N K MATHS ACADEMY

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MATHEMATICS

1: APPLICATIONS OF MATRICES AND DETERMINANTS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10X1=10)

1. If A is 3×3 Non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$
   (1)A (2)B (3)I (4) B^T

2. If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix} = \begin{bmatrix}6 & 6\\ 0 & 0\end{bmatrix}$, then A=
   (1) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$ (3) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ (4) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$

3. If $A = \begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B = \begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|adj(AB)| =$
   (1)-40 (2)-80 (3)-60 (4)-20

4. If A,B and C are invertible matrices of some order, then which one of the following is not true?
   (1) $adjA = |A| A^{-1}$ (2) $adj(AB) = (adjA)(adjB)$
   (3) $\det A^{-1} = (\det A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

5. If $A^T A^{-1}$ is symmetric,then $A^2 =$ (1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

6. If $A = \begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is
   (1) $\frac{-4}{5}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

7. The rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 6 & 8\\ -1 & -2 & -3 & -4\end{bmatrix}$ is (1)1 (2)2 (3)4 (4)3

8. If $A = (2 \quad 0 \quad 1)$, then the rank of AA^T is, (1) 1 (2) 2 (3) 3 (4) 0

9. If A is a square matrix of order n then $|adj\ A|$ is
   (1) $|A|^2$ (2) $|A|^n$ (3) $|A|^{n-1}$ (4) $|A|$

10. The rank of the matrix $\begin{pmatrix}7 & -1\\ 2 & 1\end{pmatrix}$ is (1) 9 (2) 2 (3) 1 (4) 5

PART-B:ANSWER ANY 5 QUESTIONS: (5X2=10)

11. If A be square matrix of order n, then prove that, A^{-1} exits if and only if A is non-singular.
12. Find the inverse (if it exists) of $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$
13. Find the rank of matrix $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ which is in row-echelon form.
14. Find the rank of matrix $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ by minor method
15. If $adj(A) = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ find A^{-1}
16. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

PART- C: ANSWER ANY 5 QUESTIONS :(5X3=15)

17. Find the adjoint of $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$
18. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A| I_2$
19. Find the rank of $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$
20. Solve $5x - 2y + 16 = 0, x + 3y - 7 = 0$ by Cramer's rule.
21. Solve $x + 2y + 3z = 0,\ 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$
22. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = 0_2$ Hence, find A^{-1}.

PART -D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and Hence solve $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$
24. The prices of three commodities *A*, *B*, and *C* are Rs*x*, *y*, and *z* per units respectively. A person *P* purchases 4 units of *B* and sells two units of *A* and 5 units of *C*. Person *Q* purchases 2 units of *C* and sells 3 units of *A* and one unit of *B* . Person *R* purchases one unit of *A* and sells 3 unit of *B* and one unit of *C*. In the process *P*, *Q*, and *R* earn Rs15,000, Rs1,000 and Rs4,000 respectively. Find the prices per unit of *A*, *B*, and *C*. (Use matrix inversion method to solve the problem.)

25. Solve $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$ by Cramer's rule.

26. Test for consistency and if possible solve $x-y+2z=2, 2x+y+4z=7, 4x-y+z=4$ by rank method.

27. Find value of k for which equation $kx-2y+z=1, x-2ky+z=-2, x-2y+kz=1$
(i) No solution (ii) a unique solution (iii)infinitely many solutions.

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MATHEMATICS

TEST NO -2

2. COMPLEX NUMBERS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10 x 1=10)

1. $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ is

(1)0 (2)1 (3)-1 (4) i

2. The area of the triangle formed by complex numbers z, iz and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

3. If z is a non-zero complex number, such than $2iz^2=\bar{z}$ then $|z|$ is

(1) $\frac{1}{2}$ (2)1 (3)2 (4)3

4. If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1)z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4)1

5. If Z is a complex number such that $z\in C\setminus R$ and $z+\frac{1}{z}\in R$, then $|z|$ is

(1)0 (2)1 (3)2 (4)3

6. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(1) $\frac{1}{2}$ (2)1 (3)2 (4)3

7. The Principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

(1) -110° (2) -70° (3) 70° (4) 110°

8. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(1)-2 (2)-1 (3)1 (4)2

9. If ω is a cube root of unity then the value of $(1-\omega+\omega^2)^4+(1+\omega-\omega^2)^4$ is

(1) 0 (2) 32 (3) -16 (4) -32

10. The arguments of nth roots of a complex number differ by

(1) $\frac{2\pi}{n}$ (2) $\frac{\pi}{n}$ (3) $\frac{3\pi}{n}$ (4) $\frac{4\pi}{n}$

PART-B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. Simplify $i.i^2.i^3.....i^{40}$

12. Prove that $\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$.

13. Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary part.

14. Find $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$

15. Show that $|3z-5+i|=4$ represents a circle and find its centre and radius.

16. Find the modulus and principle argument of $\sqrt{3}-i$.

PART-C: ANSWER ANY 5 QUESTIONS: (5X3=15)

17. Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

18. Prove that $|z_1+z_2| \le |z_1|+|z_2|$ (triangular inequality)

19. Show that the points $1, -\frac{1}{2}+i\frac{\sqrt{3}}{2}, -\frac{1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

20. If $|z|=3$, show that $7 \le |z+6-8i| \le 13$.

21. Obtain the Cartesian form of the locus $z=x+iy$ in $|z-4|=16$

22. If $\frac{1+z}{1-z}=\cos 2\theta + i\sin 2\theta$ show that $z=i\tan\theta$.

PART-D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Let z_1, z_2 and z_3 be complex number such that $|z_1|=|z_2|=|z_3|=r>0$ and

$z_1+z_2+z_3 \ne 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|$.

