$.

$a^2 = 9$ / $m = 4$

then $c = \pm\sqrt{9(1+16)}$

$c = \pm 3\sqrt{17}$ Ans.

27. Solution:-

$2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

$= 2 \times \frac{\pi}{3} + \frac{\pi}{6}$

$= \frac{2\pi}{3} + \frac{\pi}{6} = \frac{5\pi}{6}$ Ans.

28. Solution:-

The normal vectors of the two given planes are.

$\vec{n_1} = \hat{i} + \hat{j} - 2\hat{k}$ / $\vec{n_2} = \hat{i} - 2\hat{j} + \hat{k}$.

respectively.

$\theta = \cos^{-1}\left(\frac{|\vec{n_1}\cdot\vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}\right)$

$= \cos^{-1}\left(\frac{|1-2-2|}{|\sqrt{6}||\sqrt{6}|}\right)$

$= \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ Ans.

29. Solution:-

$$\sum_{n=1}^{10} i^{n+50} = \sum_{n=1}^{8} i^{n+50} + i^{59} + i^{60}$$

$$= 0 + (i^3) + 1 = 1 - i$$ Ans.

30. Solution:-

Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$.

Then A is a matrix of order 3×3.

So, $\rho(A) \le \min\{3,3\} = 3$,

The highest order of minors of A is 3.

$\therefore$ There is only one third order minor of A.

$|A| = 0$. So, $\rho(A) < 3$.

Next consider the second-order minors of A

$\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1 \ne 0$. So $\rho(A) = 2$ Ans.

Part – III

31. Solution:-

The sum of the coeff. of the poly. is 0,

Hence 1 is a root.

To find other roots we divide $2x^3 - 9x^2 + 10x - 3$ by $(x-1)$.

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & 0 & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

The other factor is $2x^2 - 7x + 3 = 0$.

$(2x-1)(x-3) = 0$.

Hence,

$x = \frac{1}{2}, 3$.

The roots are $1, \frac{1}{2}, 3$ Ans.

32. Solution:-

since coeff. of xy is 0,

$\boxed{p = 3}$, and the co.eff. of x^2 is equal to co.eff of y^2, $\boxed{q = 3}$.

The equation is $3x^2 + 3y^2 - 6x - 72 = 0$.

$x^2 + y^2 - 2x - 24 = 0$.

$\Rightarrow (x-1)^2 + (y+0)^2 = 5^2$.

centre is $(1, 0)$ & radius is $r = 5$ Ans.

33. Solution:-

$|z + 3 + 4i| \le |z| + |3 + 4i| = 2 + 5 = 7$.

$|z + 3 + 4i| \le 7 \longrightarrow$ (1).

$|z + 3 + 4i| \ge ||z| - |3 + 4i|| = |2 - 5| = 3$.

$|z + 3 + 4i| \ge 3 \longrightarrow$ (2).

From (1) & (2), we get,

$3 \le |z + 3 + 4i| \le 7$ Ans.

34. Solution:-

The tangent is $\perp$ to $2x + 2y + 3 = 0$.

i.e) $y = -x - \frac{3}{2}$.

$-\frac{1}{m} = -1$.

slope of the tangent $m = 1$.

Eqn. of any tangent $y = mx + \frac{a}{m}$.

$y = x + 4$.

$x - y + 4 = 0$ Ans.

35. Solution:-

$A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ adj $A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$.

$A(\text{adj}A) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I = |A|I.$

$(\text{adj}A)A = 4I = |A|I.$

Hence,

$A(\text{adj}A) = (\text{adj}A)A = |A|I_2.$

36. Solution :-

Let x & y be the numbers of questions answered correctly & incorrectly.

then $x+y=100$ $x-\frac{1}{4}y=80$.

$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -\frac{1}{4} \end{vmatrix} = -\frac{5}{4}$ $\Delta_x = \begin{vmatrix} 100 & 1 \\ 80 & -\frac{1}{4} \end{vmatrix}$

$= -25-80 = -105$.

$x = \frac{-105}{-5/4} = 84.$

Hence,

The student has answered 84 Questions correctly. Ans.

37. Solution :-

The roots of the eqn $4x^2+4px+p+2=0$.

