RK ACADEMY

FULL PORTION [COME BOOK PATTERN]

12th Standard

Maths

Exam Time : 03:00 Hrs Total Marks : 90

I. CHOOSE THE CORRECT ANSWER 20 x 1 = 20

1) If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

2) If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

3) According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

(a) -1 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

4) If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

5) The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

6) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$ and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

7) If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

8) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

9) The point of inflection of the curve $y = (x - 1)^3$ is

(a) (0, 0) (b) (0, 1) (c) (1, 0) (d) (1, 1)

10) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

11) The area between $y^2 = 4x$ and its latus rectum is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{5}{3}$

12) For any value of $n \in \mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]dx$ is

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 2

13) The solution of $\frac{dy}{dx} + p(x)y = 0$ is

(a) $y = ce^{\int pdx}$ (b) $y = ce^{-\int pdx}$ (c) $x = ce^{-\int pdy}$ (d) $x = ce^{\int pdy}$

14) The slope at any point of a curve y = f (x) is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1, 1). Then the equation of the curve is

(a) $y = x^3 + 2$ (b) $y = 3x^2 + 4$ (c) $y = 3x^4 + 4$ (d) $y = 3x^2 + 5$

15) If the function $f(x) = \frac{1}{12}$ for a < x < b, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

16) Which one is the inverse of the statement (pVq)→(p∧q)?

(a) (p∧q)→(pVq) (b) ¬(pvq)→(p∧q) (c) (¬pv¬q)→(¬p∧¬q) (d) (¬p∧¬q)→(¬pV¬q)

17) $\frac{(cos\theta+isin\theta)^6}{(cos\theta-isin\theta)^5}$ = ________

(a) cos 11θ - isin 11θ (b) cos 11θ + isin 11θ (c) cosθ + i sinθ (d) $cos\frac{6\theta}{5}+isin\frac{6\theta}{5}$

18) The pricipal value of $sin^{-1}\left(\frac{-1}{2}\right)$ is ________

(a) $\frac{\pi}{6}$ (b) $\frac{-\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{-\pi}{3}$

19) The function -3x+12 is _______ function on R.

(a) decreasing (b) strictly decreasing (c) increasing (d) strictly increasing

20) The order and degree of the differential equation $e^{\frac{d^3y}{dx^2}}+\cos(x)\frac{dy}{dx}=4$ is _________

(a) 3, not defined (b) not defined 1 (c) 3, 1 (d) none of these

II. ANSWER THE FOLLOWING (Q.NO. 30 IS COMPULSORY) 7 x 2 = 14

21) Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

22) Prove the following properties
$Re\,(z)=\frac{z+\bar{z}}{2}$ and $\text{Im}(z)=\frac{z-\bar{z}}{2i}$

23) Find the value of $sin^{-1}\left(sin\left(\frac{5\pi}{4}\right)\right)$

24) Obtain the equation of the circle for which (3, 4) and (2, -7) are the ends of a diameter.

25) Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane 5x-y+z = 8.

26) Compute the limit $\lim\limits_{x\to a}(\frac{x^n-a^n}{x-a})$

27) Evaluate the following $\int_0^{\pi/2}cos^7x\quad dx$

28) Show that y = mx + $\frac{7}{m}$, m ≠ 0 is a solution of the differential equation xy'+7$\frac{1}{y'}$-y = 0.

29) Determine whether $*$ is a binary operation on the sets given below.
(a*b) = a√b is binary on R

30) If the mean of the binomial distribution with 9 trial is 6, find the variance?

III. ANSWER THE FOLLOWING (Q.NO. 40 IS COMPULSORY) 7 x 3 = 21

31) If |z| = 1, show that $2\le\left|z^2-3\right|\le4$

32) Find solution, if any, of the equation 2cos^{2}x - 9cosx + 4 = 0

33) Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

34) Find the angle between the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

35) Prove, using mean value theorem, that $|sin\alpha-sin\beta|\le|\alpha-\beta|,\alpha,\beta\in R$

36) Assuming $\log_{10}e$ = 0.4343, find an approximate value of $\log_{10}1003$

37) Evaluate the following definite integrals:
$\int_0^{\frac{\pi}{2}}e^x\left(\frac{1+sinx}{1+cosx}\right)dx$

38) The probability density function of X is given by $f(x)=\begin{cases}kxe^{-2x} & for x>0\\ 0\quad for & x\le0\end{cases}$ Find the value of k.