23. If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

24. If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

25. Solve $z^3 + 8i = 0$, where $z \in C$

26. Find all cube roots of $\sqrt{3} + i$.

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MATHEMATICS

UNIT TEST- 3

CH 3: THEORY OF EQUATIONS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10 x 1=10)

1. A zero of $x^3 + 64$ is

(1)0 (2)4 (3)4i (4)-4

2. A polynomials equation in x of degree n always has

(1)n distinct roots (2) n real roots (3) n complex roots (4)at most one root

3. According to the rational root theorem, which number is not possible rational root of $4x^2 + 2x^4 - 10x^3 - 5$?

(1)-1 (2)$\frac{5}{4}$ (3)$\frac{4}{5}$ (4)5

4. The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k Satisfies

(1) $|k| \le 6$ (2) k=0 (3) $|k| \succ 6$ (4) $|k| \ge 6$

5. If $x^3 + 12x^3 + 10ax + 1999$ definitely has a positive root, if and only if

(1) $a \ge 0$ (2) $a \succ 0$ (3) $a < 0$ (4) $a \le 0$

6. The number of positive roots of the polynomial $\sum_{j=0}^{n} {}^nC_r (-1)^r x^r$ is

(1)0 (2)n (3)<n (4)r

7. If $-i+2$ is one is one root equation $ax^2 - bx + c = 0$, then the other root is

(1) $-i-2$ (2) $i-2$ (3) $2+i$ (4) $2i+i$

8. The equation having $4-3i$ and $4+3i$ as roots is

(1) $x^2+8x+25=0$ (2) $x^2+8x-25=0$ (3) $x^2-8x+25=0$ (4) $x^2-8x-25=0$

9. If $\frac{1-i}{1+i}$ is a root of $ax^2+bx+1=0$, where a, b are real then (a, b) is

(1) (1, 1) (2) (1, -1) (3) (0, 1) (4) (1, 0)

10. Polynomial equation P(x) = 0 admits conjugate pairs of roots only if the coefficients are

(1) imaginary (2) complex (3) real (4) either real or complex

PART-B: ANSWER ANY 5 QUESTIONS :(5X2=10)

11. Construct a cubic equation with roots 1, 1, and -2

12. Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$ as a root.

13. Solve the equation: $x^4 - 14x^2 + 45 = 0$

14. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$

15. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

16. Find the roots of $2x^3 + 3x^2 + 2x + 3 = 0$

PART-C: ANSWER ANY 5 QUESTIONS: (5X3=15)

17. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

18. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.

19. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

20.Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

21. Solve the equations $x^4 + 3x^3 - 3x - 1 = 0$

22. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, if one of its roots is $2 + \sqrt{3}i$

PART-D: ANSWER ANY 3 QUESTIONS:(3X5=15)

23. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

24. If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

25. Solve the equation $(x-2)(x-7)(x-3)(x+2) + 19 = 0$

26.Solve the equations $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

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4. INVERSE TRIGONOMETRICAL FUNCTIONS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10 x 1=10)

1. The value of $\sin^{-1}(\cos x),\ 0 \le x \le \pi$ is

(1) $\pi - x$ (2) $x - \frac{\pi}{2}$ (3) $\frac{\pi}{2} - x$ (4) $\pi - x$

2. If $\sin^{-1} x = 2\sin^{-1}\alpha$ has a solution , then

(1) $|\alpha| \le \frac{1}{\sqrt{2}}$ (2) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

3. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1}$ x is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

4. If $x = \frac{1}{5}$, the value of $\cos\left(\cos^{-1} x + 2\sin^{-1} x\right)$ is

(1) $-\sqrt{\frac{24}{25}}$ (2) $\sqrt{\frac{24}{25}}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

5. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

6. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle , then the third angle is

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

7. $\sin^{-1}\left(2\cos^2 x-1\right)+\cos^{-1}\left(1-2\sin^2 x\right)=$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

8. If $|x|\le 1$ then $2\tan^{-1}x-\sin^{-1}\frac{2x}{1+x^2}$ is equal to

(1) $\tan^{-1}x$ (2) $\sin^{-1}x$ (3)0 (4) π

9. If $\sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$, then x is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{3}}{2}$

10. $\sin\left(\tan^{-1}x\right),|x|<1$ is equal to

(1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

PART-B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. Find the principal value of $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

12. Find the period of amplitude of $y=\sin 7x$.

13. Find $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

14. Find the domain of $f(x)=\sin^{-1}(x)+\cos^{-1}(x)$

15. Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

16. If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$

PART-C: ANSWER ANY 5 QUESTIONS: (5X3=15)

17. Sketch the graph of $y=\sin\left(\frac{1}{3}x\right)$ for $0\le x<6\pi$

18. Find value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}+\cos\frac{\pi}{7}\sin\frac{5\pi}{17}\right)$

19. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x$, $|x|>1$

20. Find the value of $\cos\left[\frac{1}{2}-\cos^{-1}\left(\frac{1}{8}\right)\right]$

21. Show that $\cot(\sin^{-1} x)=\frac{\sqrt{1-x^2}}{x}, -1\le x\le 1$ and $x\neq 0$

22. Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

PART-D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Find the domain of $f(x)=\sin^{-1}\left(\frac{x^2+1}{2x}\right)$.

