12 R

Time : 3.00 hrs.

Quarterly Examination - 2024

Reg. No.

Max. Marks : 90

MATHEMATICS

SECTION - A

Answer all the questions. Choose the correct answer 20 x 1 = 20

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I^2 - A =$ a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$
2. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively
   a) $_e(\Delta_2 / \Delta_1)$, $_e(\Delta_3 / \Delta_1)$ b) $\log(\Delta_1 / \Delta_3)$, $\log(\Delta_2 / \Delta_3)$ c) $\log(\Delta_2 / \Delta_1)$, $\log(\Delta_3 / \Delta_1)$ d) $_e(\Delta_1 / \Delta_3)$, $_e(\Delta_2 / \Delta_3)$
3. If $0 \le \theta \le \pi$ and the system of equations $x - (\sin\theta)y - \cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non - trivial solution then θ is a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{4}$
4. The value of $\sum_{m=1}^{13} (i^n + i^{n-1})$ is a) $1 + i$ b) i c) 1 d) 0
5. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is a) $\frac{1}{2}$ b) 1 c) 2 d) 3
6. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is a) $\frac{2\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{2}$
7. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
   a) mn b) $m - n$ c) m^n d) n^m
8. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is a) 2 b) 4 c) 1 d) ∞
9. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \operatorname{cosec}^{-1}\frac{13}{12}$ is equal to a) 2π b) π c) 0 d) $\tan^{-1}\frac{12}{65}$
10. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to
    a) $[-1, 1]$ b) $[\sqrt{2}, 2]$ c) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ d) $[-2, -\sqrt{2}]$
11. $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$
12. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$
13. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is a) 3 b) -1 c) 1 d) 9
14. The locus of a point whose distance from $(-2, 0)$ is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is
    a) a parabola b) a hyperbola c) an ellipse d) a circle
15. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to a) 2 b) -1 c) 1 d) 0
16. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the pane $x + 3y - \alpha z + \beta = 0$, then (α, β) is
    a) $(-5, 5)$ b) $(-6, 7)$ c) $(5, -5)$ d) $(6, -7)$
17. The value of $(1 + i)^4 + (1 + i)^4$ is................ a) 8 b) 4 c) -8 d) -4
18. $y^2 - 2x - 2y + 5 = 0$ is a............ a) circle b) parabola c) ellipse d) hyperbola
19. The area of the parallelogram having diagonals $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is....................
    a) 4 b) $2\sqrt{3}$ c) $4\sqrt{3}$ d) $5\sqrt{3}$
20. The angle between the vectors $3\hat{i} + 4\hat{j} + 5\hat{k}$ and the z-axis is........... a) 30^0 b) 60^0 c) 45^0 d) 90^0

SECTION - B

Answer any seven. Q.No.30 is compulsory 7 x 2 = 14

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.
22. Find the rank of the following matrices by minor method. $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$
23. Simplify the following. $i^{59} + \frac{1}{i^{59}}$
24. If $z = (2 + 3i)(1 - i)$, then find z^{-1}.
25. If $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k.

26. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

27. Obtain the equation of the circle for which (3, 4) and (2, –7) are the ends of a diameter.

28. For any vector $\vec{a}$, prove that $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+(\hat{k}\times\vec{a})\times\hat{k}=2\vec{a}$

29. Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+2\hat{j}-2\hat{k})$ and the plane $\vec{r}=(6\hat{i}+3\hat{j}+2\hat{k})=8$

30. Find the equation of the hyperbola whose vertices are (0, $\pm$7) and $e=\frac{4}{3}$

SECTION - C

Answer any seven. Q.No.40 is compulsory **7 x 3 = 21**

31. Solve the following systems of linear equations by Cramer's rule. $5x-2y+6=0$, $x+3y-7=0$

32. If $|z|=2$ show that $3\le|z+3+4i|\le7$

33. Represent the complex number $1+i\sqrt{3}$ in polar form.

34. Show that the polynomial $9x^9+2x^5-x^4-7x^2+2=0$ has atleast six imaginary roots.

35. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

36. Find the equation of the circle with centre (2, 3) and passing through the intersection of the lines $3x-2y-1=0$, and $4x+y-27=0$

37. The equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

38. A particle acted upon by constant force $2\hat{i}+5\hat{j}+6\hat{k}$ and $-\hat{i}-2\hat{j}-\hat{k}$ is displaced from the point (4, –3, –2) at the point (6, 1, –3). Find the total work done by the forces.