The discriminant $\Delta = (4p)^2 - 4(4)(p+2)$

$= 16(p^2-p-2)$

So, we get, $= 16(p+1)(p-2)$.

$\Delta < 0$ if $-1 < p < 2$ Polynomial Imaginary roots.

$\Delta = 0$ if $p=-1$ (or) $p=2$ equal real roots.

$\Delta > 0$ if $-\infty < p < -1$ (or) $2 < p < \infty$ distinct real roots.

38. Solution:-

Let A be the point $(2,0,-1)$.

$\therefore \vec{OA} = 2\hat{i} - \hat{k}$.

$\vec{r} = \vec{AO} = -2\hat{i} + \hat{k}$.

Force $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$.

The torque is $\vec{t} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix}$

Hence, $= -\hat{i} - 2\hat{k}$.

magnitude of torque $= |-\hat{i}-2\hat{k}| = \sqrt{5}$

direction cosines of the torque are.

$-1/\sqrt{5}, 0, -2/\sqrt{5}$ Ans.

39. Solution:-

$\sin^{-1}(\sin(5\pi/4))$

$= \sin^{-1}(\sin(\pi + \pi/4))$

$\therefore 5\pi/4 \notin [-\pi/2, \pi/2]$

$= \sin^{-1}(\sin(-\pi/4))$.

$= -\pi/4$. $\therefore -\pi/4 \in [-\pi/2, \pi/2]$.

Ans.

40. Solution:-

we have $\sin\pi/4 + i\cos\pi/4$

$= i[\cos\pi/4 - i\sin\pi/4]$

$[\sin\pi/4 + i\cos\pi/4]^{12} = i^{12}[\cos\frac{\pi}{4} - i\sin\pi/4]^{12}$.

$= 1[\cos\frac{12\pi}{4} + i\sin 12\pi/4]$

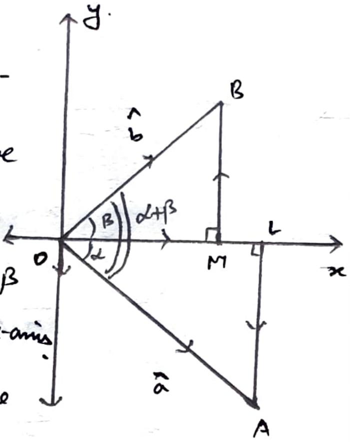
$= \cos 3\pi - i\sin 3\pi$

$= -1 - i(0)$

$= -1$ Ans.

Part – IV

41.a) Solution:-



Let $\hat{a}=\vec{OA}$, $\hat{b}=\vec{OB}$ be the unit vectors and which make angles α & β respectively, with +'ve x-axis where A & B are as in the figure. Draw AL & BM ⊥r to the x-axis.

then $|\vec{OL}| = |\vec{OA}|\cos\alpha = \cos\alpha$.

$|\vec{LA}| = |\vec{OA}|\sin\alpha = \sin\alpha$.

$\vec{OL} = |\vec{OL}|\hat{i} = \cos\alpha\,\hat{i}$

$\vec{LA} = \sin\alpha(-\hat{j})$.

$\hat{a} = \vec{OA} = \vec{OL}+\vec{LA} = \cos\alpha\,\hat{i} - \sin\alpha\,\hat{j} \rightarrow (1)$.

$\hat{b} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$. $\rightarrow (2)$.

The angle b/w $\hat{a}$ and $\hat{b}$ is $\alpha+\beta$,

$\hat{a}\cdot\hat{b} = |\hat{a}||\hat{b}|\cos(\alpha+\beta) = \cos(\alpha+\beta) \rightarrow (3)$.

On the other hand, from (1) & (2),

$\hat{a}\cdot\hat{b} = (\cos\alpha\,\hat{i} - \sin\alpha\,\hat{j})(\cos\beta\,\hat{i}+\sin\beta\,\hat{j})$.

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta \rightarrow (4)$.

From (3) & (4),

we get, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

Hence Proved.

41.b) Solution:-

From the given data,

$2x-4y+5z = 15000$

$3x+y-2z = 1000$

$-x+3y+z = 4000$

$\Rightarrow \begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$.

