

- 1 b) $\frac{1}{2} \log_2 \frac{1}{100}$
- 2 c) 512
- 3 d) $\frac{1}{2} \log_2 \frac{1}{100}$
- 4 b) (2∞)
- 5 a) $\sqrt{k^2 - 4c}$
- 6 d) 4
- 7 a) $4 + \sqrt{2}$
- 8 a) $[1 \sqrt{2}]$
- 9 a) $b \leq \sqrt{2}$ $\text{അല്ലെങ്കിൽ } b^2 - 1$
- 10 a) $1 : \sqrt{3} : 2$ (1.8 3.59)
- 11 d) 8!
- 12 c) 11
- 13 d) 1296
- 14 a) 600
- 15 d) $n(n+1)$
- 16 c) $\frac{-4}{15}$
- 17 b) $\frac{1}{2} \log_2 \frac{1}{100}$ (11-263)
- 18 b) (1 2)
- 19 a) (-3 -2)
- 20 c) $a+b=0$ (11-331)

21 $n[A \cup B] = n(A \cup B) - n(A \cap B) = 10 - 3$
 $n[P(A \cup B)] = 2^7 = 128.$

22 $\alpha + \beta = 4$ $\alpha\beta = -21$
 $x^2 - 4x - 21 = 0$

23 $\sin(45^\circ + \theta) - \sin(45^\circ - \theta)$
 $= 2 \cos 45^\circ \sin \theta = 2 \cdot \frac{1}{\sqrt{2}} \sin \theta$
 $= \sqrt{2} \sin \theta$

24) $s = 21$ $s - a = 8$ $s - b = 7$ $s - c = 6$
 $A = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{3 \times 7 \times 4 \times 2 \times 7 \times 3 \times 2}$
 $= 3 \times 7 \times 4 = 84$

25) $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ A=81
 $\frac{8 \times 9}{9!} + \frac{9}{9!} = \frac{A}{9!} \Rightarrow \frac{81}{9!} = \frac{A}{9!}$

26) $(102)^4 = (100+2)^4 = (100)^4 + 4(100)^3 \cdot 2$
 $+ 6(100)^2 \cdot 4 + 4(100) \cdot 8 + 16.$
 $= 100000000 + 80000000 + 240000$
 $+ 3200 + 16.$
 $= 108243216$

27) $e^{5x} = 1 + \frac{5x}{1!} + \frac{(5x)^2}{2!} + \frac{(5x)^3}{3!} + \frac{(5x)^4}{4!} + \frac{(5x)^5}{5!} + \dots$

28) $(0 \ 0)$ കേന്ദ്രം $5x + 12y - 3 = 0$ കേന്ദ്രം
 തന്നെ $\frac{5(0) + 12(0) - 3}{\sqrt{25 + 144}} = \frac{3}{13}$

29) $3x + 2y + 9 = 0$, $12x + 8y - 15 = 0$
 $m_1 = -\frac{3}{2}$ $m_2 = -\frac{12}{8} = -\frac{3 \times 4}{2 \times 4}$
 $m_2 = -\frac{3}{2}$
 $m_1 = m_2$ \therefore \parallel

30) $\frac{11!}{4!4!2!} = 34650$

31) $f \circ g = f(g(x)) = f(x^2 + 3) = 3x^2 + 9 - 4$
 $f \circ g = 3x^2 + 5$
 $g \circ f = g(3x - 4) = (3x - 4)^2 + 3$
 $= 9x^2 - 24x + 16 + 3 = 9x^2 - 24x + 19$

32) $x = \sqrt{2} + \sqrt{3} \Rightarrow x^2 = 5 + 2\sqrt{6}$
 $\frac{x^2 + 1}{x^2 - 2} = \frac{6 + 2\sqrt{6}}{3 + 2\sqrt{6}} \times \frac{3 - 2\sqrt{6}}{3 - 2\sqrt{6}}$
 $= \frac{18 - 12\sqrt{6} + 6\sqrt{6} - 4 \times 6}{9 - 4 \times 6}$
 $= \frac{-6 - 6\sqrt{6}}{-15} = \frac{2 + 2\sqrt{6}}{5}$