39. Find the shortest distance between the two given straight lines $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+t(-2\hat{i}+\hat{j}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=-\frac{z+2}{2}$

40. Solve by matrix inversion method $x+y=3$, $2x+3y=8$

SECTION - D

Answre all questions. **7 x 5 = 35**

41. a) If $A=\begin{bmatrix}-5&1&3\\7&1&-5\\1&-1&1\end{bmatrix}$ and $B=\begin{bmatrix}1&1&2\\3&2&1\\2&1&3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1$, $3x+2y+z=7$, $2x+y+3z=2$. **(OR)** b) Solve the following system of linear equation, by Gaussian elimination method : $4x+3y+6z=25$, $x+5y+7z=13$, $2x+9y+z=1$.

42. a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq0$ prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$ **(OR)**
b) If $z=x+iy$ is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right)=0$ show that locus of z is $2x^2+2y^2+x-2y=0$

43. a) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution. **(OR)**
b) Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy+yz+zx}\right]$

44. a) Find the equation of the circle passing through the points (1, 1), (2, –1) and (3, 2) **(OR)**
b) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

45. a) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following : $18x^2+12y^2-144x+48y+120=0$ **(OR)** b) Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

46. a) Find the parametric form of vector equation and Catesian equation of the place containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and $\vec{r}.(\hat{i}+2\hat{j}+\hat{k})=8$ perpendicular to plane **(OR)**
b) If $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$ find $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

47. a) Verify whether the given system of equation is consistent. If it is consistent $x+y+z=5$, $-x+y-z=5$, $2x-2y+2z=10$. **(OR)**
b) Find the shortest distance between the following pairs of lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

QUARTERLY EXAMINATION - 2024

MATHEMATICS

XII-STD

21/9/24

1. d) $2A^{-1}$
2. d) $e^{\Delta_1/\Delta_3}, e^{\Delta_2/\Delta_3}$
3. d) $\pi/4$
4. a) $1+i$
5. b) 1
6. d) $\pi/2$
7. a) mn
8. a) 2
9. c) 0
10. c) $[-2,-\sqrt{2}]\cup[\sqrt{2},2]$
11. d) $\dfrac{x}{\sqrt{1+x^2}}$
12. c) $\sqrt{10}$
13. d) 9
14. c) Ellipse
15. d) 0
16. b) $(-6,7)$
17. c) -8
18. b) parabola
19. d) $5\sqrt{3}$
20. c) 45°

21. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$AA^T = \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ \cos\theta\sin\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

III^{ly} $A^TA = I_2$ $AA^T = A^TA = I_2$ A is orthogonal.

22. $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -2 & 1 \end{bmatrix}$ $\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 \neq 0$ $\rho(A) = 2$

23. $i^{59} + \frac{1}{i^{59}} = (i^4)^{14} \cdot i^3 + \frac{1}{(i^4)^{14} \cdot i^3} = i^3 + \frac{1}{i^3} = -i + i = 0$

24. $z = (2+3i)(1-i) = 5+i$

$$z^{-1} = \frac{1}{z} = \frac{1}{5+i} \times \frac{5-i}{5-i} = \frac{5-i}{25+1} = \frac{5-i}{26} = \frac{5}{26} - i\frac{1}{26}$$

25. $x^2 + 2(k+2)x + 9k = 0$

$\Delta = b^2 - 4ac = 0$

$4(k+2)^2 - 4(9)k = 0$

$4(k+2)^2 - 36k = 0$

$k = 4$ or 1

26. Principal value $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\frac{\sqrt{3}}{2} = \cos x$

$\cos \pi/6 = \cos x$

$x = \pi/6$

27. $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$(x-3)(x-2) + (y-4)(y+7) = 0$

$x^2 + y^2 - 5x + 3y - 22 = 0$

28. $\hat{i}\times(\vec{a}\times\hat{i}) = (\hat{i}\cdot\hat{i})\vec{a} - (\hat{i}\cdot\vec{a})\hat{i}$

$= \vec{a} - \hat{i}(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$

$= \vec{a} - a_1\hat{i}$

$\hat{j}\times(\vec{a}\times\hat{j}) = (\hat{j}\cdot\hat{j})\vec{a} - (\hat{j}\cdot\vec{a})\hat{j}$