$AX=B$.

$|A| = \begin{vmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{vmatrix} = 68$.

$\text{adj}\,A = \begin{bmatrix} 7 & -1 & 10 \\ 19 & 7 & -2 \\ 3 & 19 & 14 \end{bmatrix}^T = \begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$.

$A^{-1} = \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$.

$X = A^{-1}B$.

$= \frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15000 \\ 1000 \\ 4000 \end{bmatrix}$.

$= \frac{1000}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 15 \\ 1 \\ 4 \end{bmatrix}$.

$= \frac{1000}{68}\begin{bmatrix} 136 \\ 68 \\ 204 \end{bmatrix} = 1000\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

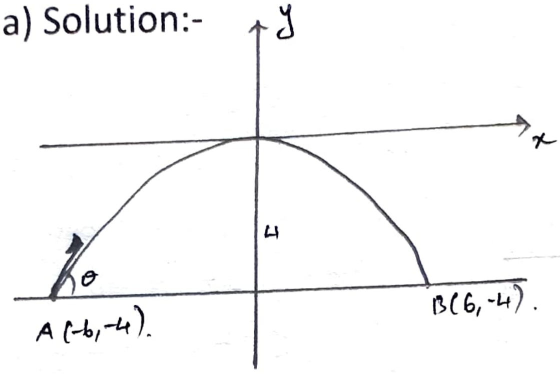
$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$.

$\therefore x=2000,\ y=1000,\ z=3000$.

Hence,

The Prices per unit of A, B & C are respectively ₹2000, ₹1000, ₹3000.

42.a) Solution:-



Eqn. of the parabola is $x^2=-4ay$.

let A(-6,-4) be the point of Projection.

$$36 = -4a(-4)$$

$$4a = 9.$$

∴ Eqn of the parabola is $x^2=-9y$.

To Find the angle of projection, find the slope of tangent at (-6,-4).

$$x^2=-9y$$

Diff. w.r.to x.

$$2x = -9\frac{dy}{dx}.$$

$$\frac{dy}{dx} = -\frac{2x}{9}.$$

$$\left(\frac{dy}{dx}\right)_{(-6,-4)} = -2\left(\frac{-6}{9}\right) = \frac{4}{3}.$$

$$\frac{dy}{dx} = \tan\theta$$

$$\tan\theta = 4/3$$

$$\theta = \tan^{-1}4/3.$$

∴ The angle of Projection is $\tan^{-1}(4/3)$ Ans.

42.b) Solution:-

$$\arg\left(\frac{z-i}{z+2}\right) = \arg(z-i) - \arg(z+2)$$

$$= \arg(x+i(y-1)) - \arg((x+2)+iy)$$

$$= \tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right).$$

$$= \tan^{-1}\left(\frac{\frac{y-1}{x} - \frac{y}{x+2}}{1+\left(\frac{y-1}{x}\right)\left(\frac{y}{x+2}\right)}\right).$$

$$= \tan^{-1}\left[\frac{(x+2)(y-1)-yx}{x(x+2)+(y-1)y}\right]$$

$$= \tan^{-1}\left[\frac{xy+x+2y-2-yx}{x^2+2x+y^2-y}\right]$$

$$= \tan^{-1}\left[\frac{-x+2y-2}{x^2+y^2+2x-y}\right].$$

$$\arg\left(\frac{z-i}{z+2}\right) = \pi/4.$$

$$\tan^{-1}\left(\frac{-x+2y-2}{x^2+y^2+2x-y}\right) = \pi/4.$$

$$\frac{-x+2y-2}{x^2+y^2+2x-y} = \tan\pi/4.$$

$$-x+2y-2 = x^2+y^2+2x-y$$

$$x^2+y^2+3x-3y+2 = 0$$

Hence,

$$x^2+y^2+3x-3y+2=0 \text{ Ans.}$$

43.a) Solution:-

$\cot^{-1}(1) = x$

$\cot x = 1.$

$\tan x = 1$

$\tan x = \tan \pi/4$

$x = \pi/4 \in [-\pi/2, \pi/2].$

$\boxed{\cot^{-1}(1) = \pi/4}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y$