24. If $a_1, a_2, a_3, ...a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+......+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$$

25. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

26. Simplify: $\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{x-y}{x+y}\right)$

27. Solve $2\tan^{-1}(x)=\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right), a<0, b>0$

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MATHEMATICS

5. TWO DIMENSIONAL ANALYTICAL GEOMETRY

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10X1=10)

1. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

2. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is
(1)1 (2)3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

3. If $P(x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2 is
(1)8 (2)6 (3)10 (4)12

4. The area of quadrilateral formed with foci of the hyberbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is
(1) $4(a^2+b^2)$ (2) $2(a^2+b^2)$ (3) a^2+b^2 (4) $\frac{1}{2}(a^2+b^2)$

5. If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of K is
(1)3 (2)-1 (3)1 (4)9

6. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is
(1)2ab (2)ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

7. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is
(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

8. The locus of a point whose distance from (-2, 0) is $\frac{2}{3}$ times its distance from the line $x=\frac{-9}{2}$ is
(1) a parabola (2)a hyperbola (3)an ellipse (4)a circle

9. The axis of the parabola $y^2=4x$ is
(1)x = 0 (2) y = 0 (3) x = 1 (4) y = 1

10. The eccentricity of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is
(1) $\sqrt{5}/3$ (2) $\sqrt{3}/5$ (3) 3 / 5 (4) 2 / 3

PART-B: ANSWER ANY 5 QUESTIONS :(5X2=10)

11. Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.

12. Find the Centre and radius of $x^2+y^2+6x-4y+4=0$.

13. Find the equation of the parabola with focus $(4,0)$ and directrix $x=-4$.

14. Find the length of latus rectum of the parabola $y^2=4ax$.

15. Find the equation of hyperbola with foci $(\pm 2,0)$, $e=\frac{3}{2}$.

16. Find the equation of tangent at t=2 to the parabola $y^2=8x$.

PART-C: ANSWER ANY 5 QUESTIONS :(5X3=15)

17. Find the equation of the circle with centre $(2,3)$ and passing through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$.

18. Find the equation of the hyperbola with vertices $(0,\pm4)$ and foci $(0,\pm6)$.

19. Find the vertex, focus, equation of directrix and length of latus rectum of $y^2 = 16x$.

20. Identify the type of conic and find center, foci, vertices and directrices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$.

21. Find the equation of tangent to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which is parallel to $10x - 3y + 9 = 0$.

22. The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex.

PART-D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Find the equation of the circle passing through the points $(1,1),(2,-1),(3,2)$.

24. Find the vertex, focus, directrix and length of the lotus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$.

25. Identify the type of conic and find center, foci,vertices and directrix of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$.

26. A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre on either sides.

27. A rod of length 1.2m moves with its end always touching the coordinates axis. The locus of a point P on the rod,which is 0.3m from the end in contact with x-axis in an ellipse find the eccentricity.

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MATHEMATICS

6. VECTOR ALGEBRA (6.1-6.3)

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10X1=10)

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a},\vec{c},\vec{b}]$ is equal to

(1)2 (2)-1 (3)1 (4)0

2. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is

(1) $|\vec{a}||\vec{b}||\vec{c}|$ (2) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (3) 1 (4) -1

3. If $\left[\vec{a},\vec{b},\vec{c}\right]=1$, then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}}+\frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}}+\frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

(1) 1 (2) -1 (3) 2 (4) 3

4. If $\vec{a}$ and $\vec{b}$ are the unit vectors such that $\left[\vec{a},\vec{b},\vec{a}\times\vec{b}\right]=\frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

5. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar , non-zero vectors such that $\left[\vec{a},\vec{b},\vec{c}\right]=3$, then $\left\{\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]\right\}^2$ is equal to

(1) 81 (2) 9 (3) 27 (3) 18

6. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors such that $\vec{a}\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$

7. If $\vec{u}=\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})$, then

(1) $\vec{u}$ is a unit vector (2) $\vec{u}=\vec{a}+\vec{b}+\vec{c}$ (3) $\vec{u}=\vec{0}$ (4) $\vec{u}\neq\vec{0}$

8. If $\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]=64$ then $\left[\vec{a},\vec{b},\vec{c}\right]$ is (1) 32 (2) 8 (3) 128

9. The value of $\left[\vec{i}+\vec{j},\vec{j}+\vec{k},\vec{k}+\vec{i}\right]$ is equal to (1) 0 (2) 1 (3) 2 (4) 4

10. The vector $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$ is

(1) Perpendicular to $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$

(2) Parallel to the vectors $(\vec{a}\times\vec{b})$ and $(\vec{c}\times\vec{d})$

(3) Parallel to the line of intersection of the plane containing $\vec{a}$ and $\vec{b}$ and the plane containing $\vec{c}$ and $\vec{d}$

(4) Perpendicular to the line of intersection of the plane containing $\vec{a}$ and $\vec{b}$ and the plane containing $\vec{c}$ and $\vec{d}$

PART- B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. A particle acted upon by constant forces $2\hat{i}+5\hat{j}+6\hat{k}$ and $-\hat{i}-2\hat{j}-\hat{k}$ is displaced from the point $(4,-3,-2)$ to the point $(6,1,-3)$. Find the total work done by the forces.

12. If $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

13. The volume of the parallel piped whose cortex minus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$, $-3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of λ .

14. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

15. If $\hat{a}=-3\hat{i}-\hat{j}+5\hat{k}$, $\hat{b}=\hat{i}-2\hat{j}+\hat{k}$, $\hat{c}=4\hat{j}-5\hat{k}$ find $\hat{a}.\left(\hat{b}\times\hat{c}\right)$.

16. If $\hat{a},\hat{b},\hat{c}$ are three vectors prove that $\left[\hat{a}+\hat{c},\hat{a}+\hat{b},\hat{a}+\hat{b}+\hat{c}\right]=\left[\hat{a},\hat{b},\hat{c}\right]$.