$= \vec{a} - a_2\hat{j}$

$\hat{k}\times(\vec{a}\times\hat{k}) = (\hat{k}\cdot\hat{k})\vec{a} - (\hat{k}\cdot\vec{a})\hat{k}$ 3

$= \vec{a} - a_3\hat{k}$

$= 3\vec{a} - (a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$

$= 3\vec{a} - \vec{a}$

$= 2\vec{a}$

29. $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$ $\quad \vec{n} = 6\hat{i}+3\hat{j}+2\hat{k}$

$\vec{b}\cdot\vec{n} = 8 \quad |\vec{b}| = 3 \quad |\vec{n}| = 7$

$\theta = \sin^{-1}\left(\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}\right)$

$= \sin^{-1}\left(\frac{8}{3\times 7}\right)$

$= \sin^{-1}\left(\frac{8}{21}\right)$

30. Vertices $(0,\pm a) = (0,\pm 7)$ $\quad e = \frac{4}{3}$.

$a = 7 \quad e = \frac{4}{3} \quad c = ae = \frac{28}{3}$.

$a^2 = 49 \quad c^2 = a^2+b^2$

$b^2 = c^2 - a^2 = \left(\frac{28}{3}\right)^2 - 49$

Equation of hyperbola $\quad = \frac{784}{9} - 49$

$= \frac{784-441}{9} = \frac{343}{9}$

$\frac{x^2}{49} + \frac{9y^2}{343} = 1$

$\pi/3$

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31. $5x - 2y + 6 = 0$ $x + 3y - 7 = 0$

$$\begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 7 \end{pmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$$

$$\text{adj } A = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$$

$$X = \frac{1}{17}\begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} -6 \\ 7 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} -18+14 \\ 6+35 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} -4 \\ 41 \end{bmatrix}$$

$(x, y) = -4/17, \ 41/17$

Cramer's rule

$$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$$

$$\Delta_x = \begin{vmatrix} -6 & -2 \\ 7 & 3 \end{vmatrix} = -18 + 14 = -4$$

$$\Delta_y = \begin{vmatrix} 5 & -6 \\ 1 & 7 \end{vmatrix} = 35 + 6 = 41$$

$$x = \frac{\Delta x}{\Delta} = \frac{-4}{17}$$

$$y = \frac{\Delta y}{\Delta} = \frac{41}{17}$$

32. $|z + 3 + 4i| \leq |z| + |3 + 4i|$

$|z| = 2$ $\quad = 2 + 5$

$= 7$

$|z + 3 + 4i| \leq 7 \longrightarrow (1)$

$|z + 3 + 4i| \geq ||z| - |3 + 4i||$

$= |2 - 5|$

$= 3$

$|z + 3 + 4i| \geq 3 \longrightarrow (2)$

$3 \leq |z + 3 + 4i| \leq 7$

33. $r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$\arg(z) = \pi/3$

$$1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 2\left[\cos(\pi/3 + 2k\pi) + i\sin(\pi/3 + 2k\pi)\right]$$

34. $P(x) = 9x^9 + 2x^5 - x^4 - 7x^2 + 2 = 0$

number of sign changes in $P(x) = 2$ so 2 positive roots cannot be more than 2.

$P(-x) = 9(-x)^9 + 2(-x)^5 - (-x)^4 - 7(-x)^2 + 2$

$= -9x^9 - 2x^5 - x^4 - 7x^2 + 2$

Number of sign changes in $P(-x) = 1$ So 1 negative root cannot be more than 1.

0 is not a root. Maximum no. of real roots $= 3$

No. of Imaginary roots $= 9 - 3 = 6$

36. $3x - 2y = 1$

$4x + y = 27$ $\Rightarrow$

$3x - 2y = 1$

$8x + 2y = 54$

$11x = 55$

$x = 5$

$3(5) - 2y = 1$

$-2y = 1 - 15$

$-2y = -14$

$\boxed{y = 7}$

point of intersection $(5, 7)$

Eqn of circle centre $(2, 3)$ $(x-2)^2 + (y-3)^2 = r^2$

It passes $(5, 7)$

$(5-2)^2 + (7-3)^2 = r^2$

$3^2 + 4^2 = r^2$

$9 + 16 = r^2$

$r^2 = 25$

$r = \pm 5$

$(x-2)^2 + (y-3)^2 = 25$

$x^2 + y^2 - 4x - 6y - 12 = 0.$

35. $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

$= \sin^{-1}\left[\sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right)\right]$

