Tsl12M

Tenkasi District

Common Quarterly Examination - 2024

23-09-2024

Standard 12

MATHEMATICS

Time Allowed: 3.00 Hours

Maximum Marks: 90

PART - A

Answer all the questions. Choose the correct answer: **20×1=20**

1) If $A\begin{pmatrix}1 & -2\\ 1 & 4\end{pmatrix}=\begin{pmatrix}6 & 0\\ 0 & 6\end{pmatrix}$ then A =

a) $\begin{pmatrix}1 & -2\\ 1 & 4\end{pmatrix}$ b) $\begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix}$ c) $\begin{pmatrix}4 & 2\\ -1 & 1\end{pmatrix}$ d) $\begin{pmatrix}4 & -1\\ 2 & 1\end{pmatrix}$

2) If $\rho(A) = \rho(A/B)$ then the system AX = B of linear equations is

a) consistent and has a unique solution

b) consistent

c) consistent and has infinitely many solution

d) inconsistent

3) The augumented matrix of a system of linear equations is $\begin{pmatrix}1 & 2 & 7 & 3\\ 0 & 1 & 4 & 6\\ 0 & 0 & \lambda-7 & \mu+5\end{pmatrix}$

the system has infinitely many solutions if

a) $\lambda = 7, \mu \neq -5$ b) $\lambda = -7, \mu = 5$ c) $\lambda \neq 7, \mu \neq -5$ d) $\lambda = 7, \mu = -5$

4) If A is a 3×3 matrix such that $|3\ adjA| = 3$ then |A| is equal to

a) $\pm\frac{1}{9}$ b) $\pm\frac{1}{3}$ c) $\frac{1}{3}$ d) $\frac{1}{9}$

5) The area of the triangle formed by the complex number z, iz and z+iz in the Argand's diagram is

a) $\frac{1}{2}|z|^2$ b) $|z|^2$ c) $\frac{3}{2}|z|^2$ d) $2|z|^2$

6) The principal argument of $\frac{3}{-1+i}$ is

a) $\frac{-5\pi}{6}$ b) $\frac{-2\pi}{3}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{2}$

7) If $(1+i)(1+2i)(1+3i).....(1+ni) = x+iy$ then $2.5.10.....(1+n^2)$ is

a) 1 b) i c) x^2+y^2 d) $1+n^2$

8) If $x = \frac{-1+i\sqrt{3}}{2}$ then the value of x^2+x+1 is

a) 0 b) 1 c) -1 d) i

9) If α, β and γ are the roots of x^3+px^2+qx+r then $\Sigma\frac{1}{\alpha}$ is

a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

10) A zero of x^3+64 is

a) 0 b) 4 c) 4i d) −4

11) The polynomial x^3-Kx^2+9x has three real zeros if and only if K satisfies

a) $|K| \le 6$ b) $K = 0$ c) $|K| > 6$ d) $|K| \ge 6$

12) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution then

a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $|\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| > \frac{1}{\sqrt{2}}$

13) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right) =$

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

14) If $\sin^{-1}\frac{x}{5}+\text{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$ then the value of x is

a) 4 b) 5 c) 2 d) 3

15) The radius of the circle $3x^2+by^2+4bx-6by+b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

16) If $x+y = k$ is a normal to the parabola $y^2 = 12x$ then the value of k is

a) 3 b) −1 c) 1 d) 9

17) The eccentricity of the ellipse $(x-3)^2+(y-4)^2 = \frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

18) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}\ \vec{b}\ \vec{c}] = 3$ then $\left\{[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]\right\}^2 =$

a) 81 b) 9 c) 27 d) 18

19) If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ then the value of $[\vec{a}\ \vec{b}\ \vec{c}] =$

a) $|\vec{a}|\ |\vec{b}|\ |\vec{c}|$ b) $\frac{1}{3}|\vec{a}|\ |\vec{b}|\ |\vec{c}|$ c) 1 d) −1

20) If the planes $\vec{r}.(2\hat{i}-\lambda\hat{j}+\hat{k}) = 3$ and $\vec{r}.(4\hat{i}+\hat{j}-\mu\hat{k}) = 5$ are parallel then the value of λ and μ are

a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, -2$ d) $\frac{1}{2}, 2$

PART - B

Answer any 7 questions. Qn. No. 30 is compulsory: **7×2=14**

21) Find the rank of the matrix $\begin{pmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ which is in row-e chelon form.

