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HIGHER SECONDARY SECOND YEAR

12th - Maths / UNIT 3 - Theory of Equations

EXERCISE 3.7

Choose the correct or the most suitable answer from the given four alternatives :

1. A zero of x^3+64 is
   (1) 0 (2) 4 (3) $4i$ (4) -4

2. If f and g are polynomials of degrees m and n respectively, and if $h(x)=(f\circ g)(x)$, then the degree of h is
   (1) mn (2) $m+n$ (3) m^n (4) n^m

3. A polynomial equation in x of degree n always has
   (1) n distinct roots (2) n real roots (3) n complex roots (4) at most one root.

4. If α,β, and γ are the zeros of x^3+px^2+qx+r, then $\sum\frac{1}{\alpha}$ is
   (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

5. According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?
   (1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

6. The polynomial x^3-kx^2+9x has three real zeros if and only if, k satisfies
   (1) $|k|\le 6$ (2) $k=0$ (3) $|k|>6$ (4) $|k|\ge 6$

7. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x-2\sin^2 x+1$ is
   (1) 2 (2) 4 (3) 1 (4) ∞

8. If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if
   (1) $a\ge 0$ (2) $a>0$ (3) $a<0$ (4) $a\le 0$

9. The polynomial x^3+2x+3 has
   (1) one negative and two imaginary zeros (2) one positive and two imaginary zeros
   (3) three real zeros (4) no zeros

10. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is
    (1) 0 (2) n (3) $<n$ (4) r

Example 3.2

If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

Example 3.4

Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$, $a \neq 0$

Example 3.7

If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

1. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio $3:2$.

9. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

11. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was left standing.

Example 3.14

Prove that a line cannot intersect a circle at more than two points.

2. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.

3. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root.

5. Prove that a straight line and parabola cannot intersect at more than two points.

Example 3.19

Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P.

Example 3.21

If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P., prove that $9pqr = 27r^2 + 2q^3$.

Assume $p, q, r \neq 0$

2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

3. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

5. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

Example 3.24

Solve the equation $(2x-3)(6x-1)(3x-2)(x-2) - 5 = 0$.

Theorem 3.6

A polynomial equation $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$, $(a_n \neq 0)$ is a reciprocal equation if, and only if, one of the following two statements is true:

(i) $a_n = a_0$, $a_{n-1} = a_1$, $a_{n-2} = a_2 \cdots$

(ii) $a_n = -a_0$, $a_{n-1} = -a_1$, $a_{n-2} = -a_2, \cdots$

3. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

4. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$.

6. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.

3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.