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CHAPTER 7
APPLICATION OF DIFFERENTIAL CALCULUS

5 MARKS

1. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds? (a) At what time the particle is at rest? (b) At what time the particle changes direction? (c) Find the total distance travelled by the particle in the first 2 seconds. **Eg. 7. 6**
2. The price of a product is related to the number of units available (supply) by the equation $Px + 3P - 16x = 234$, where P is the price of the product per unit in Rupees(`) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week. **(EG 7.8)**
3. **(Two variable related rate problem):** A road running north to south crosses a road going east to west at the point P . Car A is driving north along the first road and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and travelling at 80 km/hr, while car B is 15 kilometres to the east of P and travelling at 100 km/hr. How fast is the distance between the two cars changing? **(EG 7.10)**
4. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
   (i) At what times the particle changes direction?
   (ii) Find the total distance travelled by the particle in the first 4 seconds.
   (iii) Find the particle's acceleration each time the velocity is zero. **Ex. 7.1 – 1 June'23**
5. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   `(iii) What is the instantaneous velocity of the camera when it hits the ground? **Ex. 7.1 - 2**
6. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
   (i) At what times the particle changes direction?
   (ii) Find the total distance travelled by the particle in the first 4 seconds.
   (iii) Find the particle's acceleration each time the velocity is zero. **(EX 7.1 – 3)**
7. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 $cubic\ m/min$, how fast is the depth of the water increases when the water is 8 metres deep? **Ex. 7.1 - 8**
8. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall. (i) How fast is the top of the ladder moving down the wall? **Ex. 7.1 - 9**
   (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?
9. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? **Ex. 7.1 - 10**
10. Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t$, $t \in \mathbb{R}$ at any point on the curve. **(EX 7.2 – 8)**
11. Find the acute angle between $y = x^2$ and $y = (x-3)^2$. **Eg. 7. 14**
12. If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. **(EG 7.17)**
13. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally. **(EG 7.18)**

14. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. **Ex. 7.7 - 3**
15. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume? **Eg. 7. 62**
16. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from $(1,1)$. **Eg. 7. 63**
17. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire. **Ex. 7.8 - 4**
18. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum. **Ex. 7.8 - 5**
19. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 $sq.mtrs$ in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material? **Ex. 7.8- 6**
20. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm. **Ex. 7.8 - 7**
21. A manufacturer wants to design an open box having a square base and a surface area of 108 $sq.cm$. Determine the dimensions of the box for the maximum volume. **Ex. 7.8 - 10**
22. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone **Ex. 7.8 - 11**.
23. Find the maximum value of $\frac{\log x}{x}$ (*creative*) **March '23**
24. Show that the volume of largest right circular cone that can be inscribed in a spare of radius 'a' is $\frac{8}{27}$(volume of the sphere) **(creative)**
25. Te top and bottom margins of a poster are each 6 cms. and the margins are each 4 cms. If the area of printed material on the poster is fixed at 384 cms^2, find the dimension of the poster with the smallest area. **(creative)**
26. Prove that the sum of intercepts on the co-ordinate axes of any tangent to the curve $x = a\,cos^3\theta; y = a\,sin^3\theta$ at θ is $x\,cos\theta - y\,sin\theta = a\cos 2\theta$ **(creative)**
27. If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $8k^2 = 1$. **(creative)**

CHAPTER 8
DIFFERENTIALS AND PARTIAL DERIVATIVES

1. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate: (i) Absolute error (ii) Relative error (iii) Percentage error **Ex. 8. 1– 4**
2. Find the f_x, f_y and show that $f_{xy} = f_{yx}$. (i) $f(x,y) = \frac{3x}{y+\sin x}$ **(EX 8. 4 – 2)**
3. Find the f_x, f_y and show that $f_{xy} = f_{yx}$. (ii) $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$ **(EX 8. 4 – 2)**
4. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. **(EG 8.22)** **June '23**
5. If $v(x,y) = \log\left(\frac{x^2+y^2}{x=y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$. **(EX 8.7 – 5)**
6. If $w(x,y,z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$. **(EX 8.7 – 6)**
7. Using Euler's theorem $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.**(creative)**
8. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ if $u = \frac{x}{y^2} - \frac{y}{x^2}$**(creative)**
9. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ if $u = sin\,3x\cos 4y$**(creative)**

