VILLUPURAM DIST.

Class : 12 Register Number

COMMON QUARTERLY EXAMINATION-2024-25

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

Part - I

Note : (i) Answer All the questions. 20×1=20

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2023} + y^{2024} + z^{2025} + \frac{1}{x^{2023}y^{2024}z^{2025}}$ is

   a) 0 b) 1 c) 2 d) 4

2. If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1}x$ is

   a) $-\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{5}$

3. If $\tan^{-1}2$ and $\tan^{-1}3$ are two angles of a triangle, then the third angle is

   a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$

4. The radius of the circle passing through the point (6,2) two of those diameter are $x + y = 6$ and $x + 2y = 4$ is

   a) 10 b) $2\sqrt{5}$ c) 6 d) 4

5. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

6. If the coordinates at one end of a diameter of the circle $3x^2 + 3y^2 - 9x + 6y + 5 = 0$ are (1,2), the coordinates of the other end are

   a) (-4, 2) b) (2, -4) c) (4, -2) d) (-2, 4)

7. If $[\vec{a}, \vec{b}, \vec{c}] = 1$, then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}} + \frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

   a) 1 b) -1 c) 2 d) 3

8. The coordinates of the point where the line $\vec{r} = (6\vec{i} - \vec{j} - 3\vec{k}) + t(-\vec{i} + 4\vec{k})$ meets the plane $\vec{r}.(\vec{i} + \vec{j} - \vec{k}) = 3$ are

   a) (2, 1, 0) b) (7, -1, -7) c) (1, 2, -6) d) (5, -1, 1)

9. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

   a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

10. If $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$ then $[\vec{a}, \vec{b}, \vec{c}]$ is

    a) 1 b) -1 c) 2 d) 3

11. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|\text{adj } B|}{|C|} =$

    a) 2 b) 8 c) 72 d) 16

12. If A is a non - singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

13. If $0 \leq \theta \leq \pi$ and the systems of equations $x + (\sin\theta) y - (\cos\theta) z = 0$, $(\cos\theta) x - y + z = 0$ $(\sin\theta) x + y - z = 0$ has a non trivial solution then θ is

a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{4}$

14. $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then the value of K is

a) $\frac{1}{19}$ b) 19 c) $\frac{-1}{19}$ d) -19

15. The solution of the equation $|Z| - z = 1 + 2i$ is

a) $\frac{3}{2} - 2i$ b) $\frac{-3}{2} + 2i$ c) $2 - \frac{3i}{2}$ d) $2 + \frac{3i}{2}$

16. If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$, then $|z|$ is

a) 0 b) 1 c) 2 d) 3

17. $\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6 =$

a) -2 b) -1 c) 1 d) 2

18. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (fog)(x)$, then the degree of h is

a) mn b) m + n c) m^n d) n^m

19. The number of real numbers in $[0, \frac{\pi}{2}]$ satisfying $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$ is

a) 2 b) 1 c) 4 d) ∞

20. The number of positive zeros of the polynomial $\Sigma_{r=0}^{n} {}^nC_r (-1)^r x^r$ is

a) 0 b) n c) <n d) r

PART - II

Note: (i) Answer any 7 questions. (ii)Question No: 30 is compulsory: **7x2=14**

21. Find the rank of the matrix $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

22. Show that $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real.

23. Find the modulus and principal argument ,when $z = -\sqrt{3} - i$.

24. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.

25. Find the principal value of $\sec^{-1}(-2)$.

26. Prove that: $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

27. Find the vertices, foci for the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$

28. Show that the points (2,3,4),(-1,4,5) and (8,1,2) are collinear.

29. Find the distance between the planes $x + 2y - 2z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$

30. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ prove that $A^{-1} = A^T$

PART - III

Note:(i) Answer any Seven Questions. (ii)Question No.40 is compulsory **7x3=21**

31. If $A \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ Verify that $(AB)^{-1} = B^{-1} A^{-1}$

32. Solve the system of linear equations by matrix inversion method, $5x + 2y = 3$, $3x + 2y = 5$

33. If z_1 z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, Find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

34. Solve : $x^3 - 3x^2 - 33x + 35 = 0$

35. Find the value of $\sin^{-1}[\sin 5]$

36. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

37. If the equation $3x^2 + (3 - p)xy + qy^2 - 2px = 8pq$ represents a circle, Find p and q. also determine the centre and radius of the circle.

38. Prove that the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$

39. Let $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i}$ and $\vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$. If $C_1 = 1$ and $C_2 = 2$, find C_3 such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar

40. Prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$

PART - IV

Answer all questions of the following: **7x5=35**

41. (a) Prove by vector method that Cos(A - B) = CosA cosB + SinA sinB

(OR)

(b) A rod of length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with -axis is an ellipse. Find the eccentricity.

42. (a) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4, 3, 2) to the plane $x + 2y + 3z = 2$.

(OR)

(b) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$.

43. (a) Find the centre, foci, and eccentricity of the hyperbola $9x^2 - y^2 - 36x - 6y + 18 = 0$

(OR)

(b) Draw the graph of $y = \sin x$ in $\left[-\pi/2, \pi/2\right]$ and $y = \sin^{-1}(x)$ in $[-1,1]$

44. (a) Solve the equations. $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

(OR)

(b) Solve : $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$

45. (a) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1} \text{ and } \frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$$

(OR)

(b) Solve the following system of linear equations, by Gaussian elimination method:

$4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$

46. (a) Investigate for what values of λ and μ the system of linear equations,

$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(OR)**

(b) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$

(OR)

47. (a) If $2 + i$ and $3 - \sqrt{2}$ are the roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ Find all roots.

(b) If $a_1, a_2, a_3 \ldots\ldots a_n$ is an arithmetic progression with common difference d, then prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1 + a_1a_n}$$