39) Prove that q → p ≡ ¬p → ¬q

40) Solve : ydx+(x-y^2)dy=0

IV. ANSWER THE FOLLOWING 7 x 5 = 35

41) a)

(a) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

(OR)

b)

Simplify: $\left(-\sqrt{3}+3i\right)^{31}$

42) a)

If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

(OR)

b)

Find the value of $tan\left[\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

43) a)

A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x -axis is an ellipse. Find the eccentricity.

(OR)

b)

If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $\left|\vec{AB}\right|^2+\left|\vec{AC}\right|^2=2(\left|\vec{AD}\right|^2+\left|\vec{BD}\right|^2)$

44) a)

Find the parametric form of vector equation and Cartesian equations of the plane containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to plane $\vec{r}.(\hat{i}+2\hat{j}+\hat{k})=8$

(OR)

b)

Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

45) a)

Let w(x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}},(x,y,z)\neq(0,0,0)$. Show that $\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial y^2}+\frac{\partial^2w}{\partial z^2}=0$

(OR)

b)

Find the area of the region bounded between the curves y = sin x and y = cos x and the lines x = 0 and x = π

46) a)

Solve the differential equation $ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y\right)dy$

(OR)

b)

The equation of electromotive force for an electric circuit containing resistance and self inductance is E = Ri + L$\frac{di}{dt}$, Where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when E = 0.

47) a)

If X is the random variable with probability density function f(x) given by,

$$f(x)=\begin{cases} x+1 & -1\leq x<0 \\ -x+1 & 0\leq x<1 \\ 0 & otherwise \end{cases}$$

then find

(i) the distribution function F(x)

(ii) P(-0.5 ≤X ≤ 0.5)

(OR)

b)

Let A be Q\{1}. Define $*$ on A by x*y = x + y − xy . Is $*$ binary on A? If so, examine the existence of identity, existence of inverse properties for the operation $*$ on A.

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I. CHOOSE THE CORRECT ANSWER

20 x 1 = 20

1) (c) 19

2) (b) 1

3) (c) $\frac{4}{5}$

4) (b) $\frac{3\pi}{4}$

5) (c) $\sqrt{10}$

6) (b) $\vec{b}$

7) (a) $2\sqrt{3}$

8) (b) 2.5

9) (c) (1, 0)

10) (d) 4.8 cu.cm

11) (c) $\frac{8}{3}$

12) (c) 0

13) (b) $y = ce^{-\int pdx}$

14) (a) $y = x^3 + 2$

15) (d) 16 and 24

16) (d) $(\neg p \wedge \neg q) \to (\neg p \vee \neg q)$

17) (b) cos 11θ + isin 11θ

18) (a) $\frac{\pi}{6}$

19) (b) strictly decreasing

20) (a) 3, not defined

II. ANSWER THE FOLLOWING (Q.NO. 30 IS COMPULSORY)

7 x 2 = 14

21) Let A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Then, A^T = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

So, we get

AA^T = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

= $\begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I_2

Similarly, we get $A^TA = I_2$. Hence $AA^T = A^TA = I_2 \Rightarrow$ A is orthogonal.

22) $Re(z) = \frac{z+\bar{z}}{2}$ and $\text{Im}(z) = \frac{z-\bar{z}}{2i}$

Let z = x+iy where x is the Re(x) and y is the Im (z)

Then $\bar{z}$ = x-iy

z+$\bar{z}$ = x + iy + x - iy = 2x

$\therefore \frac{z+\bar{x}}{2}$ = x

$\frac{z+\bar{x}}{2}$ = Re(z)

Also z-$\bar{z}$ = x+iy-(x-iy)

= x+iy-x+iy = 2iy

$\frac{z-\bar{z}}{2i}$ = y

$\therefore \frac{z-\bar{z}}{2i}$ = Im(z)

Hence proved

23) $sin^{-1}\left(sin\left(\frac{5\pi}{4}\right)\right)$

= $sin^{-1}\left(sin\left(\pi+\frac{\pi}{4}\right)\right) \because \frac{5\pi}{4} \notin \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

= $sin^{-1}\left(sin\left(-\frac{\pi}{4}\right)\right)$

= $\frac{-\pi}{4} \epsilon \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

24) Given ends of diameter are (3, 4)(2, -7)

∴ Equation of the circle is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

⇒ (x - 3)(x - 2) + (y - 4)(y + 7) = 0

⇒ $x^2 - 2x - 3x + 6 + y^2 + 7y - 4y - 28 = 0$

⇒ $x^2 + y^2 - 5x + 3y - 22 = 0$

25) Here (x_1, y_1, z_1) = (3, -4, -3) and direction ratios of the given straight line are (a, b, c) = (-4, -7, 12).