Chapter 9
Applications of Integration

1. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$(**EX 9.3 – 2)**
2. Find the area of the region bounded between the parabolas $y^2 = 4x$and $x^2 = 4y$. **Eg. 9.54**
3. Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$. **Eg. 9.56**
4. The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$. Find the area of the smaller segment. **Eg. 9.57**
5. Find the area of the region in the first quadrant bounded by the parabola $y^2 = 4x$, the line $x + y = 3$ and $y -$ axis. **Eg. 9.58**
6. Find, by integration, the area of the region bounded by the lines $5x - 2y = 15, x + y + 4 = 0$ and the x –axis **Eg. 9.59. June '23**
7. Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are $(-1,1), (3, 2)$, and $(0,5)$ respectively. **Eg. 9.60**
8. Using integration, find the area of the region which is bounded by x –axis, the tangent and normal to the circle $x^2 + y^2 = 4$ drawn at $(1, \sqrt{3})$. **Eg. 9.61**
9. Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x –axis, $x = -3$ and $x = 3$. **Ex. 9. 8– 3**
10. Find the area bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$. **Ex. 9. 8– 4**
11. Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$. **Ex. 9. 8– 5**
12. Find the area of bounded by $y = \tan x$, $y = \cot x$ and the lines $x = 0$, $x = \frac{\pi}{2}$, $y = 0$. **Ex. 9. 8– 6**
13. Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$. **Ex. 9. 8– 7**
14. Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them. **Ex. 9. 8– 8**
15. The curve $y = (x - 2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ. **Ex. 9. 8– 9**
16. Find the area of the common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$. **Ex. 9. 8– 10**
17. Find the area of the region common to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$**(Creative) March '23**
18. Find the common area enclosed by the parabolas $y^2 = x$ and $x^2 = y$. **(Creative)**

Chapter 10
ORDINARY DIFFFERNTIAL EQUAIONS

1. Solve $\frac{dy}{dx} = \tan^2(x + y)$ **(EX 10.5 – 4 x)**
2. If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Express V in terms of t given that $V = 0$ when $t = 0$. **(EX 10.5 – 1)**
3. Solve $(y^2 - 2xy)dx = (x^2 - 2xy)dy$ **(EX 10.6 – 5)**
4. Solve $(x^2 + y^2)dy = xydx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0. **(EX 10.6 – 8)**
5. Solve: $(1 + x^3)\frac{dy}{dx} + 6x^2y = 1 + x^2$. **(EG 10.25)**
6. $(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$ **(EX 10.7 – 4)**
7. $\left(y - e^{\sin^{-1}x}\right)\frac{dx}{dy} + \sqrt{1 - x^2} = 0$ **(EX 10.7 – 7)**
8. The growth of a population is proportional to the number present.
9. If the population of a colony doubles in 50 years, in how many years will the population become triple? **(EG 10.27)**
10. A radioactive isotope has an initial mass 200 mg, which two years later is 50 mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life

means the time taken for the radioactivity of a specified isotope to fall to half its original value). **(EG 10.28)**

11. In a murder investigation, a corpse was found by a detective at exactly 8 $p.m$. Being alert, the detective also measured the body temperature and found it to be $70°F$. Two hours later, the detective measured the body temperature again and found it to be $60°F$. If the room temperature is $50°F$, and assuming that the body temperature of the person before death was $98.6°F$, at what time did the murder occur? **(EG 10.29)**
12. A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t. **(EG 10.30)**
13. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours? **(EX 10.8 – 1)**
14. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. **(EX 10.8 – 2)**
15. The equation of electromotive force for an electric circuit containing resistance and selfinductance is $E = Ri + L\frac{di}{dt}$, where E is the lectromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$. **(EX 10.8 – 3)**
16. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years? **(EX 10.8 – 6)**
17. Water at temperature $100°C$ cools in 10 minutes to $80°C$ in a room temperature of $25°C$. Find (i) The temperature of water after 20 minutes (ii) The time when the temperature is $40°C$. $\left(\log_e \frac{11}{15} = -0.3101, \log_e 5 = 1.6094\right)$ **(EX 10.8 – 7)**
18. A pot of boiling water at $100°C$ is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to $80°C$, and another 5 minutes later it has dropped to $65°C$. Determine the temperature of the kitchen. **(EX 10.8 – 9)**
19. A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$. **(EX 10.8 – 10)**
20. The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal? $(log 2 = 06931)$ **(creative)**
21. Radium disappears at the rate proportional to the amount present. 5% of the amount disappears in 50 years, how much will remain at the end of 100 years *(Take A_0 as the initial amount)* **(creative)**

CHAPTER 11
PROBABILITY DISTRIBUTIONS LAPLACE

1. A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws. (i) Find the probability mass function. (ii) Find the cumulative distribution function. (iii) Find $P(3 \leq X < 6)(iv)$ Find $P(X \geq 4)$. **(EG 11.8)**
2. A random variable X has the following probability mass function.

