**COMMON QUARTERLY EXAMINATION - 2024** Reg. No.

# XII - MATHEMATICS

*Time Allowed : 3.00 Hrs.* *Maximum Marks: 90*

## Part - I

$20 \times 1 = 20$

**I. Choose the correct answer:**

1. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then A =

a) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ c) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ d) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$ and $A(\text{adj } A)=\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$, then k =

a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1

3. If $0 \le \theta \le \pi$ and the system of equations ,$x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then $\theta$ is

a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{4}$

4. The value of $\sum_{i=1}^{13}\left(i^n + i^{n-1}\right)$ is

a) 1 + i b) i c) 1 d) 0

5. If |z| = 1, then the value of $\frac{1+z}{1+\bar{z}}$ is

a) z b) $\bar{z}$ c) $\frac{1}{z}$ d) 1

6. If $\omega \neq 1$ is a cubic root of unity and $(1 + \omega)^7 = A + B\omega$, then (A, B) equals

a) (1, 0) b) (–1,1) c) (0,1) d) (1,1)

7. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4x - 2\sin^2x + 1$ is

a) 2 b) 4 c) 1 d) $\infty$

8. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\pi$

9. If $|x| \le 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

a) $\tan^{-1}x$ b) $\sin^{-1}x$ c) 0 d) $\pi$

10. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

11. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

a) 3 b) –1 c) 1 d) 9

12. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

13. If $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

a) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ b) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ c) 1 d) –1

14. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

15. The distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0 is

a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

16. A zero of $x^3 + 64$ is

a) 0 b) 4 c) 4i d) –4

17. Which of the following are correct according to Grammar's rules?

i) $\Delta = 0$ ii) $\Delta \neq 0$ ii) Only one solution iv) infinitely many solution

a) (i) and (iv) b) (ii) and (iii) c) (i) and (ii) d) (ii) and (iii)

18. Select the incorrect statement

a) $|z|^2=1 \Rightarrow \frac{1}{z}=\bar{z}$ b) $Re(z) \leq |z|$ c) $||z_1|-|z_2|| \geq |z_1+z_2|$ d) $|z^n| = |z|^n$

19. If y = mx + c tangent to the parabola $y^2 = 4ax$ then

a) $c=\frac{a}{m}$ b) $c=\frac{m}{a}$ c) $c^2 = a^2m^2 + m^2s$ d) m = c

20. If $e^{ix}$ is a periodic function then its period is

a) 0 b) $\pi$ c) $2\pi$ d) $4\pi s$

**Part - II**

**II. Answer any 7 questions. (Q.No.30 is compulsory)** **7 x 2 = 14**

21. If $adj(A)=\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find $A^{-1}$

22. Show that $(2+i\sqrt{3})^{10}-(2-i\sqrt{3})^{10}$ is purely imaginary

23. Find the polynomial equation of minimum degree with rational coefficients having $2-\sqrt{3}i$ as a root.

24. If |z| = 2, show that $3 \leq |z + 3 + 4i| \leq 7$.

25. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$

26. Identify the type of conic sections for each of the equations $y^2 + 4x + 3y + 4 = 0$

27. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$

28. For any vector $\vec{a}$, prove that $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$

29. If $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

30. Solve the following system of linear equations by matrix inversion method: .
$3x + 2y = 3$, $6x + 4y = 7$

## Part - III

**III. Answer any 7 questions. (Q.No.40 is compulsory)** **7 x 3 = 21**

31. $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$, Find the rank of the matrices by row reduction method and show that rank is 3.

32. Find the cube roots of unity.

33. Simplify $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

34. Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

35. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

36. Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

37. The volume of the parallelopiped whose coterminus edges are
$7\hat{i}+\lambda\hat{j}-3\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$, $-3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of $\lambda$

38. Find the exact number of real zeros and imaginary of the polynomial
$x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$

39. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$

40. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of *l, m, n*

## Part - IV

**IV. Answer all the questions.** **7 x 5 = 35**

41. a) Investigate for what values of and the system of linear equations
$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

***(OR)***

b) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta \sin\gamma = 0$, show that

i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and

ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

42. a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$

***(OR)***

b) If $z_1$, $z_2$ and $z_3$ are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$. Show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$

43. a) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

***(OR)***

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

44 a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

***(OR)***

b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

45. a) Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$.

***(OR)***

b) Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection.