33) LHS

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9}$$

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{2\pi}{9} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{18} \right) + \sin^2 \frac{7\pi}{18}$$

$$= \sin^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18}$$

$$= 1 + \sin^2 \left(\frac{\pi}{2} - \frac{7\pi}{18} \right) + \sin^2 \frac{7\pi}{18}$$

$$= 1 + \cos^2 \frac{\pi}{18} + \sin^2 \frac{7\pi}{18} = 1+1$$

$$= 2 \text{ RHS}$$

$$8-5a = a-a^2$$

$$a^2 - 6a + 8 = 0$$

$$(a-4)(a-2) = 0 \quad a = 2, 4$$

$$a = 2 \quad b = -1 \quad a = 4, b = -3$$

$$\frac{x}{2} + \frac{y}{-1} = 1 \quad \frac{x}{4} - \frac{y}{3} = 1$$

$$3x - 4y = 12$$

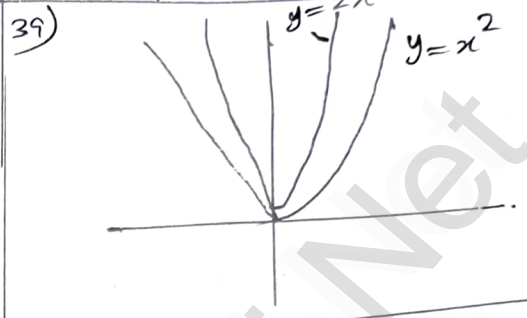
$$x - 2y = 2$$

34)

$$\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}$$

$$= \frac{2 \cos 45^\circ \sin 60^\circ}{2 \cos 90^\circ \cos 60^\circ} = \frac{\sin 60^\circ}{\cos 60^\circ}$$

$$= \tan 60^\circ = \sqrt{3}$$



35)

$$\text{LHS} = {}^{2n}C_n = \frac{(2n)!}{n!n!}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdots 2n}{n! \cdot n!}$$

$$= \frac{2 \cdot 4 \cdot 6 \cdots 2n \cdot 1 \cdot 3 \cdots 2n-1}{n!}$$

$$= \frac{2^n \cdot 1 \cdot 3 \cdot 5 \cdots 2n-1}{n! \cdot n!}$$

$$= \frac{2^n \cdot 1 \cdot 3 \cdot 5 \cdots 2n-1}{n!}$$

40)

$$-5t^2 + 100t = 495$$

$$5t^2 - 100t + 495 = 0$$

$$t^2 - 20t + 99 = 0$$

$$(t-11)(t-9) = 0$$

$$t = 9, 11$$

36)

$$6 + 66 + 666 + \dots$$

$$= 6[1 + 11 + 111 + \dots]$$

$$= \frac{6}{9}[9 + 99 + 999 + \dots]$$

$$= \frac{6}{9}[10 - 1 + 100 - 1 + 1000 - 1 + \dots]$$

$$= \frac{6}{9} \left[\frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{2}{3} \left[\frac{10(10^n - 1)}{9} - n \right]$$