Direction ratios of the normal to the given plane are (A, B, C) = (5, -1, 1).

We observe that, the given point (x_1, y_1, z_1) = (3, 4, -3) satisfies the given plane 5x-y+z = 8

Next, aA+bB+cC = (-4)(5)+(-7)(-1)+(12)(1) = -1 $\neq$ 0.

So, the normal to the plane is not perpendicular to the line.

Hence, the given line does not lie in the plane.

26) If we put directly x = a we observe that the given function is in an indeterminate form $\frac{0}{0}$.

As the numerator and the denominator functions are polynomials they both are differentiable.

Hence by an application of the l'Hôpital Rule we get,

$\lim\limits_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = \lim\limits_{x\to a}\left(\frac{n\times x^{n-1}}{1}\right)$

= $n \times a^{n-1}$.

27) $I_n = \int_0^{\pi/2} cos^7 x dx = \frac{n-1}{n} I_{n-2}, n \geq 2$

$\therefore I_7 = \int_0^{\pi/2} cos^7 x dx = \frac{6}{7} \times \frac{4}{5} \times \frac{2}{3} \times 1 = \frac{16}{35}$

28) The given function is y mx +$\frac{7}{m}$, where m is an arbitrary constant ....(1)

Differentiating both sides of equation (1) with respect to x, we get y' = m.

Substituting the values of y' and y in the given differential equation

we get xy'$\frac{1}{y'}$-y = xm +$\frac{7}{m}$- mx -$\frac{7}{m}$ = 0

Therefore, the given function is a solution of the differential equation xy' + 7$\frac{1}{y'}$ - y = 0

29) √b is not defined for negative values, b which also ∈ R.

Hence, a√b is not defined for all a, b ∈R

* is not a binary operation on R

30) Given n = 9 and

mean = 6

np = 6

$\Rightarrow\ p = \frac{6}{9} = \frac{2}{3}$

$\therefore q = 1 - p$

$= 1 - \frac{2}{3} = \frac{1}{3}$

Variance = npg = $9 \times \frac{2}{3} \times \frac{1}{3} = 2$

III. ANSWER THE FOLLOWING (Q.NO. 40 IS COMPULSORY) 7 x 3 = 21

31) $|z^2-3| \leq |z^2|+|-3|$ [Triangle law of inequality]

$\leq |z|^2+3 \leq 1+3$ [∴ |z| = 1]

$|z^2-3| \leq 4$ ....(1)

Also, $|z^2- 3| \geq ||z^2|-|-3||$

$\geq ||z|^2-3|$ [∵ |-3| = 3]

$\geq |1^2-3|$ [∵ |z| = 1]

≥ |-2| .

$|z^2-3| \geq 2$............(2)

From (1) and (2) we get $2 \leq |z^2-3| \leq 4$

Hence proved.

32) $2cos^2x$ - 9cosx + 4 = 0 .... (1)

The left hand side of this equation is not a polynomial in x. But it looks like a polynomial. In fact, we can say that this is a polynomial in cos x. However, we can solve the equation (1) by using our knowledge on polynomial equations. If we replace cos x by y, then we get the polynomial equation $2y^2 - 9y + 4 = 0$ for which 4 and $\frac{1}{2}$ are solutions.

From this we conclude that x must satisfy cos x = 4 or cos x = $\frac{1}{2}$.

But cos x = 4 is never possible, if we take cos x = $\frac{1}{2}$, then we get infinitely many real numbers x satisfying cos x = $\frac{1}{2}$; in fact, for all n $\in Z$, $x = 2n\pi \pm \frac{\pi}{3}$ are solutions for the given equation (1).

If we repeat the steps by taking the equation $\cos^2 x - 9\cos x + 20 = 0$, we observe that this equation has no solution.

33) The Latus rectum LL′ of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through S(ae, 0)

Hence L is (ae, y_1)

Therefore, $\frac{a^2e^2}{a^2} + \frac{{y_1}^2}{b^2} = 1$

$\frac{{y_1}^2}{b^2} = 1 - e^2$

${y_1}^2 = b^2(1-e^2)$

$= b^2\left(\frac{b^2}{a^2}\right)\left(since, e^2 = 1 - \frac{b^2}{a^2}\right)$

$y_1 = \pm\frac{b^2}{a}$

That is the end points of Latus rectum L and L′ are $\left(ae, \frac{b^2}{a}\right)$ and $\left(ae - \frac{b^2}{a}\right)$