PART- C: ANSWER ANY 5 QUESTIONS :(5X3=15)

17. Prove by vector method then an angle in a semi-circle is a right-angle.
18. Prove by vector method that the area of the quadrilateral ABCD having diagonals AC and BD, $\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$.
19. Find the attitude of a parallelepiped determine by the vector $\hat{a}=-2\hat{i}+5\hat{j}+3\hat{k}$, $\hat{b}=\hat{i}+3\hat{j}-2\hat{k}$ and $\hat{c}=-3\hat{i}+\hat{j}+4\hat{k}$ if the base is taken as the parallelogram determined by $\hat{b}$ and $\hat{c}$.
20. Prove that $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$
21. For any vector $\vec{a}$ prove that $\vec{i}\times\left(\vec{a}\times\vec{i}\right)+\vec{j}\times\left(\vec{a}\times\vec{j}\right)+\vec{k}\times\left(\vec{a}\times\vec{k}\right)=2\vec{a}$.
22. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=l\vec{a}+m\vec{b}+n\vec{c}$ Find the value of l, m, n

PART -D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.
24. Prove that vector method that the perpendicular (attitudes) from the vertices to the opposite sides of a triangle is concurrent.
25. If $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$, verify that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\left[\vec{a},\vec{b},\vec{d}\right]\vec{c}-\left[\vec{a},\vec{b},\vec{c}\right]\vec{d}$.
26. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{c}=-\hat{i}-2\hat{j}+3\hat{k}$ verify that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}$.

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MATHEMATICS

6. VECTOR ALGEBRA (6.4-6.9)

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10X1=10)

1. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}, \frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

2. The angle between the line $\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+t(2\hat{i}+\hat{j}-2\hat{k})$ meets the plane $\vec{r}.(\hat{i}+\hat{j})+4=0$ is

(1) $0°$ (2) $30°$ (3) $45°$ (4) $90°$

3. Distance from the origin to the plane $3x-6y+2z+7=0$ is

(1) 0 (2)1 (3)2 (4)3

4. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

5. If the distance of the point (1,1,1)from the origin is half its distance from the plane $x+y+z+k=0$, then the value of k are

(1) ± 3 (2) ± 6 (3)-3,9 (4)3,-9

6. If the length of the perpendicular from the origin to the plane $2x+3\lambda+\lambda z=1, \lambda>0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3)0 (4)1

7. The shortest distance of the point (2, 10, 1) from the plane $\vec{r}\cdot(3\vec{i}-\vec{j}+4\vec{k})=2\sqrt{26}$ is

(1) $2\sqrt{26}$ (2) $\sqrt{26}$ (3) 2 (4) $\frac{1}{\sqrt{26}}$

8. $\vec{r}=s\vec{i}+t\vec{j}$ is the equation of

(1)a straight line joining the points $\vec{i}$ and $\vec{j}$ (2)xoy plane
(3)yoz plane (4)zox plane

9. The point of intersection of the line $\vec{r}=(\vec{i}-\vec{k})+t(3\vec{i}+2\vec{j}+7\vec{k})$ and the plane $\vec{r}\cdot(\vec{i}+\vec{j}-\vec{k})=8$ is

(1) (8, 6, 22) (2) (-8, - 6, -22) (3) (4, 3, 11) (4) (-4, -3, -11)

10. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is,

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2\sqrt{6}}$

PART- B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. Find the acute angle between the lines $\vec{r}=(\hat{i}+2\hat{j}+4\hat{k})+t(2\hat{i}+2\hat{j}+\hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.
12. Show that the lines $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ and $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ are parallel.
13. Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$.
14. Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=5$.
15. Find the distance between the planes $\vec{r}.(2\hat{i}-\hat{j}-2\hat{k})=6$ and $\vec{r}.(6\hat{i}-3\hat{j}-6\hat{k})=27$.
16. Find the angle between the planes $\vec{r}.(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$.

PART- C: ANSWER ANY 5 QUESTIONS :(5X3=15)

17. The vector equation in parametric form of a line is $\vec{r}=(3\hat{i}-2\hat{j}+6\hat{k})+(2\hat{i}-\hat{j}+3\hat{k})$ Find (i) The direction cosines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equation of the line.
18. The vertices of ΔABC are A$(7,2,1)$, B$(6,0,3)$ and C$(4,2,4)$ Find $\angle ABC$.
19. If the two lines $\frac{x-1}{2}=\frac{y+1}{2}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point. Find the value of m.
20. Find the direction cosines of the normal to the plane and length of the perpendicular form the origin to the plane $\vec{r}.(3\hat{i}-4\hat{j}+12\hat{k})=5$.
21. Find the direction cosines of the normal to the plane $12x+3y-4z=65$. Also find the non-parametric form of vector equation of a plane and the length of the perpendicular to the plane from the origin.
22. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(2\hat{i}-7\hat{j}+4\hat{k})=3$, $3x-5y+4z+11=0$ and the point $(-2,1,3)$.

PART -D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Find the shortest distance between the two given straight line $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+t(-2\hat{i}+\hat{j}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$.
24. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also find the equation of the perpendicular.

25. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$.
26. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2)$, $(-1,-2,6)$ and $(6,-4,-2)$.

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MATHEMATICS

7. APPLICATIONS OF DIFFERENTIAL CALCULUS

MARKS: 50 **TIME: 1.30 HRS**

I. CHOOSE THE BEST ANSWER: 10 X 1 = 10

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3/\sec$. The rate of change of its radius when radius is $\frac{1}{2}cm$

   (1)3 cm/s (2)2 cm/s (3) 1 cm/s (4) $\frac{1}{2}cm/s$
2. The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangents is -0.25?