41) a) $a \neq 0, m \in \mathbb{Z}$

$$m - m = 0 = 0(1)$$

$$\Rightarrow mRm$$

b) $mRn \Rightarrow m - n = 7k$

$$n - m = -7k = 7(-k)$$

$$nRm$$

c) mRn, nRl

$$m - n = 12k_1 \quad n - l = 12k_2$$

$$m - n + n - l = 12(k_1 + k_2)$$

$$m - l = 12(k_1 + k_2)$$

$$mRl$$

37)

$$\sqrt[3]{65} = (64 + 1)^{\frac{1}{3}}$$

$$= (64)^{\frac{1}{3}} \left[1 + \frac{1}{64} \right]^{\frac{1}{3}}$$

$$= 4 \left(1 + \frac{1}{3} \cdot \frac{1}{64} \right) \approx 4.02$$

$$= 4 + \frac{1}{48} = 4.02$$

41 b)

$$A+B = A5$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

$$\tan A + \tan B + \tan A \tan B = 1$$

$$\tan A (1 + \tan B) + 1 + \tan B = 2$$

$$(1 + \tan A)(1 + \tan B) = 2$$

38)

$$\frac{x}{a} + \frac{y}{1-a} = 1 \quad \frac{8}{a} + \frac{3}{1-a} = 1$$

$$\frac{8-8a+3a}{a-a^2} = 1$$

42a) $\frac{7+x}{(1+x)(1+x^2)} = \frac{A(1+x) + B}{1+x^2} + \frac{C}{1+x}$

$7+x = (A(1+x) + B)(1+x^2) + C(1+x)$

$A = -3 \quad B = 4 \quad C = 3$

$\frac{7+x}{(1+x)(1+x^2)} = \frac{4-3x}{1+x^2} + \frac{3}{1+x}$

42a) $\frac{x+1}{x+3} < 3$

$\frac{x+1}{x+3} - 3 < 0$

$\frac{x+1-3x-9}{x+3} < 0$

$\frac{-2x-8}{x+3} < 0 \quad \frac{x+4}{x+3} > 0$

42b)

G	A	R	D	E	N
5!	4!	3!	2!	1!	0!
3	0	3	0	0	0

$3 \times 5! + 3 \times 3! + 1 = 3 \times 120 + 3 \times 6 + 1 = 360 + 18 + 1 = 379$

D	A	N	G	E	R
1	0	2	1	0	0

$1 \times 5! + 2 \times 3! + 1 \times 2! + 1 = 120 + 12 + 2 + 1 = 135$

Number line: $\leftarrow \infty \quad -4 \quad -3 \quad \infty \rightarrow$

	$x+4$	$x+3$	Sign
$(-\infty, -4)$	-	-	+
$(-4, -3)$	+	-	-
$(-3, \infty)$	+	+	+

$x \in (-\infty, -4) \cup (-3, \infty)$

43a) $A+B+C = \pi \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2}$

$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \cot \frac{C}{2}$

$\frac{\tan A + \tan B}{2} = \cot \frac{C}{2} \left(\frac{1 - \tan A \tan B}{2} \right)$

44b) $\left(x^2 - \frac{1}{x^3}\right)^6$

$T_{r+1} = {}^6C_r (x^2)^{6-r} \left(\frac{-1}{x^3}\right)^r$

$= {}^6C_r x^{12-2r} (-1)^r x^{-3r}$

$= {}^6C_r x^{12-5r} (-1)^r$

x^2 term: $2 = 12 - 5r \Rightarrow r = 2$

x^6 term: $6 = 12 - 5r \Rightarrow r = 6/5$

$\therefore x^6$ term does not exist.

x^2 term: ${}^6C_2 (-1)^2 = \frac{6 \times 5}{1 \times 2} = 15$

$\tan \frac{C}{2} \tan \frac{A}{2} + \tan \frac{C}{2} \tan \frac{B}{2} = 1 - \tan \frac{A}{2} \tan \frac{B}{2}$