Hence the length of latus rectum LL′ = $\frac{b^2}{a}$

34) If $\hat{b} = \frac{2\hat{i}+2\hat{j}-\hat{k}}{\left|2\hat{i}+2\hat{j}-\hat{k}\right|} = \frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$. Therefore from the definition of direction cosines of $\hat{b}$, we have

$cos\alpha = \frac{2}{3}, cos\beta = \frac{2}{3}, cos\gamma = -\frac{1}{3}$

where α, β, γ are the angles made by $\hat{b}$ with the positive x -axis, positive y -axis, and positive z -axis, respectively. As the angle between the given straight line with the coordinate axes are same as the angles made by $\hat{b}$ with the coordinate axes, we have $\alpha = cos^{-1}\left(\frac{2}{3}\right), \beta = cos^{-1}\left(\frac{2}{3}\right), \gamma = cos^{-1}\left(\frac{-1}{3}\right)$ respectively.

35) Let f (x) = sin x which is a differentiable function in any open interval. Consider an interval $[\alpha, \beta]$. Applying the mean value theorem there exists $c \in (\alpha, \beta)$ such that,

$\frac{sin\beta - sin\alpha}{\beta - \alpha} = f'(c) = cos(c)$

Therefore, $\frac{sin\beta - sin\alpha}{\beta - \alpha} = |cos(c)| \leq 1$

Hence, $|sin\alpha - sin\beta| \leq |\alpha - \beta|$

Remark

If we take $\beta = 0$ in the above problem, we ge $|sin\alpha| \leq |\alpha|$

36) $\log_{10}e = 0.4343$ to find $\log_{10}$g 1003

$f(1000) = \log_{10}1000 = \log_{10}10^3 = 3\log_{10}10^3 = 3\log_{10}10$

= 3(1) = 3

$f'(x) = \frac{1}{x}.\log10^e$

$f'(1000) = \frac{1}{1000}(0.4343)$

$\therefore L(x) = f(x_0) f'(x_0)(x - x_0)$

$= 3 + \frac{1}{1000}(0.4343)(3)$

$= 3 + \frac{1.3029}{1000}$

= 3 + 0.0013029

$\log_{10}1003 = 3.0013029$

37) $Let \int_0^{\frac{\pi}{2}} e^x\left(\frac{sinx}{1+cosx}\right)dx$

$\int_0^{\frac{\pi}{2}} e^x\left(\frac{1+sinx}{cos^2\frac{x}{2}}\right)dx$

$[\because cosx = 2cos^2x - 1 \Rightarrow 1 + cos2x = 2cos^2x]$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} e^x\left(\frac{1}{cos^2\frac{x}{2}} + \frac{sinx}{cos^2\frac{x}{2}}\right)dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} e^x\left(sec^2\frac{x}{2} + \frac{2sin\frac{x}{2}cos\frac{x}{2}}{cos^2\frac{x}{2}}\right)dx$

$[\because sin2x = 2sinx \quad cosx]$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} e^x\left(sec^2\frac{x}{2}2tan\frac{x}{2}\right)dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} e^x(f'(x) + f(x)dx$

Where $f(x) = 2tan\frac{x}{2}$

$= \frac{1}{2}.e^x.f(x) = \left[\left(\frac{1}{2}e^x.2tan\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$

$= \left[e^x tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$= e^{\frac{\pi}{2}} tan\frac{\pi}{4} - e^0 tan0 = e^{\frac{\pi}{2}}(1)$

$\therefore I = e^{\frac{\pi}{2}}$

38) Given $f(x) = \begin{cases} kxe^{-2x} & for x > 0 \\ 0 \quad for & x \leq 0 \end{cases}$

Since the given function is a probability density function

$\int_{-\infty}^{\infty} f(x)dx$ = 1

$\Rightarrow k\int_0^{\infty} xe^{-2x}dx = 1$

$\Rightarrow k\frac{1!}{(2)^2} = 1$

$[\int_0^{\infty} x^n e^{-ax} = \frac{n!}{a^{+1}}$, Here a = 2, n = 1]

$\Rightarrow \frac{k}{4} = 1$

$\Rightarrow k = 4$

39)

p	q	q → p	~p	~q	~q → ~p
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

The entries in the columns corresponding q → p and ~p → ~q are identical and hence they are equivalent.

q → p ≡ ~p → ~q

Hence proved

40) $xy = \frac{y^4}{4} + c$

IV. ANSWER THE FOLLOWING

7 x 5 = 35

41) a)

Given A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

AB = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

= $\begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-3 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$

= $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ = 4. I_3

BA = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

= $\begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix}$

= $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ = 4. I_3.