   (1) -8 (2) -4 (3) -2 (4) 0
3. What is the value of the limit $\lim_{x\to 0}\left(\cot x-\frac{1}{x}\right)$?

   (1) 0 (2)1 (3) 2 (4) ∞
4. The number given by the mean value theorem for the function $\frac{1}{x}, x\in[1,9]$ is

   (1) 2 (2)2.5 (3) 3 (4)3.5
5. The maximum value of the function $x^2e^{-2x}, x>0$ is

   (1) $\frac{1}{e}$ (2) $\frac{1}{2e}$ (3) $\frac{1}{e^2}$ (4) $\frac{4}{e^2}$
6. The point of inflection of the curve $y=(x-1)^3$ is

   (1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

7. One of the closet point on the curve $x^2 - y^2 = 4$ to the point (6,0) is

(1) (2, 0) (2) $(\sqrt{5},1)$ (3) $(3,\sqrt{5})$ (4) $(\sqrt{13},-\sqrt{3})$

8. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(1) 2 (2)2.5 (3)3 (4)3.5

9. The gradient of the curve $y = -2x^3 + 3x + 5$ at $x = 2$ is

(1) -20 (2) 27 (3) -16 (4) -21

10. $\lim_{x \to \infty} \frac{x^2}{e^x}$ is =

(1) 2 (2) 0 (3) ∞ (4) 1

II. ANSWER ANY 5 QUESTIONS: 5 X 2 = 10

11. Find the slope of the tangent to the curve $y = x^4 + 2x^2 - x$ at x =1.
12. Explain why Rolle's Theorem is not applicable to the following function in the respectiveintervals. $f(x) = \tan x$, $x \in [0, \pi]$
13. Evaluate: $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$
14. Find the asymptotes of the function $f(x) = \frac{1}{x}$
15. Write the Maclaurin series expansion of e^x
16. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$

III.ANSWER ANY 5 QUESTIONS: 5 X 3 = 15

17. The temperature T in Celsius in a long rod of length 10 m, insulated at both ends, is a function of Length$x$ given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of therod is zero
18. Find the tangent and normal to $y = x \sin x$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$
19. Expand sin x in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms
20. Find the absolute extrema of the function $f(x) = 3cosx$ on the closed interval $[0, 2\pi]$
21. Determine the intervals of concavity of the curve $y = 3 + sinx$
22. Find the intervals of monotonicity and hence find the local extrema for the function $f(x) = x^{\frac{2}{3}}$

IV. ANSWER ANY 3 QUESTIONS: 3 X 5 = 15

23. A police jeep, approaching an orthogonal intersection from the northern direction, is chasinga speeding car that has turned and moving straight east. When the jeep is 0.6 km north of theintersection and the car is 0.8 km to the east. The police determine with a radar that the distancebetween them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at theinstant of measurement, what is the speed of the car?
24. Prove that $x^2 + 4y^2 = 8$ andthe hyperbolaintersect orthogonally $x^2 - 2y^2 = 4$
25. Find the dimensions of the largest rectangle that can be inscribed in a semi-circle of radius*r* cm
26. Evaluate $\lim_{x \to 0^+} (\cos x)^{\frac{1}{x^2}}$

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MATHEMATICS

CH 8: DIFFERENTIALS AND PARTIAL DERIVATIVES

MARKS: 50 **TIME: 1.30 HRS**

I. CHOOSE THE BEST ANSWER: 10 X 1 = 10

1. A circular template has a radius of 10 cm. The measurement of radius has an approximateerror of 0.02 cm. Then the percentage error in calculating area of this template is
   (1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%
2. If $v(x,y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
   (1) $e^x + e^y$ (2) $\frac{1}{e^x + e^y}$ (3) 2 (4) 1
3. If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
   (1) xye^{xy} (2) $(1+xy)e^{xy}$ (3) $(1+y)e^{xy}$ (4) $(1+x)e^{xy}$
4. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is
   (1) $12x_0 + dx$ (2) $12x_0 dx$ (3) $6x_0 dx$ (4) $6x_0 + dx$

5. If $g(x,y)=3x^2-5y+2y^2, x(t)=e^t$ and $y(t)=\cos t, then \dfrac{dr}{dt}$ is equal to

(1) $6e^{2t}+5\sin t-4\cos t\sin t$ (2) $6e^{2t}-5\sin t-4\cos t\sin t$

(3) $3e^{2t}+5\sin t-4\cos t\sin t$ (4) $3e^{2t}-5\sin t-4\cos t\sin t$

6. If $f(x)=\dfrac{x}{x+1}$, then its differential is given by

(1) $\dfrac{-1}{(x+1)^2}dx$ (2) $\dfrac{1}{(x+1)^2}dx$ (3) $\dfrac{1}{x+1}dx$ (4) $\dfrac{-1}{x+1}dx$

7. Linear approximation for $g(x)=\cos x$ at $x=\dfrac{\pi}{2}$ is

(1) $x+\dfrac{\pi}{2}$ (2) $-x+\dfrac{\pi}{2}$ (3) $x-\dfrac{\pi}{2}$ (4) $-x-\dfrac{\pi}{2}$

8. If u = $\dfrac{1}{\sqrt{x^2+y^2}}$, then x $\dfrac{\partial u}{\partial x}$ + y $\dfrac{\partial u}{\partial y}$ is equal to

(1) $\frac{1}{2}$u (2) u (3) $\frac{3}{2}$u (4) - u

9. If $u(x,y)=x^4+y^3+3x^2y^2+3x^2y$ then $\dfrac{\partial^2 u}{\partial y^2}$ is

(1) $6y+6x^2$ (2) $12xy-6x$ (3) $12x^2y-6x$ (4) $3y^2+6x^2y+3x^2$

10. If $f(x,y,z)=xy+yz+zx$, then f_x-f_2 is equal to

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

II. ANSWER ANY 5 QUESTIONS: 5 X 2 = 10

11. Use the linear approximation to find approximate values of $(123)^{\frac{2}{3}}$

12. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

13. Evaluate $\lim\limits_{(x,y)\to(1,2)} g(x,y)$, if limit exists, where $g(x,y)=\dfrac{3x^2-xy}{x^2+y^2+3}$

14. Determine whether the function $h(x,y)=\dfrac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$ is homogeneous or not. If it is, so find the degree.