$\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

45a) $3x + 4y + 2 = 0$

$-5x - 12y + 5 = 0$

$\frac{3x + 4y + 2}{\sqrt{9+16}} = \pm \frac{-5x - 12y + 5}{\sqrt{25+144}}$

$\frac{3x + 4y + 2}{5} = \pm \frac{-5x - 12y + 5}{13}$

$39x + 52y + 26 = \pm (-25x - 60y + 25)$

$64x + 112y + 1 = 0$

43b) $\sqrt{\frac{1-x}{1+x}} = (1-x)^{1/2} (1+x)^{-1/2}$

$= \left(1 - \frac{x}{2} + \frac{1}{2} \left(\frac{-1}{2}\right) x^2\right)$

$\left(1 - \frac{x}{2} + \frac{1}{2} \cdot \frac{3}{2} x^2\right)$

$= \left(1 - \frac{x}{2} - \frac{x^2}{8}\right) \left(1 - \frac{x}{2} + \frac{3}{8} x^2\right)$

$= 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x}{2} + \frac{3x^2}{8} - \frac{x^2}{8}$

$= 1 - x + \frac{x^2}{2}$

45b) $f(x) = 3x - 5$

$g(x) = \frac{x+5}{3}$

$f \circ g = f\left(\frac{x+5}{3}\right) = 3\left(\frac{x+5}{3}\right) - 5 = x + 5 - 5 = x$

$g \circ f = g(3x - 5) = \frac{3x - 5 + 5}{3} = \frac{3x}{3} = x$

$f^{-1} = g \Rightarrow f$ and g are inverse functions.

46 a

$$LHS = \log_{16} 75 - 2 \log_9 5 + \log_{243} 32$$

$$= \log_{16} \frac{25}{16} \times \frac{32}{81} - \log_{81} 25$$

$$= \log_{81} \frac{25 \times 32}{16} - \log_{81} 25$$

$$= \log_{81} 2 \times \frac{25 \times 81}{81} = \log 2$$

= RHS

ii)

$$a^2 - 4(a+2) = 0$$

$$a^2 - 4a - 8 = 0$$

$$a = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm \sqrt{48}}{2}$$

$$= \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$$

46 b

$$abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 2(-3)(-20) + 2(19)(-3) - 1$$

$$- 2 \left(\frac{19}{4} \right)^2 - (-3)(-3) + 20(-1)^2$$

= 0
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$$2x^2 - xy - 3y^2 = 2x^2 + 2xy - 3xy$$

$$= 2x(x+y) - 3y(x+y)$$

$$= (2x - 3y)(x+y)$$

$$2x - 3y + l = 0, \quad x + y + m = 0$$

$$2m + l = -6, \quad -3m + l = 19$$

$$\therefore 5m = -25 \quad \boxed{m = -5}$$

$$\boxed{l = 4}$$

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$$2x - 3y + 4 = 0$$

$$x + y - 5 = 0$$

47 a

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a} \cot \frac{B}{2}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\frac{a-b}{a+b} \cot \frac{C}{2} = \frac{2R \sin A - 2R \sin B}{2R \sin A + 2R \sin B} \cot \frac{C}{2}$$

$$= \frac{2R \sin \frac{A+B}{2} \sin \left(\frac{A-B}{2}\right)}{2R \sin \frac{A+B}{2} \cos \left(\frac{A-B}{2}\right)} \cot \frac{C}{2}$$

$$= \frac{\sin \frac{A+B}{2} \sin \left(\frac{A-B}{2}\right)}{\cos \left(\frac{A-B}{2}\right)} \cot \frac{C}{2}$$

$$= \cot\left(\frac{A+B}{2}\right) \tan\left(\frac{A-B}{2}\right) \cot \frac{C}{2}$$

$$= \tan\left(\frac{A-B}{2}\right)$$

47 b

$$p(n) = x - y$$

$$p(1) = x^2 - y^2 = (x+y)(x-y)$$

p(1) දෙසලස

p(k) දෙසලස

$$x - y = (x+y)^m$$

$$x = (x+y)^m + y$$

$$p(k+1) = x \cdot x - y \cdot y$$

$$= (x+y)^m \cdot x - y \cdot y$$

$$= x^m (x+y) + x \cdot y - y \cdot y$$

$$= (x+y)^{2k}$$

p(k+1) දෙසලස
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