So, we get AB = BA = 4. I_3

⇒ $\left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = 1$

⇒ B^{-1} = $\frac{1}{4}$ = 1

Writing the given set of equations in matrix form we get,

$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

⇒ $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

⇒ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = [\frac{1}{4}A] \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

= $\frac{1}{4} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

= $\frac{1}{4}\begin{bmatrix}-5+7+6\\7+7-10\\1-7+2\end{bmatrix}=\frac{1}{4}\begin{bmatrix}8\\4\\-4\end{bmatrix}=\begin{bmatrix}2\\1\\-1\end{bmatrix}$

∴ x = 2, y = 1, z = -1

Hence, the solution set is {2, 1, - 1}.

(OR)

b) $\left(-\sqrt{3}+3i\right)^{31}$

Let $-\sqrt{3}+3i=r\left(cos\theta+isin\theta\right)$. Then, we get

$r=\sqrt{\left(-\sqrt{3}\right)^2+3^2}=\sqrt{12}=2\sqrt{3}$

$\alpha=tan^{-1}\left|\frac{3}{-\sqrt{3}}\right|=tan^{-1}\sqrt{3}=\frac{\pi}{3}$

$\theta=\pi-\alpha=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ ($\because$ $\sqrt{3}+3i$ lies in II Quadrant)

Therefore,$-\sqrt{3}+3i=2\sqrt{3}\left(cos\frac{2\pi}{3}+isin\frac{2\pi}{3}\right)$

Raising power 31 on both sides,

$\left(-\sqrt{3}+3i\right)^{31}=\left(2\sqrt{3}\right)^{31}\left(cos\frac{2\pi}{3}+isin\frac{2\pi}{3}\right)^{31}$

= $\left(2\sqrt{3}\right)^{31}\left(cos\left(20\pi+\frac{2\pi}{3}\right)+isin\left(20\pi+\frac{2\pi}{3}\right)\right)$

= $\left(2\sqrt{3}\right)^{31}\left(cos\frac{2\pi}{3}+isin\frac{2\pi}{3}\right)$

= $\left(2\sqrt{3}\right)^{31}\left(cos\left(\pi-\frac{\pi}{3}\right)+isin\left(\pi-\frac{\pi}{3}\right)\right)$

= $\left(2\sqrt{3}\right)^{31}\left(-cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)=\left(2\sqrt{3}\right)^{31}\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$.

42) a) Given equation are $x^2px+q=0$ ....(1)

and $x^2+p'x+q'=0$ ....(2)

Let ∝ be the common root for (1) and (2)

∴ $\propto^2$ + p∝ + q = 0 ....(3)

and $\propto^2$+ p'∝ + q' = 0 ....(4)

Solving (3) and (4) by cross multiplication method we get

p q 1 p

p' q' 1 p'

$\Rightarrow\frac{\alpha^2}{pq'-p'q}=\frac{\alpha}{q-q'}=\frac{1}{p'-p}$

consider $\frac{\alpha^2}{pq'-p'q}=\frac{\alpha}{q-q'}$

$\Rightarrow\frac{\alpha^2}{\alpha}=\frac{pq'-p'q}{q-q'}$

$\alpha=\frac{pq'-p'q}{q-q}$

Consider $\frac{\alpha}{q-q'}=\frac{1}{p'-p}\Rightarrow\alpha=\frac{q-q'}{p'-p}$

Hence its roots are $\frac{pq'-p'q}{q-q'}or\ \frac{q-q'}{p'-p}$

(OR)

b) $tan\left[\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

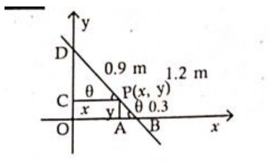
Let a = tan 0

Now, $tan\left[\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]=tan\left[\frac{1}{2}sin^{-1}\left(\frac{2tan\theta}{1+tan^2\theta}\right)+\frac{1}{2}cos^{-1}\left(\frac{1-tan^2\theta}{1+tan^2\theta}\right)\right]$

$tan\left[\frac{1}{2}sin^{-1}(sin2\theta)+\frac{1}{2}cos^{1}(cos2\theta)\right]=tan[2\theta]=\frac{2tan\theta}{1-tan^2\theta}=\frac{2a}{1-a^2}$

43) a) Let AB be the rod and P(x_1, y_1) be a point on the rod such that AP = 0.3 m.

Draw PD ⊥ x-axis and PC ⊥ y - axis.