$= \sin^{-1}\left[\sin\left(\frac{6\pi}{9}\right)\right]$

$= \sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \sin^{-1}\sin\left(\pi - \frac{\pi}{3}\right) = \sin^{-1}\sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3}.$

37. $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$

$a^2=484 \quad b^2=64$

$c^2=a^2-b^2$

$=484-64$

$=420$

$c^2=420$

$c=20.5$ cm

38. $\vec{F}=\hat{i}+3\hat{j}+5\hat{k}$

$A(4,-3,-2) \quad B(6,1,-3)$

$d=\vec{AB}=\vec{OB}-\vec{OA}$

$=(6\hat{i}+\hat{j}-3\hat{k})-(4\hat{i}-3\hat{j}-2\hat{k})$

$=2\hat{i}+4\hat{j}-\hat{k}$

$W=\vec{F}\cdot\vec{d}=(\hat{i}+3\hat{j}+5\hat{k})\cdot(2\hat{i}+4\hat{j}-\hat{k})$

$=9$ units

39. Given $\vec{r}=\vec{a}+t\vec{b} \quad \vec{r}=\vec{c}+s\vec{d} \quad \vec{c}=3\hat{i}-2\hat{k}$

$\vec{a}=2\hat{i}+3\hat{j}+4\hat{k} \quad \vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$

$\vec{d}=2\hat{i}-\hat{j}+2\hat{k}$

$d=\frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|}$

$(\vec{c}-\vec{a})\times\vec{b}=12\hat{i}+14\hat{j}-5\hat{k}$

$d=\frac{|12\hat{i}+14\hat{j}-5\hat{k}|}{|-2\hat{i}+\hat{j}-2\hat{k}|}$

$=\frac{\sqrt{365}}{3}$

40. $x+y=3$

$2x+3y=8$

$\Delta=\begin{vmatrix}1&1\\2&3\end{vmatrix}=3-2=1 \quad \Delta_y=\begin{vmatrix}1&3\\2&8\end{vmatrix}=8-6=2$

$\Delta_x=\begin{vmatrix}3&1\\8&3\end{vmatrix}=9-8=1 \quad x=\frac{\Delta x}{\Delta}=\frac{1}{1}=1,\ y=\frac{\Delta y}{\Delta}=\frac{2}{1}=2$

SEC - D

41. a)

$$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$$BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4I_3$$

$AB = BA = 4I_3$

$$B^{-1} = \frac{1}{4}A = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \rightarrow (1)$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$BX = C$

$X = B^{-1}C$

$$X = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

b)

$$\left[\begin{array}{ccc|c} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{array}\right] \xleftrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 4R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & -1 & -13 & -25 \end{array}\right] \xrightarrow[R_3 \to R_3 \div (-1)]{R_2 \to R_2 \div (-1)} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 1 & 13 & 25 \end{array}\right]$$

$$\xrightarrow{R_3 \to 17R_3 - R_2} \begin{bmatrix} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 0 & 199 & 398 \end{bmatrix}$$

$$x + 3y + 7z = 13$$
$$17y + 22z = 27$$
$$199z = 398$$
$$z = 2 \qquad y = -1, \ x = 4$$

42.

a) $|z_1| = |z_2| = |z_3| = r \qquad z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = r^2$

$$z_1 = \frac{r^2}{\bar{z}_1} \qquad z_2 = \frac{r^2}{\bar{z}_2} \qquad z_3 = \frac{r^2}{\bar{z}_3}$$

$$z_1 + z_2 + z_3 = \frac{r^2}{\bar{z}_1} + \frac{r^2}{\bar{z}_2} + \frac{r^2}{\bar{z}_3}$$

$$= r^2\left[\frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}\right]$$

$$= r^2\left[\frac{\bar{z}_2\bar{z}_3 + \bar{z}_1\bar{z}_3 + \bar{z}_1\bar{z}_2}{\bar{z}_1\bar{z}_2\bar{z}_3}\right]$$