$\sin y = -\sqrt{3}/2.$

$\sin y = -\sin \pi/3.$

$= \sin(-\pi/3)$

$\sin y = \sin(-\pi/3)$

$-\pi/3 \in [-\pi/2, \pi/2]$

$y = -\pi/3.$

$\boxed{\sin^{-1}(-\sqrt{3}/2) = -\pi/3}$

$\sec^{-1}(-\sqrt{2}) = z.$

$\sec z = -\sqrt{2}.$

$\frac{1}{\cos z} = -\sqrt{2}.$

$\cos z = -\frac{1}{\sqrt{2}}.$

$\cos z = -\cos(\pi/4).$

$\cos z = \cos(\pi - \pi/4)$

$\cos z = \cos 3\pi/4. \quad 3\pi/4 \in [0, \pi].$

$z = 3\pi/4.$

$\boxed{\sec^{-1}(-\sqrt{2}) = 3\pi/4}$

$\therefore \cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

$= \pi/4 - \pi/3 - (3\pi/4).$

$= -\pi/3 - 2\pi/4.$

$= \frac{-4\pi - 6\pi}{12} = -10\pi/12.$

$= -\frac{5\pi}{6}$ Ans.

43.b) Solution:-

$1+2i$, & $\sqrt{3}$ are the given zeros. since the co-eff. are real, $1-2i$ is also a zero of the polynomial.

The factors corresponding to the zeros $1+2i$ & $1-2i$ is $x^2 - s_1 x + s_2$, where,

$s_1 = 1+2i + 1-2i = 2.$

$s_2 = (1+2i)(1-2i) = 5.$

The factor is $x^2 - 2x + 5$.

since, the co.eff. are rational, $-\sqrt{3}$ is also a zero.

The factor corresponding to $\sqrt{3}$ and $-\sqrt{3}$ is $x^2 - s_1 x + s_2$ where.

$s_1 = \sqrt{3} + (-\sqrt{3}) = 0$

$s_2 = (\sqrt{3})(-\sqrt{3}) = -3.$

$\therefore x^2 - 3$ is another factor.

$\therefore$ combined factor is $(x^2-2x+5)(x^2-3)$

$\therefore x^4 - 2x^3 + 2x^2 + 6x - 15.$

$\therefore x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$

$\equiv (x^4 - 2x^3 + 2x^2 + 6x - 15)(x^2 + Px - 9)$

Equate x-term,

$-39 = -15P - 54.$

$P = -1$

$\therefore$ The other factor is $x^2 - x - 9$

The corresponding zeros are.

$\frac{1 \pm \sqrt{1+36}}{2}, \quad \frac{1 \pm \sqrt{37}}{2}.$

Hence, the zeros are,

$1 \pm 2i, \ \pm\sqrt{3}$, & $\frac{1 \pm \sqrt{37}}{2}$ Ans.

44.a) Solution:-

This eqn is Type I even degree reciprocal eqn. Hence, it can be rewritten as,

$$x^2\left[\left(x^2+\frac{1}{x^2}\right)-10\left(x+\frac{1}{x}\right)+26\right]=0.$$

since $x\neq 0$,

we get.

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+26=0.$$

Let $y=x+\frac{1}{x}$.

$$(y^2-2)-10(y)+26=0.$$

$$y^2-10y+24=0.$$

$$(y-6)(y-4)=0.$$

$y=6$ (or) $y=4$.

case(i) $y=6 \Rightarrow x+\frac{1}{x}=6$.

$x=3+2\sqrt{2},\ 3-2\sqrt{2}$.

case(ii) $y=4 \Rightarrow x+\frac{1}{x}=4$.

$x=2+\sqrt{3},\ 2-\sqrt{3}$.

Hence,

the roots are $3\pm2\sqrt{2},\ 2\pm\sqrt{3}$ Ans.

44.b) Solution:-

by definition,

the domain of $y=\cos^{-1}x$ is

$-1\le x\le 1$ (or) $|x|\le 1$.