15. If $v(x,y)=x^2-xy+\dfrac{1}{4}y^2+7,\quad x,y\in R$ find the differential dv

16. Find Δf and df for the function $f(x)=x^3-2x^2$ at $x=2, \Delta x=dx=0.5$ and compare

III. ANSWER ANY 5 QUESTIONS: 5 X 3 = 15

17. The time T, taken for a complete oscillation of a single pendulum with length l , is given by the equation $T=2\pi\sqrt{\dfrac{l}{g}}$,where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

18. Let $f, g:(a,b) \to R$ be differentiable functions. Show that $d(fg) = fdg + gdf$

19. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$

20. Let $F(x, y) = x^3y + y^2x + 7$ for all $(x, y) \in R^2$. Calculate $\frac{\partial F}{\partial x}(-1,3)$ *and* $\frac{\partial F}{\partial y}(-2,1)$

21. Let $u(x, y) = e^{-2y}\cos(2x)$ for all $(x, y) \in R^2$. Prove that u is harmonic function in R^2

22. Let $g(x, y) = x^2 - yx + \sin(x + y), x(t) = e^{3t}, y(t) = t^2, t \in R$ find $\frac{dg}{dt}$

IV. ANSWER ANY 3 QUESTIONS: 3 X 5 = 15

23. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

24. Let $z(x, y) = x^3 - 3x^2y^3, x = se^t, y = se^{-t}$ $s,t \in R$, find $\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}$

25. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm.

    (i) Approximately, how much did the tree's diameter grow?

    (ii) What is the percentage increase in area of the tree's cross-section?

26. If $f(x, y) = \frac{3x}{y + \sin x}$, find f_x, f_y and show that $f_{xy} = f_{yx}$

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MATHEMATICS

9. APPLICATIONS OF INTEGRAL CALCULUS

MARKS: 50 TIME: 1.30 Hrs

I. CHOOSE THE BEST ANSWER: 10 X 1 = 10

1. The value of $\int_{-1}^{2}|x|dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

2. The value of $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x dx$ is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 0 (4) $\frac{2}{3}$

3. If $f(x) = \int_0^x t \cos t dt$, then $\frac{df}{dx}$ is

(1) $\cos X - X \sin X$ (2) $\sin X + X \cos X$ (3) $X \cos X$ (4) $X \sin X$

4. The value of $\int_0^1 x(1-x)^{99} dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

5. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$, then n is

(1) 10 (2) 5 (3)8 (4) 9

6. The value of $\int_0^{\pi} \sin^4 x^2 dx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

7. The value of $\int_0^{\infty} e^{-3x} x^2 dx$ is

(1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

8. The value of $\int_0^1 \left(\sin^{-1} x\right)^2 dx$ is

(1) $\frac{\pi^2}{4} - 1$ (2) $\frac{\pi^2}{4} + 2$ (3) $\frac{\pi^2}{4} + 1$ (4) $\frac{\pi^2}{4} - 2$

9. The value of $\int_0^{\pi} \sin^4 x dx$ is

(1) $3\pi/16$ (2) 3 / 16 (3) 0 (4) $3\pi/8$

10. The area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between x = 0 and x = $\pi/4$ is.

(1) $\sqrt{2}+1$ (2) $\sqrt{2}-1$ (3) $2\sqrt{2}+1$ (4) $2\sqrt{2}+2$

II. ANSWER ANY 5 QUESTIONS: 5 X 2 = 10

11. Estimate the value of $\int_0^{0.5} x^2 \, dx$ using the Riemann sums corresponding to 5 subintervals of equal width and applying the mid-point rule

12. Evaluate $\int_0^1 [2x] \, dx$ where $[.]$ the greatest integer function

13. Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$

14. Evaluate $\int_0^{\frac{\pi}{2}} \sin^{10} x \, dx$

15. Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves $y = \log x$, $y = 0$, $x = 0$ and $y = 2$

16. Prove that: $\int_0^{\infty} \frac{1}{a^2 + x^2} dx = \frac{\pi}{2a}$

III. ANSWER ANY 5 QUESTIONS: 5 X 3 = 15

17. Evaluate $\int_0^{a} \frac{f(x)}{f(x) + f(a-x)} dx$

18. Evaluate $\int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{1+\cos x} \right) dx$

19. Find the volume of a sphere of radius *a*.

20. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x} dx$

21. Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum

22. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4\sin^2 x}$

IV. ANSWER ANY 3 QUESTIONS: 3 X 5 = 15

24. Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$

25. Find the volume of a right-circular cone of base radius r and height h

26. Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$

27. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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MATHEMATICS

10. DIFFERENTIAL EQUATIONS

MARKS: 50 **TIME: 1.30 Hrs**

I. CHOOSE THE BEST ANSWER: **10 X 1 = 10**

1. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{3}}+x^{\frac{1}{4}}=0$ are respectively