Δ ADP ≅ Δ PCB

$\therefore\frac{PC}{DA}=\frac{PB}{AP}=\frac{BC}{PD}$

$\Rightarrow\frac{x_1}{DA}=\frac{0.9}{0.3}=\frac{BC}{y_1}$

$\Rightarrow DA=\frac{0.3x_1}{0.9}=\frac{x_1}{3}$

and BC = $\frac{0.9y_1}{0.3}=\frac{9}{3}y_1=3y_1$

Now OA = OD + DA

= $x_1+\frac{x_1}{3}=\frac{4x_1}{3}$

OB = OC + BC = y_1 + $3y_1$ = $4y_1$

But OA^2 + OB^2 = AB^2

$\Rightarrow \left(\frac{4x_1}{3}\right)^2 + (4y_1)^2 = (1.2)^2$

$\Rightarrow \frac{x_1{}^2}{9} + \frac{y_1{}^2}{9} = \frac{1.44}{16} = 0.09 \cong 1$

$\therefore$ Locus of (x_1, y_1) is $\frac{x^2}{9} + \frac{y^2}{1} = 1$

Here $a^2 = 9$, $b^2 = 1$

$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{9-1}{9}}$

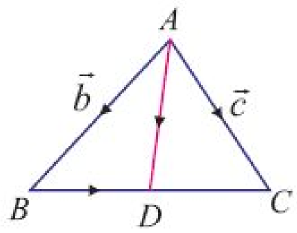
$= \sqrt{\frac{8}{9}}$

$e = \frac{2\sqrt{2}}{3}$

(OR)

b) Let A be the origin, $\vec{b}$ be the position vector of B and $\vec{c}$ be the position vector of C .

Now D is the midpoint of BC , and so the position vector of D $\frac{\vec{b}+\vec{c}}{2}$. There, we get



$\left|\overrightarrow{AD}\right|^2 = \overrightarrow{AD}.\overrightarrow{AD} = \left(\frac{\vec{b}+\vec{c}}{2}\right).\left(\frac{\vec{b}+\vec{c}}{2}\right) = \frac{1}{4}(\left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\vec{b}.\vec{c})$....(1)

Now, $\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \frac{\vec{b}+\vec{c}}{2} - \vec{b} = \frac{\vec{c}-\vec{b}}{2}$

Then, we get, $= \left|\overrightarrow{BD}\right|^2 = \overrightarrow{BD}.\overrightarrow{BD} = \left(\frac{\vec{c}-\vec{b}}{2}\right).\left(\frac{\vec{c}-\vec{b}}{2}\right) = \frac{1}{4}(\left|\vec{b}\right|^2 + |\vec{c}|^2 - 2\vec{b}.\vec{c})$....(2)

Now, adding (1) and (2), we get

Therefore, $\left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \frac{1}{4}(\left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\vec{b}.\vec{c}) + \frac{1}{4}(\left|\vec{b}\right|^2 + |\vec{c}|^2 - 2\vec{b}.\vec{c}) = \frac{1}{2}(\left|\vec{b}\right|^2 + |\vec{c}|^2)$

$\Rightarrow \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BD}\right|^2 = \frac{1}{2}(\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2)$

Hence, $\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 = 2(\left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BD}\right|^2)$

44) a) The plane containing the line

$\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$

$\therefore$ The required plane is passing through the point $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and parallel to a vector $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$ Also, the plane is perpendicular to the plane

$\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$\therefore$ The parametric form of vector equation of the plane passing through one point $(\vec{a})$ and parallel to two vectors $\vec{b}$ and $\vec{c}$

$\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$ where s, t ∈ R

$\Rightarrow \vec{r}.(\hat{i} - \hat{j} + 3\hat{k}) + s(2\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})$ s, t ∈ R

Cartesian equation is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x - 1 & y + 1 & z - 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

⇒ (x - 1)(-1 - 8) - (y + 1)(2 - 4) + (z - 3)(4 + 1) = 0

⇒ (x - 1)(-9) - (y + 1)(-2) + (z - 3)5 = 0

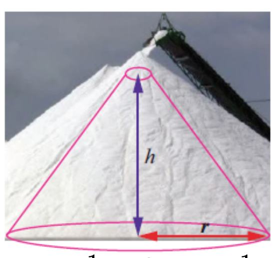
⇒ -9x + 9 +2y + 2 + 5z - 15 = 0

⇒ -9x + 2y + 5z - 4 = 0

⇒ 9x - 2y - 5z + 4 = 0

(OR)

b) Let h and r be the height and the base radius. Therefore h = 2r. Let V be the volume of the salt cone.



$V = \frac{1}{3}\pi r^2 h = \frac{1}{12}\pi h^3; \frac{dV}{dt} = 30$ mtr^3 / min.