This leads to,

$$-1\le\frac{2+\sin x}{3}\le 1.$$

$$-3\le 2+\sin x\le 3.$$

$$-5\le \sin x\le 1.$$

reduces to

$$-1\le \sin x\le 1.$$

which gives

$$-\sin^{-1}(1)\le x\le \sin^{-1}(1)$$

(or).

$$-\pi/2\le x\le \pi/2.$$

Hence,

the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ Ans.

45.a) Solution:-

$2\cos\alpha = x+\frac{1}{x} \Rightarrow \frac{x^2+1}{x} = 2\cos\alpha$.

$x^2 - 2\cos\alpha\, x + 1 = 0$.

$\therefore\ x = \cos\alpha \pm i\sin\alpha$.

Let $x = \cos\alpha + i\sin\alpha$.

$||^{ly}$ $y = \cos\beta + i\sin\beta$.

i) $\frac{x}{y} = \frac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = \cos(\alpha-\beta) + i\sin(\alpha-\beta)$.

$||^{ly}$ $\frac{y}{x} = \cos(\alpha-\beta) - i\sin(\alpha-\beta)$.

$$\boxed{\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)}$$

ii). $xy = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$.

$\frac{1}{xy} = \cos(\alpha+\beta) - i\sin(\alpha+\beta)$.

$$\boxed{xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)}$$

45.b) Solution:-

$\vec{a} = \hat{i} - \hat{j}$ $\quad \vec{b} = \hat{i} - \hat{j} - 4\hat{k}$

$\vec{c} = 3\hat{j} - \hat{k}$ $\quad \vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$.

$\vec{a}\times\vec{b} = 4\hat{i} + 4\hat{j}$

$\vec{c}\times\vec{d} = 8\hat{i} - 2\hat{j} - 6\hat{k}$.

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = -24\hat{i} + 24\hat{j} - 40\hat{k}$ $\rightarrow$ (1).

$[\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

$= 28(3\hat{j} - \hat{k}) - 12(2\hat{i} + 5\hat{j} + \hat{k})$.

$= -24\hat{i} + 24\hat{j} - 40\hat{k} \longrightarrow$ (2).

From (1) & (2), we get,

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$.

Hence Proved.

46.a) Solution:-

Here the no. of unknown is 3.

The matrix form of the system is

$$AX = B$$

where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & \lambda \\ 1 & 3 & -5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

$$B = \begin{bmatrix} 7 \\ \mu \\ 5 \end{bmatrix}.$$

Applying elementary row operations on the augmented matrix $[A/B]$, we get,

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 1 & \lambda & \mu \\ 1 & 3 & -5 & 5 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 1 & 3 & -5 & 5 \\ 1 & 1 & \lambda & \mu \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & -1 & \lambda-1 & \mu-7 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & \lambda-7 & \mu-9 \end{array}\right] \quad R_3 \to R_3 + R_2.$$

i] If $\lambda = 7$ & $\mu \neq 9$.

then $\rho(A) = 2$, $\rho([A/B]) = 3$,

$\rho(A) \neq \rho([A/B])$.

Hence, system is inconsistent, $\boxed{\text{No solution}}$

ii] If $\lambda \neq 7$ & μ is any real number.

$\rho(A) = 3$ & $\rho([A/B]) = 3$.

$\rho(A) = 3 = \rho([A/B]) =$ no. of unknowns.

They system is consistent, $\boxed{\text{Unique solution}}$

iii] $\lambda = 7$, $\mu = 9$, $\rho(A) = 2$, $\rho([A/B]) = 2$.

$\rho(A) = \rho([A/B]) = 2 < 3$ (no. of unknowns)

system is consistent, $\boxed{\text{Infinitely many solution}}$

46.b) Solution:-

Let the general eqn. of the circle be,

$$x^2 + y^2 + 2gx + 2fy + c = 0, \to (1)$$

It passes through points $(1,1)$ $(2,-1)$ & $(3,2)$.

$$\therefore \quad 2g + 2f + c = -2 \to (2)$$

$$4g - 2f + c = -5 \to (3).$$

$$6g + 4f + c = -13 \to (4).$$

$(2)-(3) \Rightarrow \quad -2g + 4f = 3. \to (5)$

$(4)-(3) \Rightarrow \quad 2g + 6f = -8 \to (6)$

$(5)+(6) \Rightarrow \quad \boxed{f = -\tfrac{1}{2}}$

Substituting, $f = -\frac{1}{2}$ in (6)

$\Rightarrow \boxed{g = -\tfrac{5}{2}}$.