22) Solve the system of linear equations.
$2x-y = 8$, $3x+2y = -2$ by matrix inversion method.

23) Show that $|2z+2-4i| = 2$ represent a circle and find its centre and radius.

24) Find a polynomial equation of minimum degree with rational coefficients having $2i+3$ as a root.

25) Find the exact number of real zeros and imaginary of the polynomial $x^9+9x^7+7x^5+5x^3+3x$.

26) Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

27) Determine whether the point $(-4, -3)$ lie outside, on or inside the circle.
$x^2+y^2-5x+2y-5 = 0$

28) Find the equation of the hyperbola with vertices $(0, \pm 4)$ and foci $(0, \pm 6)$.

29) Find the length of the perpendicular from the point $(1, -2, 3)$ to the plane $x-y+z = 5$.

30) For any vector $\bar{a}$ prove that $\hat{i}\times(\bar{a}\times\hat{i})+\hat{j}\times(\bar{a}\times\hat{j})+\hat{k}\times(\bar{a}\times\hat{k}) = 2\bar{a}$.

PART - C

Answer any 7 questions. Qn.No. 40 is compulsory: **7×3=21**

31) Verify the property $(AT)^{-1} = (A^{-1})^T$ with $A = \begin{pmatrix} 2 & 9 \\ 1 & 7 \end{pmatrix}$.

32) If $z = 2-2i$ find the rotation of z by θ radians in the counts clockwise direction about the origin when $\theta = \frac{2\pi}{3}$.

33) Find the square root of $-6+8i$.

34) If α, β, γ and σ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8 = 0$ find a quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\sigma$ and $\alpha\beta\gamma\sigma$.

35) Find all the values of x such that $-3\pi \le x \le 3\pi$ and $\sin x = -1$.

36) Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$.

37) If $y = 4x+c$ is a tangent to the circle $x^2+y^2 = 9$ find c.

38) A particle acted upon by constant forces $2\hat{i}+5\hat{j}+6\hat{k}$ and $-\hat{i}-2\hat{j}-\hat{k}$ is displaced from the point (4, −3, −2) to the point (6, 1, −3). Find the total work done by the forces.

39) Find the angle between the lines $\vec{r} = (4\hat{i}-\hat{j})+t(\hat{i}+2\hat{j}-2\hat{k})$, $\vec{r} = (\hat{i}-2\hat{j}+4\hat{k})+s(-\hat{i}-2\hat{j}+2\hat{k})$.

40) P(x, y) be any point on $4x^2+9y^2 = 36$ with foci $F_1(5, 0)$ and $F_2(-\sqrt{5}, 0)$ then find PF_1+PF_2.

PART - D

Answer all the questions: **7×5=35**

41) a) If $F(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$ show that $(F(\alpha)]^{-1} = F(-\alpha)$.

(OR)

b) Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$ is $(at_1t_2, a(t_1+t_2))$.

42) a) Test the consistency of the following system of linear equations and if possible solve $x-y+2z = 2$, $2x+y+4z = 7$, $4x-y+z = 4$.

(OR)

b) Show that the lines $\vec{r} = (6\hat{i} + \hat{j} + 2\hat{k}) + s(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (3\hat{i} + 2\hat{j} - 2\hat{k}) + t(2\hat{i} + 4\hat{j} - 5\hat{k})$ are skew lines and hence find the shortest distance between them.

43) a) Find the vertex, focus, equation of directrix and length of latus rectum of the parabola $y^2-4y-8x+12 = 0$.

(OR)

b) If $z = x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ show that $x^2+y^2 = 1$.

44) a) Find all cube roots of $\sqrt{3}+i$.

(OR)

b) Solve the equation $6x^4-5x^3-38x^2-5x+6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

45) a) Solve the equation $x^3-9x^2+14x+24 = 0$ if it is given that two of its roots are in the ratio 3:2.

(OR)

b) Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$.

46) a) Solve: $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

(OR)

b) Find the parametrix form of vector equation and cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x+6y+6z = 9$.

47) a) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$.

(OR)

b) Assuming that water issuing from the end of horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond their vertical line will the water strike the ground?