Hence, $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

Therefore, $\frac{dh}{dt} = 4\frac{dV}{dt} \cdot \frac{1}{\pi h^2}$

That is, $\frac{dh}{dt} = 4 \times 30 \times \frac{1}{100\pi}$

$= \frac{6}{5\pi}$ mtr / min.

45) a) Given w(x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}}$

$= (x^2 + y^2 + z^2)^{-\frac{1}{2}}$

$\frac{\partial w}{\partial x} = \frac{-1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2x)$

$= (-x^2 + y^2 + z^2)^{-\frac{1}{2}}$

$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right)$

$= -[x\left(\frac{-3}{2}\right)(x^2 + y^2 + z^2)^{-\frac{3}{2}}$

$(\cancel{2}x) + (x^2+y^2+z^2)^{\frac{3}{2}}$

$= (x^2 + y^2 + z^2)^{-\frac{5}{2}} [-3x^2 + x^2 + y + z^2]$

$= -(x^2 + y^2 + z^2)^{-\frac{5}{2}} [y^2 + z^2 - 2x^2]$ ....(1)

$\frac{\partial w}{\partial y} = \frac{-1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2y)$

$= -y(x^2 + y^2 + z^2)^{-\frac{3}{2}}$

$\frac{\partial^2 w}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right)$

$= -\left[y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{\frac{5}{2}}(\cancel{2}y) + (x^2+y^2+z^2)^{\frac{3}{2}}(1)\right]$

$= -(x^2 + y^2 + z^2)^{-\frac{5}{2}}$

$= -(x^2 + y^2 + z^2)^{-\frac{5}{2}} [3y^2 + x^2 + y^2 + z^2]$

$= -(x^2 + y^2 + z^2)^{-\frac{5}{2}} [x^2 + y^2 + 2z^2]$ ....(2)

Now $\frac{\partial w}{\partial z} = \frac{-1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2z)$

$= -z(x^2 + y^2 + z^2)^{-\frac{3}{2}}$

$\therefore \frac{\partial^2 w}{\partial z^2} = -(x^2 + y^2 + z^2)^{-\frac{5}{2}} [x^2 + y^2 - 2z^2]$ ...(3)

(1)+(2)+(3)⟶

$\therefore \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} = -(x^2 + y^2 + z^2)^{-\frac{5}{2}} [y^2 + z^2 - 2x^2 + x^2 + z^2 - 2y + x^2 + y - 2z^2]$

$= -(x^2 + y^2 + z^2)^{-\frac{5}{2}}(0) = 0$

Hence proved

(OR)

b) Equation of the given curves are y = sin x ..(1)

Y = cos x ...(2)

from (1) and (2), sin x = cos x

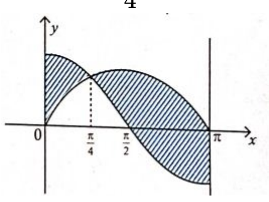
y = sin x

x	0	$\pi/2$
y	0	1

y = cos x

x	0	$\pi/2$
y	1	0

$\Rightarrow x = \frac{\pi}{4}$



$\therefore$ Required area = $2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (sinx - cosx)dx$

[$\because$ the area is symmetrical about X - axis]

$= 2[-cosx - sinx]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$= -2\left[\left(cos\frac{3\pi}{4} + sin\frac{3\pi}{4}\right) - \left(cos\frac{\pi}{4} + sin\frac{\pi}{4}\right)\right]$

$= -2\left[\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

[cos 135° = cos(180°- 45) = -cos 45° sin135° = sin(180°- 45) = -sin 45°]

$= -2\left[\frac{-2}{\sqrt{2}}\right] = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

46) a)

$ye^{\left(\frac{1}{r}\right)} \cdot dx = \left(xe^{\frac{1}{y}} + y\right) dy$

$\frac{dx}{dy} = \frac{x \cdot e^{\left(\frac{6}{y}\right)} + y}{ye^{\left(\frac{x}{y}\right)}}$

$\frac{dx}{dy} = \left(\frac{x}{y}\right) + \frac{1}{e^{\left(\frac{6}{y}\right)}}$

Put x = vy $\Rightarrow \left(\frac{x}{y}\right) = \text{v}$ and $\frac{dx}{dy} = \cdot v + y \cdot \frac{dv}{dy}$

$(1) \Rightarrow \; v + y \cdot \frac{dv}{dy} = v + \frac{1}{e^y}$

$\text{e}^v \cdot \text{dv} = \frac{dy}{y}$

Integrating on both sides,

ie) $\int e^v \cdot dv = \int \frac{dy}{y}$

$e^v = \log|y| + \log|c|$

$e^{\left(\frac{x}{y}\right)} = \log|cy|$

(OR)

b)