(1) 2, 3 (2) 3, 3 (3) 2,6 (4) 2,4

2. The order of the differential equation of all circle with center at (h, k) and radius 'a' is

(1) 2 (2) 3 (3) 4 (4) 1

3. The general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$ is

(1) $xy=k$ (2) $y=k\log x$ (3) $y=kx$ (4) $\log y=kx$

4. The solution of $\frac{dy}{dx}-p(x)y=0$ is

(1) $y=ce^{\int pdx}$ (2) $y=ce^{-\int pdx}$ (3) $x=ce^{-\int pdy}$ (4) $x=ce^{\int pdy}$

5. The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{\lambda}$ is

(1) $\frac{x}{e^{\lambda}}$ (2) $\frac{e^{\lambda}}{x}$ (3) λe^x (4) e^x

6. The degree of the differential equation $y(x)=1+\frac{dy}{dx}+\frac{1}{1.2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3+........$ is

(1) 1 (2) 3 (3) 1 (4) 4

7. The solution of the differential equation $\frac{dy}{dx}=2xy$ is

(1) $y=Ce^{x^2}$ (2) $y=2x^2+c$ (3) $y=Ce^{-x^2}+C$ (4) $y=x^2+C$

8. If sin X is the integrating factor of the linear differential equation $\frac{dy}{dx}+py=Q$, then P is

(1) log sin X (2) cos X (3)tan X (4) cot X

9. The population P in any year t is such that the rate of increase in the population is proportional

to the population then

(1) $p = Ce^{kt}$ (2) $p = Ce^{-kt}$ (3) $p = Ckt$ (4) $p = C$

10. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(1) 2 (2) -2 (3) 1 (4) -1

II. ANSWER ANY 5 QUESTIONS: 5 X 2 = 10

11. Find the differential equation of the family of parabolas $y^2 = 4ax$, where *a* is an arbitrary constant

12. Show that $y = ae^x + be^{-x}$ is a solution of the differential equation $y'' - y = 0$

13. Solve $\frac{dy}{dx} + 2y = e^{-x}$

14. Solve $\sin\left(\frac{dy}{dx}\right) = a$, $y(0) = 1$

15. Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$

16. Express the following physical statement in the form of differential equation: Radium decays at a rate proportional to the amount Q present

III. ANSWER ANY 5 QUESTIONS: 5 X 3 = 15

17. Find the differential equation of the family of circles passing through the points $(a,0)$ and $(-a,0)$

18. Solve $\left(e^y + 1\right)\cos x dx + e^y \sin x dy = 0$

19. Solve $x\frac{dy}{dx} + 2y - x^2 \log x = 0$

20. Solve $\left(ydx - xdy\right)\cot\left(\frac{x}{y}\right) = ny^2 dx$

21. Solve $(1-x^2)\frac{dy}{dx} - xy = 1$

22. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$

IV. ANSWER ANY 3 QUESTIONS: 3 X 5 = 15

23. Solve $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$

24. Solve $\left(1 + 3e^{\frac{y}{x}}\right)dy + 3e^{\frac{y}{x}}\left(1 - \frac{y}{x}\right)dx = 0$, given that $y = 0$ when $x = 1$

25. Solve $\frac{dy}{dx} = \frac{\sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3} y$

26. The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

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MATHEMATICS

11. PROBABILITY DISTRIBUTIONS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10X1=10)

1. A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x) = \begin{cases} \frac{1}{l} 0 < x < l \\ 0 l \le x < 2l \end{cases}$ The mean and variance of the shorter of the two pieces are respectively

(1) $\frac{1}{2}, \frac{l^2}{3}$ (2) $\frac{1}{2}, \frac{l^2}{6}$ (3) $1, \frac{l^2}{12}$ (4) $\frac{1}{2}, \frac{l^2}{12}$

2. A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of$X$ is

(1) 6 (2)4 (3)3 (4)4

3. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is

(1)0.11 (2) 1.1 (3) 11 (4)1

4. If $p\{X=0\} = 1 - p\{X=1\}$. If $E[X] = 3\text{var}(X)$, then $p\{X=0\}$

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$

5. The random variable X has the probability density function $F(x)=\begin{cases} ax+b & 0<x<1 \\ 0 & otherwise \end{cases}$ and $E(X)=\frac{7}{2}$, then a and b are respectively

(1) 1 and $\frac{1}{2}$ (2) $\frac{1}{2}$ and 1 (3) 2 and 1 (4) 1 and 2

6. The probability function of a random variable is defined as:

x	-2	-1	0	1	2
f(x)	k	2k	3k	4k	5k

Then E(X) is equal to: (1) $\frac{1}{15}$ (2) $\frac{1}{10}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

7. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is

(1) 0.24 (2) 0.48 (3) 0.6 (4) 0.96

8. Variance of the random variable X is 4. Its mean is 2. Then $E(X^2)$ is

(1) 2 (2) 4 (3) 6 (4) 8

9. If X is a discrete random variable then $P(X \geq a)=$

(1) $P(X<a)$ (2) $1-P(X \leq a)$ (3) $1-P(X<a)$ (4) 0

10. Mean and variance of binomial distribution are

(1) nq, npq (2) np, $\sqrt{npq}$ (3) np, nq (4) np, npq

PART- B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images

12. Prove that $E(ax+b)=aE(x)+b$, where a and b are constants

13. Find the mean and variance of a random variable X, whose probability mass function is

$$f(x)=\begin{cases} \frac{4-x}{6}, & x=1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

14. Find the binomial distribution function for five fair coins are tossed once and X denotes the number of heads.

15. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

16. Using binomial distribution find the mean and variance of X for a fair coin is tossed 100 times, and X denotes the number of heads.