46. a) Find the non parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

***(OR)***

b) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, Find the products AB and BA and hence solve the system of equations

$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$

47. a) Show that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

***(OR)***

b) If $a_1, a_2, a_3, \ldots\ldots\ldots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots.\tan^{-1}\left(\frac{d}{1+a_n a_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1 a_n}$$

# MATHEMATICS

## XII - Quarterly Exam - 2024 - Key

### Part-I

1. (c) $\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$
2. (d) 1
3. (d) $\pi/4$
4. (a) $1+i$
5. (a) $z$
6. (d) (1,1)
7. (a) 2
8. (b) $\pi/3$
9. (c) 0
10. (c) $\sqrt{10}$
11. (d) 9
12. (b) $1/3$
13. (a) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$
14. (d) $\pi/2$
15. (a) $\frac{\sqrt{7}}{2\sqrt{2}}$
16. (d) $-4$
17. MA.
18. (c) $||z_1|-|z_2|| \geq |z_1+z_2|$
19. (a) $c=\frac{a}{m}$
20. (c) $2\pi$

### Part-II

21. $|\text{adj}\,A| = 36$

$A^{-1} = \pm\frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

22. $\bar{z} = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$

$= -[(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}]$

$\bar{z} = -z \Rightarrow z$ is Imaginary

z ஒரு கற்பனையெண்

23. SR = 4

PR = 7

Eqn: $x^2-4x+7$

24. $||z_1|-|z_2|| \leq |z_1+z_2| \leq |z_1|+|z_2|$

$|3-10| \leq |z+6-8i| \leq 3+10$

$7 \leq |z+6-8i| \leq 13$

25. $p+q = -n/l$ ; $pq = n/l$

LHS: $\sqrt{p/q} + \sqrt{q/p} + \sqrt{n/l}$

$= \frac{p+q}{\sqrt{pq}} + \sqrt{n/l}$

$= -\frac{n/l}{\sqrt{n/l}} + \sqrt{n/l}$

$= 0 =$ RHS.

26. $A=0$, $E=3$, $c=1$, $F=4$, $D=4$

$B=0$ and either A or C is 0

$\therefore$ parabola.

27. $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

28. $\sum \vec{i}\times(\vec{a}\times\vec{i})$

$= 3\vec{a} - (a_1\vec{i}+a_2\vec{j}+a_3\vec{k})$

$= 3\vec{a} - \vec{a}$

$= 2\vec{a}$

29. $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0$

$\Rightarrow m = -3$

30. $|A| = 0$

$A^{-1}$ does not exist

$\therefore$ No solution.

தீர்வு இல்லை

32. $z = (1)^{1/3}$

$z = cis\frac{2k\pi}{3}$

$z = 1, \frac{-1\pm i\sqrt{3}}{2}$

33. $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

$= i^3 - (-i)^3$

$= -i - i = -2i$

34. $\frac{2\pi}{3} + \frac{3\pi}{4} = \frac{17\pi}{12}$

35. $p = q = 3$

(வட்டம்)

centre C (1,0)

Radius = 5

(ஆரம்)

36. $L(ae, y_1)$

$y_1 = \frac{b^2}{a}$

$LR = 2y_1 = \frac{2b^2}{a}$

37. $\begin{vmatrix} 7 & \lambda & -3 \\ 1 & 2 & -1 \\ -3 & 7 & 5 \end{vmatrix} = 0$

$\Rightarrow \lambda = -5$

38. $P(x)$ and $P(-x)$ has no sign change

$\therefore P(x)$ has no +ve & -ve roots

### Part-III

31. $A \sim \begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix}$

$\sim \begin{pmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{pmatrix}$

$\sim \begin{pmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$

$\rho(A) = 3$

**(39)** $\alpha^2+p\alpha+q=0$

$\alpha^2+p'\alpha+q'=0$

$\alpha=\dfrac{pq'-p'q}{q-q'}$

$\alpha'=\dfrac{q-q'}{p'-p}$

**(40)**

$l\vec{a}+m\vec{b}+n\vec{a}$

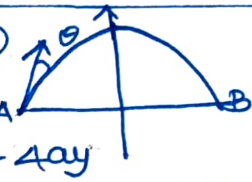
$=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

$\therefore l=0$

$m=10$

$n=-3$

**(44) (a)**




$x^2=-4ay$

$(-6)^2=-4a(4)\Rightarrow a=-9/4$

$\therefore x^2=-9y$

$2x=-9\,dy/dx$

$\therefore \tan\theta=\dfrac{2x}{-9}$ $\therefore \theta=\tan^{-1}\dfrac{4}{5}$ At $(-6,4)$