Given E = Ri + L $\frac{di}{dt}$

$\frac{E}{L} = \frac{Ri}{L} + \frac{di}{dt}$

$\Rightarrow \frac{Ri}{L} + \frac{di}{dt} = \frac{E}{L}$

This is a linear differential equation

$Here \quad P = \frac{R}{L} and \quad Q = \frac{E}{L}$

$\therefore \int pdt = \int \frac{R}{L} dt = \frac{R}{L} t$

$\therefore I.F = e^{\int pdt} = e^{\frac{Rt}{L}}$

$\therefore$ Solution is $ie^{\int pdt} = \int Qe^{\int pdt} dt + C$

$\Rightarrow ie^{\frac{Rt}{L}} = \int \frac{E}{L} . e^{\frac{Rt}{L}} dt + C$

$\therefore ie^{\frac{Rt}{L}} = \frac{E}{L} \frac{e^{\frac{Rt}{L}}}{\frac{R}{L}} dt + C$

$i = \frac{E}{R} e^{\frac{Rt}{L}} + C$

$i = \frac{E}{R} + ce^{-\frac{Rt}{L}}$

When E = 0,

$i = 0 + ce^{-\frac{Rt}{L}}$

$\Rightarrow i = ce^{-\frac{Rt}{L}}$

47) a)

$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ -x+1 & 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

(i) Distribution function

Case 1 : x < -1

F(x) = $\int_{-\infty}^{x} f(u)du$ = 0

Case 2 : -1 ≤ x < 0

$\int_{-\infty}^{x} f(u)du$

= $\int_{-\infty}^{x} f(x)dx = \left[\frac{x^2}{2} + x\right]_{-1}$

= $\left(\frac{u^2}{2} + u\right) = \frac{x^2}{2} + x - \left(\frac{1}{2} + 1\right)$

= $\frac{x^2}{2} + x + \frac{1}{2}$

Case 3 : 0 ≤ x < 1,

$F(X) = \int_0^x (-x+1)dx = \left[-\frac{x^2}{2} + x\right]_0^x$

= $\left(-\frac{x^2}{2} + x\right) - (0) = \frac{x^2}{2} + x$

When 1 ≤ x,

$F(x) = \int_1^x f(x)dx = \int_1^x 0dx$

= $\therefore F(X) = \begin{cases} \frac{x^2}{2} + x + \frac{1}{2} & -1 \le x < 0 \\ -\frac{x^2}{2} + x & 0 \le x < 1 \\ 0 & otherwise \end{cases}$

(ii) p(0.5 ≤ X ≤ 0.5)

= $\int_{-0.5}^{0.5} f(x)dx = \int_{0.5}^{0} f(x)dx + \int_0^{0.5} f(x)dx$

= $\int_{-0.5}^{0} (x+1)dx + \int_0^{0.5} (-x+1)dx$

= $\left[\frac{x^2}{2} + x\right]_{-0.5}^{0} + \left[\frac{-x^2}{2} + x\right]_0^{0.5}$

= $0-\left(\frac{0.5^2}{2}-0.5\right)+\left(-\frac{(0.5)^2}{2}+0.5\right)-0$

= $-\left(\frac{.25}{2}-0.5\right)+\left(\frac{-0.25}{2}+0.5\right)$

= $\frac{.25}{2}+0.5-\frac{0.25}{2}+0.5=0.25+1$

= 0.75

(OR)

b) Existence of identity:

We have to find an element a' ∈ A such that

a*a' = a'*a = e

⇒ a + e-ae = a

⇒ a + e-ae = a

⇒ e-ae = 0

⇒ e(1-a) = 0

⇒ e = $\frac{0}{1-a}$ = 0∈A

∴ A has an identity under *.

Existence of inverse:

For every a ∈ A, there exist a' ∈ A such that a*a' = a' * a = e

⇒ a+a'-aa' = 0 [∵e=0]

⇒ a+a'(1-a) = 0

⇒ a'(1-a) = -a

⇒ a' = $\frac{-a}{1-a}$

To prove that $\frac{-a}{1-a}$≠ 1

Suppose $\frac{-a}{1-a}$ = 1

⇒ -a = 1-a

⇒ -a + a = 1 ⇒ 0 ≠ 1

∴ Our assumption is wrong

⇒ $\frac{-a}{1-a}$ ≠ 1

∴ A has inverse for every element x ∈ A