PART- C: ANSWER ANY 5 QUESTIONS :(5X3=15)

17. An urn contains 2 white balls and 3 red balls. Samples of 3 balls are chosen at random from theurn. If X denotes the number of red balls chosen, find the values taken by the random variable $X$and its number of inverse images

18. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find theprobability mass function for number of heads occurred

19. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{2}(x^2 + x) & , 0 \le x < 1 \\ 1 & , 1 \le x < \infty \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.3 \le X \le 0.6)$

20. Find the mean and variance of a random variable X , whose probability density function is

$$f(x) = \begin{cases} \frac{1}{2} e^{\frac{-x}{2}} & for \quad x > 0 \\ 0 & , \ otherwise \end{cases}$$

21. Four fair coins are tossed once. Find the probability mass function, mean and variance fornumber of heads occurred

22. The mean and variance of a binomial variate$X$ are respectively 2 and 1.5. Find

(i) $P(X = 0)$ (ii) $P(X = 1)$

PART -D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$

24. If X is the random variable with probability densityfunction $f(x)$ given by,

$$f(x) = \begin{cases} x - 1 & , 1 \le x < 2 \\ -x + 3 & , 2 \le x < 3 \\ 0 & , \ otherwise \end{cases}$$

Find (i) the distribution function $F(x)$ (ii) $P(1.2 \le X \le 2.5)$

25. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200liters and a maximum of 600 liters with probability density function $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & otherewise \end{cases}$

Find (i) the value of k (ii) the distribution function(iii) the probability that daily sales will fall between 300 liters and 500 liters?

26. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find theprobabilities that among 12 such lights

(i) Exactly 10 will have a useful life of at least 600 hours;

(ii) At least 11 will have a useful life of at least 600 hours;

(iii) At least 2 will *not* have a useful life of at least 600 hours

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MATHEMATICS

12. DISCRETE MATHEMATICS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10X1=10)

1. Subtraction is not a binary operation in

(1) $\mathbb{R}$ (2) $\mathbb{Z}$ (3) $\mathbb{N}$ (4) $\mathbb{Q}$

2. The operation * defined $a*b=\frac{ab}{7}$ is not a binary operation on

(1) $\mathbb{Q}^+$ (2) $\mathbb{Z}$ (3) $\mathbb{R}$ (4) $\mathbb{C}$

3. If $a*b=\sqrt{a^2+b^2}$ on the real numbers then * is

(1)commutative but not associative (2) associative but not commutative

(3) both commutative and associative (4) neither commutative nor associative

4. If a compound statement involves 3 simple statements, then the number of rows in the truth

table is

(1)9 (2) 8 (3) 6 (3) 3

5. Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$?

(1) $\neg r \rightarrow (\neg p \wedge \neg q)$ (2) $\neg r \rightarrow (p \wedge q)$

(3) $r \rightarrow (p \wedge q)$ (4) $p \rightarrow (q \vee r)$

6. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

(1) 1 (2) 2 (3) 3 (4) 4

7. which of the following is not true?

(1) Negation of a negation of a statement is the statement itself.

(2) If the last column of the truth table contains only T then it is a tautology.

(3) If the last column of its truth table contains only F then it is a contradiction

(4) If p and q are any two statements then $p \leftrightarrow q$ is a tautology

8. If p is true and q is unknown then

(1) $\sim p$ is true (2) $p \vee (\sim p)$ is false (3) $p \wedge (\sim p)$ is true (4) $p \vee q$ is true

9. Which of the following are binary operation on R?

i) $a*b = \min\{a,b\}$ ii) $a*b = \max\{a,b\}$ iii) $a*b = a$ iv) $a*b = b$

10. '$\div$' is a binary operation on (1) N (2) R (3) Z (4) $C - \{0\}$

PART- B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. Examine the binary operation (closure property) of the following operations on the respectivesets (if it is not, make it binary) $a * b = a + 3ab - 5b^2 \;\forall\; a, b \in z.$

12. Prove that i n an algebraic structure the identity element (if exists) must be unique

13. Determine whether $*$ is a binary operation on the set $(a * b) = a\sqrt{b}$ is binary onR

14. Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$

15. Determine the truth value of each of the following statements

(i) If 6 + 2 = 5, then the milk is white. (ii) China is in Europe or $\sqrt{3}$ is an integer

16. Construct the truth table for $\neg p \wedge \neg q$

PART- C: ANSWER ANY 5 QUESTIONS :(5X3=15)

17. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existenceof identity and (v) existence of inverse for the arithmetic operation $-$ on Z

18. Verify (i) closure property (ii) commutative property, and (iii) associative property of thefollowing operation on the given set. $(a*b)=a^b \ \forall \ a,b \in N$ (Exponentiation property)

19. Let $*$ be defined on Rby $(a*b)=a+b+ab-7$ is $*$ binary on R? If so , find $3*\left(\frac{-7}{15}\right)$

20. Define an operation $*$ on Q as follows: $a*b=\left(\frac{a+b}{2}\right); a,b \in Q$ Examine the existenceof identity and the existence of inverse for the operation $*$ on Q

21. Construct the truth table for $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$

22. Show that $\neg(p \rightarrow q) \equiv p \wedge \neg q$

PART -D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Let $M=\begin{pmatrix} x & x \\ x & x \end{pmatrix}: x \in R-\{0\}$ and let $*$ be the matrix multiplication. Determinewhether M is closed under $*$. If so, examine the commutative,associative, the existence of identity, existence ofinverse properties for the operation $*$ on M

24. Let A be $Q \setminus \{1\}$ .Define $*$ on A by $x*y=x+y-xy$. Is $*$ binary on A? If so,examine the commutative,associative, existence of identity, existence of inverse properties for the operation $*$ on A

25. Establish the equivalence property connecting the bi-conditional with conditional: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

26. Verify whether the $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow q)$ compound propositions are tautologies or contradictions orcontingency

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