**Part - IV**

**(41) (a)** $[A,B]=\begin{bmatrix}1&2&1&7\\1&3&-5&5\\1&1&\lambda&\mu\end{bmatrix}$

$\sim\begin{bmatrix}1&2&1&7\\0&1&-6&-2\\0&0&\lambda-7&\mu-9\end{bmatrix}$

(i) No sol: $\lambda=7,\ \mu\neq 9$

(ii) unique sol: $\lambda\neq 7,\ \mu\in R$

(iii) Many sol: $\lambda=7,\ \mu=9$

$\tan^{-1}\left(\dfrac{2y}{x^2+y^2-1}\right)=\dfrac{\pi}{2}$

$\Rightarrow x^2+y^2=1$

**(44) (b)** $m=1,\ c=4$

$a^2=12,\ b^2=4$

$c^2=16$

$a^2m^2+b^2=12(1)+4=16$

$\therefore$ Tangent

Tangential point $(-3,1)$

**(42) (b)** $z_1=\dfrac{1}{\bar{z}_1}$

$z_2=\dfrac{4}{\bar{z}_2}$ $z_3=\dfrac{9}{\bar{z}_3}$

$|9z_1z_2+4z_1z_3+z_2z_3|$

$=|z_1z_2z_3|\left|\dfrac{9}{z_3}+\dfrac{4}{z_2}+\dfrac{1}{z_1}\right|$

$=|1\cdot2\cdot3|\ |\bar{z}_3+\bar{z}_2+\bar{z}_1|$

$=6\ (1)$

$=6.$

**45 (a)** Diagram

$\vec{a}=\cos\alpha\,\vec{i}+\sin\alpha\,\vec{j}$

$\vec{b}=\cos\beta\,\vec{i}+\sin\beta\,\vec{j}$

$\vec{b}\times\vec{a}=\vec{k}(\sin\alpha\cos\beta-\cos\alpha\sin\beta)$

$\vec{b}\times\vec{a}=\vec{k}\,[\sin(\alpha-\beta)]$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta.$

**(41) (b)** $a=cis\,\alpha$

$b=cis\,\beta$

$c=cis\,\gamma$

$a+b+c=0$

$cis\,\alpha+cis\,\beta+cis\,\gamma=0$

$\Rightarrow a^3+b^3+c^3=3abc$

$(cis\,\alpha)^3+(cis\,\beta)^3+(cis\,\gamma)^3$

$=3\,[(cis\,\alpha)(cis\,\beta)(cis\,\gamma)]$

$\Rightarrow$ Equating Real and Imaginary parts,

$\Rightarrow \cos3\alpha+\cos3\beta+\cos3\gamma$

$=3\cos(\alpha+\beta+\gamma)$

and

$\sin3\alpha+\sin3\beta+\sin3\gamma$

$=3\sin(\alpha+\beta+\gamma)$

**(43) (a)**

The roots are

$1+2i,\ 1-2i,\ \sqrt{3},\ -\sqrt{3}$

$\alpha,\ \beta.$

$S_1=\alpha+\beta+2=3$

$\alpha+\beta=1\to(1)$

$S_6=\alpha\beta\times(-15)=135$

$\alpha\beta=-9\to(2)$

From (1) and (2),

$\alpha^2-\alpha-9=0$

$\alpha=\dfrac{1\pm\sqrt{37}}{2}$ and $\beta$

**(45) (b)**

$\vec{a}=3\vec{i}+3\vec{j}+\vec{k}$

$\vec{b}=6\vec{i}+2\vec{j}+\vec{k}$

$\vec{u}=3\vec{i}-\vec{j}$

$\vec{v}=2\vec{i}-3\vec{k}$

$\begin{vmatrix}3&-1&0\\3&-1&0\\2&0&3\end{vmatrix}=0$

$\therefore$ Intersecting.

Intersecting point $=(6,2,1).$

**(42) (a)** $z=x+iy$

$\therefore \dfrac{z-1}{z+1}=\dfrac{(x-1)+iy}{(x+1)+iy}\times\dfrac{(x+1)-iy}{(x+1)-iy}$

$=\dfrac{(x^2+y^2-1)+i(2y)}{(x+1)^2+y^2}$

**(43) (b)**

Roots are

$1/3,\ 3,\ -2,\ -1/2$

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46(a)

$\vec{a} = 2\vec{i}+2\vec{j}+\vec{k}$

$\vec{b} = 9\vec{i}+3\vec{j}+6\vec{k}$

V.E: $\vec{v} = 2\vec{i}+6\vec{j}+6\vec{k}$

$\vec{r} = 2\vec{i}+2\vec{j}+\vec{k} + s(7\vec{i}+\vec{j}+4\vec{k}) + t(2\vec{i}+6\vec{j}+6\vec{k})$

C.E:

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 4 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$\Rightarrow 3x+4y-5z-9=0$

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46 (b) $AB = BA = 8I$

$B^{-1} = \frac{1}{8}A$

$\therefore \quad x = 3, y = -2, z = -1$

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47 (a)

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}(z)$

$= \tan^{-1}\left[\frac{\frac{x+y}{1-xy}+z}{1-\left(\frac{x+y}{1-xy}\right)z}\right]$

$= \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

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47 (b)

$\tan^{-1}\left[\frac{d}{1+a_1a_2}\right] = \tan^{-1}\left[\frac{a_2-a_1}{1+a_1a_2}\right] = \tan^{-1}a_2 - \tan^{-1}a_1$

$\tan^{-1}\left[\frac{d}{1+a_2a_3}\right] = \tan^{-1}\left[\frac{a_3-a_2}{1+a_2a_3}\right] = \tan^{-1}a_3 - \tan^{-1}a_2$

$|||^{ly}$ $\tan^{-1}\left[\frac{d}{1+a_na_{n-1}}\right] = \tan^{-1}\left[\frac{a_n-a_1}{1+a_1a_n}\right] = \tan^{-1}a_n - \tan^{-1}a_{n-1}$

$\tan\left[\sum_{n=1}^{n}\frac{d}{1+a_na_{n-1}}\right] = \tan\left[\tan^{-1}\left[\frac{a_n-a_1}{1+a_1a_n}\right]\right] = \frac{a_n-a_1}{1+a_1a_n}$