Substituting, $f = -\frac{1}{2}$, $g = -\frac{5}{2}$ in (2).

$\therefore \boxed{c = 4}$

$\therefore$, The required eqn. of the circle is

$$x^2 + y^2 + 2\left(-\tfrac{5}{2}\right)x + 2\left(-\tfrac{1}{2}\right)y + 4 = 0.$$

$$\Rightarrow x^2 + y^2 - 5x - y + 4 = 0$$

47.a) Solution:-

$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = 9\hat{i} + 3\hat{j} + 6\hat{k}$.

let $\vec{v} = 2\hat{i} + 6\hat{j} + 6\hat{k}$.

Parametric Form :- (vector Equation)

$\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{v}, \ \forall s,t \in \mathbb{R}$.

$\vec{r} = (1-s)(2\hat{i} + 2\hat{j} + \hat{k}) + s(9\hat{i} + 3\hat{j} + 6\hat{k}) + t(2\hat{i} + 6\hat{j} + 6\hat{k})$.

The Cartesian equation of the plane is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0.$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0.$$

$(x-2)[6-30] - (y-2)[42-10] + (z-1)[42-2] = 0$.

$(x-2)(-24) - (y-2)(32) + (z-1)(40) = 0$.

$-24x + 48 - 32y + 64 + 40z - 40 = 0$.

$-24x - 32y + 40z + 72 = 0$.

$-8[3x + 4y - 5z - 9] = 0$.

$3x + 4y - 5z - 9 = 0$

47.b) Solution:-

$4x^2 + y^2 + 24x - 2y + 21 = 0$.

$4x^2 + 24x + y^2 - 2y + 21 = 0$.

$4(x^2 + 6x + 9 - 9) + (y^2 - 2y + 1 - 1) + 21 = 0$.

$4(x+3)^2 - 36 + (y-1)^2 - 1 + 21 = 0$.

$4(x+3)^2 + (y-1)^2 = 16$,

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1.$$

$\therefore$ centre is $(-3, 1)$,

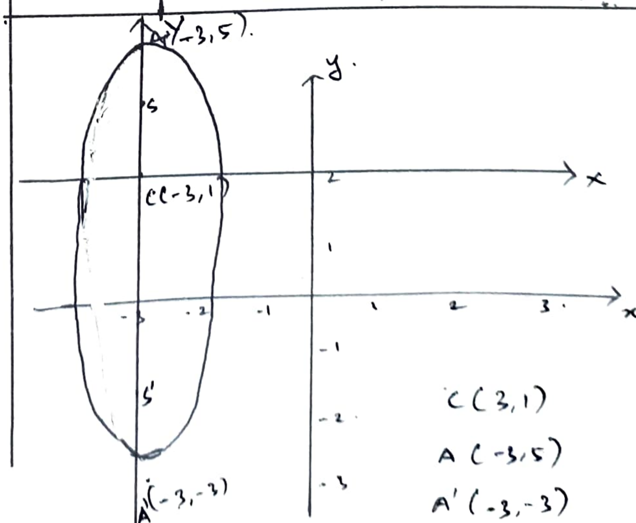
$\boxed{a = 4}$ / $\boxed{b = 2}$.

The major axis is Parallel to y-axis.

$c^2 = a^2 - b^2 = 4^2 - 2^2 = 16 - 4 = 12$.

$\boxed{c = \pm 2\sqrt{3}}$

centre	$(h, k) = (-3, 1)$.	
vertices.	$(h, k \pm a) = (-3, 1 \pm 4)$	$(-3, 5)$ & $(-3, -3)$
Foci.	$(h, k \pm c) = (-3, 1 \pm 2\sqrt{3})$	$(-3, 1+2\sqrt{3})$ & $(-3, 1-2\sqrt{3})$



C(3,1)

A(-3,5)

A'(-3,-3)

S(-3, 1+2√3)

S'(-3, 1-2√3).

...ALL THE BEST...

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