$$= |r^2|\left|\frac{\overline{z_2z_3 + z_1z_3 + z_1z_2}}{\overline{z_1z_2z_3}}\right|$$

$$= r^2\left|\frac{z_2z_3 + z_1z_3 + z_1z_2}{z_1z_2z_3}\right|$$

$$= r^2\frac{|z_2z_3 + z_1z_3 + z_1z_2|}{r^3}$$

$$|z_1 + z_2 + z_3| = \frac{|z_2z_3 + z_1z_3 + z_1z_2|}{r}$$

$$\left|\frac{z_2z_3 + z_1z_3 + z_1z_2}{z_1 + z_2 + z_3}\right| = r$$

b) $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = 0$

$z = x + iy$

$$\operatorname{Im}\left(\frac{2(x+iy)+1}{i(x+iy)+1}\right) = 0 \qquad \operatorname{Im}\left(\frac{2x+1+2iy}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}\right) = 0$$

$$\operatorname{Im}\left(\frac{-x(2x+1) + 2y(1-y)}{(1-y)^2 + x^2}\right) = 0$$

$$-2x^2 - x + 2y - 2y^2 = 0$$

$$2x^2 + x - 2y + 2y^2 = 0$$

43 a) $x = \frac{1}{3},\ x = 3$

$$\begin{array}{r|rrrrr}
\frac{1}{3} & 6 & -5 & -38 & -5 & 6 \\
 & 0 & 2 & -1 & -13 & -6 \\ \hline
3 & 6 & -3 & -39 & -18 & \boxed{0} \\
 & 0 & 18 & 45 & 18 & \\ \hline
 & 6 & 15 & 6 & \boxed{0} &
\end{array}$$

$6x^2+15x+6=0$

$2x^2+5x+2=0$

$(x+\frac{1}{2})(x+2)=0$

$x=-2, -\frac{1}{2}.$

b) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{x+y}{1-xy}\right)+\tan^{-1}z$

$$=\tan^{-1}\left[\frac{\frac{x+y}{1-xy}+z}{1-\frac{(x+y)}{(1-xy)}z}\right]$$

$$=\tan^{-1}\left[\frac{x+y+z(1-xy)}{(1-xy)-(x+y)z}\right]$$

$$=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-xz}\right]$$

44 a) General eqn of circle

$x^2+y^2+2gx+2fy+c=0$

$2g+2f+c=-2$

$4g-2f+c=-5$

$6g+4f+c=-13$

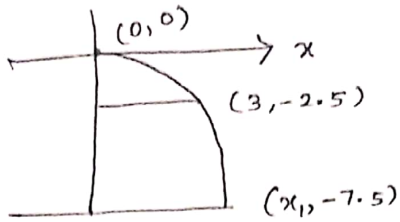
② - ③ $\quad -2g+4f=3$

④ - ③ $\quad 2g+6f=-8$

$g=-\frac{5}{2} \quad f=-\frac{1}{2} \quad c=4$

$x^2+y^2-5x-y+4=0$

b)



$x^2=-4ay$

$(3,-2.5)$ $\quad a=9/10$

$x^2=-\frac{36}{10}y \longrightarrow (2)$

$(x_1,-7.5)$

$x_1^2=-\frac{36}{10}\times -7.5$

$x_1^2=\frac{270}{10}$

$x_1^2=27$

$x_1=3\sqrt{3}\ m$

---

45. a) $18x^2+12y^2-144x+48y+120=0$

$18(x-4)^2+12(y+2)^2=216$

$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$

$(h,k)=(4,-2)$

$a^2=18 \quad b^2=12$

$a=3\sqrt{2} \quad c^2=a^2-b^2=18-12=6 \quad c=\sqrt{6}$

$ae=\sqrt{6}$

$e=\frac{\sqrt{6}}{3\sqrt{2}}=\frac{1}{\sqrt{3}}$

Major axis || el to y

foci $(h,k+c)=(4,-2+\sqrt{6})$

$(h,k-c)=(4,-2-\sqrt{6})$

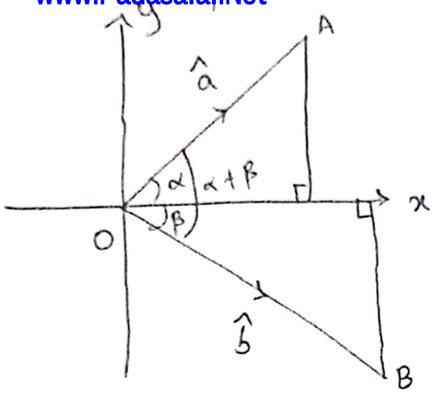
vertices $(h,k+a)=(4,-2+3\sqrt{2})$

$(h,k-a)=(4,-2-3\sqrt{2})$

Directrix $y=k\pm a/e=-2\pm 3\sqrt{2}\sqrt{3}$

$=-2\pm 3\sqrt{6}$.

b) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$



$\vec{OA} = \hat{a}$ $\quad \vec{OB} = b$

$\hat{a} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\hat{b} = \cos\beta\,\hat{i} - \sin\beta\,\hat{j}$

$\hat{b}\times\hat{a} = |\hat{b}||\hat{a}|\sin(\alpha+\beta)\,\hat{k}$

$= \sin(\alpha+\beta)\,\hat{k} \rightarrow (1)$

$$\hat{b}\times\hat{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & -\sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$$

$= \hat{k}(\sin\alpha\cos\beta + \cos\alpha\sin\beta) \rightarrow (2)$

From (1) & (2)

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$

46. a) $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ $\quad \vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$ $\quad \vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

Vector eqn: $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$= (\hat{i} - \hat{j} + 3\hat{k}) + s(2\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})$

Cartesian eqn: $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$9x - 2y - 5z + 4 = 0.$

b) $\vec{a}\times\vec{b} = 4\hat{i} + 4\hat{j}$

$\vec{c}\times\vec{d} = 8\hat{i} - 2\hat{j} - 6\hat{k}$

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = -24\hat{i} + 24\hat{j} - 40\hat{k} \rightarrow (1)$

$[\vec{a}\ \vec{b}\ \vec{d}] = 28$ $\quad [\vec{a}\ \vec{b}\ \vec{c}] = 12$

$[\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = 28(3\hat{j} - \hat{k}) - 12(2\hat{i} + 5\hat{j} + \hat{k})$

$= -24\hat{i} + 24\hat{j} - 40\hat{k} \rightarrow (2)$

from (1) & (2) identity verified.

47.

a)
$x+y+z=5$

$-x+y-z=5$

$2x-2y+2z=10$

$$\begin{bmatrix}1 & 1 & 1\\ -1 & 1 & -1\\ 2 & -2 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5\\ 5\\ 10\end{bmatrix}$$

$$[A|B]=\left[\begin{array}{ccc|c}1 & 1 & 1 & 5\\ -1 & 1 & -1 & 5\\ 2 & -2 & 2 & 10\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1 & 1 & 1 & 5\\ 0 & 2 & 0 & 10\\ 0 & -4 & 0 & 0\end{array}\right]\quad \begin{array}{l}R_2\to R_2+R_1\\ R_3\to R_3-2R_1\end{array}$$

$$\sim\left[\begin{array}{ccc|c}1 & 1 & 1 & 5\\ 0 & 2 & 0 & 10\\ 0 & 0 & 0 & 20\end{array}\right]\quad R_3\to R_3+2R_2$$

$\rho(A)=2 \quad \rho(A|B)=3$

$\rho(A)\neq\rho(A|B)$ The given system is inconsisten

b)
$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

$\vec{a}=3\hat{i}+8\hat{j}+3\hat{k}$ $\quad\quad$ $\vec{c}=-3\hat{i}-7\hat{j}+6\hat{k}$

$\vec{b}=3\hat{i}-\hat{j}+\hat{k}$ $\quad\quad$ $\vec{d}=-3\hat{i}+2\hat{j}+4\hat{k}$

$\vec{c}-\vec{a}=-6\hat{i}-15\hat{j}+3\hat{k}$

$$\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}=\hat{i}(-6)-\hat{j}(15)+\hat{k}(3)$$

$$=-6\hat{i}-15\hat{j}+3\hat{k}$$

$(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})=36+225+9=270$

$|\vec{b}\times\vec{d}|=\sqrt{36+225+9}=\sqrt{270}$

$$d=\frac{|(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})|}{|\vec{b}\times\vec{d}|}=\frac{270}{\sqrt{270}}=\sqrt{270}$$