x	1	2	3	4	5	6

$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X \le 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$ **(EG 11.10)**

3. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii) the cumulative distribution function (iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$ **(EX 11.2 – 2)**
4. If X is the random variable with probability density function $f(x)$ given by, $f(x) = \begin{cases} x-1, & 1 \le x < 2 \\ -x+3, & 2 \le x < 3 \\ 0, & Otherwise \end{cases}$
   find (i) the distribution function $F(x)$ (ii) $P(1.5 \le X \le 2.5)$ **(EG 11.12)**
5. The probability density function of random variable X is given by
   $f(x) = \begin{cases} k, & 1 \le x \le 5 \\ 0, & Otherwise \end{cases}$. Find (i) Distribution function
   (ii) $P(X < 3)$ (iii) $P(2 < X < 4)$ (iv) $P(3 \le X)$ **(EG 11.14)**
6. Let X be a random variable denoting the life time of an electrical equipment having probability density function $f(x) = \begin{cases} ke^{-2x}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$
   Find (i) the value of k (ii) Distribution function (iii) $P(X < 2)$
   (iv) calculate the probability that X is at least for four unit of time (v) $P(X = 3)$. **(EG 11.15)**
7. The probability density function of X is given by $f(x) = \begin{cases} ke^{\frac{-x}{3}}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$ Find
   (i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv) $P(5 \le X)$ (v) $P(X \le 4)$. **(EX 11.3 – 4)**
8. If X is the random variable with probability density function $f(x)$ given by,
   $f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ 0, & Otherwise \end{cases}$
   then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \le X \le 0.5)$ **(EX 11.3 – 5)**
9. Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

   Find (i) the value of c (ii) Mean and variance. **(EG 11.16)**
10. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
    (i) exactly 10 will have a useful life of at least 600 hours;
    (ii) at least 11 will have a useful life of at least 600 hours;
    (iii) at least 2 will not have a useful life of at least 600 hours. **(EX 11.5 – 6)**
11. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. **(EG 11.22)**

CHAPTER 12
DISCRETE MATHEMATICS

1. Verify (i) closure property, (ii) commutative property,(iii) associative property, (iv) existence of identity, and(v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5. **(EG 12.9)**
2. Verify (i) closure property, (ii) commutative property,(iii) associative property, (iv) existence of identity, and(v) existence of inverse for the operation X_{11} on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$. **(EG 12.10)**

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3. Define an operation$*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{a+b}{2}, a, b \in \mathbb{Q}$. Examine the closure, commutative,, associative ,examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(EX 12.1 – 5)**
4. Define an operation$*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{ab}{3}, a, b \in \mathbb{Q}$. Examine the closure, commutative,, associative ,examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(similar creative)**
5. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three Boolean matrices. Find $(i)\ A \vee B\ (ii)\ A \wedge B\ \ (iii)\ (A \vee B) \wedge C\ (iv)\ (A \wedge B) \vee C$. **(EX 12.1 – 8)**
6. Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M closed under $*$. If so, examine commutative and associative ,examine the existence of identity, existence of inverse properties for the operation $*$ on M . **(EX 12.1 – 9) June '23**
7. Let $M = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R} - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M closed under $*$. If so, examine commutative and associative ,examine the existence of identity, existence of inverse properties for the operation $*$ on M . **(similar creative)**
8. Let A be $\mathbb{Q}\backslash\{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so, examine the Commutative, examine the existence of identity, existence of inverse properties for the operation $*$ on A. **(EX 12.1 – 10)**
9. Let A be $\mathbb{Q}\backslash\{-1\}$. Define $*$ on A by $a * b = a + b + ab$. Is $*$ binary on A? If so, examine the Commutative, examine the existence of identity, existence of inverse properties for the operation $*$ on A. **(similar creative))**
10. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table. **(EX 12.2 – 15)**

3 MARKS

CHAPTER 7
APPLICATION OF DIFFERENTIAL CALCULUS

1. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$. (a) Compute the maximum height of the particle reached (b) What is the velocity when the particle hits the ground? **Eg. 7.5**
2. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second. At what rate the radius of the balloon changes when the radius is $7cm$? Also compute the rate at which the surface area changes. **Eg. 7.7**
3. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high **Eg. 7.9**
4. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water? **(EX 7.1 – 6)**
5. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore? **Ex. 7.1 - 7**
6. Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t, t \in \mathbb{R}$. **Eg. 7.13**
7. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection $(0,0), (1,1)$. **Eg. 7.15**

8. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$. **(EX 7.2 – 2)**
9. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$. **(EX 7.2 – 3)**
10. Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$. **(EX 7.2 – 9)**
11. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally. **Ex. 7.2 - 10**
12. Without actually solving show that the equation $x^4 + 2x^3 - 2 = 0$ has only one real root in the interval $(0,1)$. **(EG 7.22)**
13. Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots\dots\dots\dots + a_1 x + a_0$ there is a zero of the polynomial $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \dots\dots + a_1$. **(EG 7.23)**
14. Prove that there is a zero of the polynomial, $2x^3 - 9x^2 - 11x + 12$ in the interval $(2,7)$ given that 2 and 7 are the zeros of the polynomial $x^4 - 6x^3 - 11x^2 + 24x + 28$. **(EG 7.24)**
15. Find the values in the interval $(1, 2)$ of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$. **(EG 7.25)**
16. Suppose $f(x)$ is a differentiable function for all x with $f'(x) \le 29$ and $f(b) = 17$. What the maximum value is of $f(7)$? **(EG 7.27)**
17. Prove, using mean value theorem, that $|\sin\alpha - \sin\beta| \le |\alpha - \beta|, \alpha, \beta \in \mathbb{R}$. **(EG 7.28)**
18. A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from $-10°C$ to $100°C$. Show that the rate of change of temperature at some time t is $5°C$ per second. **(EG 7.29)**
19. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval: $(i)\ f(x) = x^3 - 3x + 2, x \in [-2,2]$ **(EX 7.3 – 4)**
20. Show that the value in the conclusion of the mean value theorem for
    $(i)\ f(x) = \frac{1}{x}$ on a closed interval of positive numbers $[a, b]$ is $\sqrt{ab}$.
    $(ii)\ f(x) = Ax^2 + Bx + C$ on any interval $[a, b]$ is $\frac{a+b}{2}$. **(EX 7.3 – 5)**
21. A race car driver is racing at 20^{th} km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours. **(EX 7.3 – 6)**
22. Suppose that for a function $f(x), f'(x) \le 1$ for all $1 \le x \le 4$. Show that $f(4) - f(1) \le 3$. **(EX 7.3 – 7)**
23. Does there exist a differentiable function $f(x)$ such that $f(0) = -1, f(2) = 4$ and $f'(x) \le 2$ for all x. Justify your answer. **(EX 7.3 – 8)**
24. Show that there lies a point on the curve $f(x) = x(x+3)e^{\frac{-\pi}{2}}, -3 \le x \le 0$ where tangent drawn is parallel to the x –axis. **(EX 7.3 – 9)**
25. Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$. **(EX 7.3 – 10)**
26. Expand $\log(1 + x)$ as a Maclaurin's series upto 4 non-zero terms for $-1 < x \le 1$. **(EG 7.30)**
27. Write the Taylor series expansion of $\frac{1}{x}$ about $x = 2$ by finding the first three non-zero terms. **(EG 7.32)**
28. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$. **(EX 7.4 –2)**
29. If $\lim\limits_{\theta\to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$. **(EG 7.37)**
30. Using the l'Hôpital Rule prove that, $\lim\limits_{x\to 0^+}(1+x)^{\frac{1}{x}} = e$. **(EG 7.43)**
31. Evaluate: $\lim\limits_{x\to 1} x^{\frac{1}{1-x}}$. **(EG 7.45)**
32. $\lim\limits_{x\to 0^+}(\cos x)^{\frac{1}{x^2}}$ **(EX 7.5 – 11)**

33. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0 + \left(1+\frac{r}{n}\right)^{nt}$ If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$. **(EX 7.5 – 12)**
34. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$. **Eg. 7.48**
35. Find the absolute extrema of the function $f(x) = 3\cos x$ on the closed interval $[0,2\pi]$. **(EG 7.49)**
36. Discuss the monotonicity and local extrema of the function $f(x) = \log(1+x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $\log(1+x) > \frac{x}{1+x}$. **(EG 7.53)**
37. Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$. **(EG 7.56)**
38. Determine the intervals of concavity of the curve $f(x) = (x-1)^3.(x-5), x \in \mathbb{R}$ and, points of inflection if any. **(EG 7.57)**
39. Find the local maximum and minimum of the function x^2y^2 on the line $x + y = 10$. **(EG 7.61)**
40. Prove that among all the rectangles of the given area square has the least perimeter. **Eg. 7.65**
41. Find two positive numbers whose sum is 12 and their product is maximum. **Ex. 7.8- 1 June '23**
42. Find two positive numbers whose product is 20 and their sum is minimum. **Ex. 7.8 - 2**
43. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$. **(EX 7.8 – 3)**
44. Prove that among all the rectangles of the given perimeter, the square has the maximum area. **Ex. 7.8 - 8**
45. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius $r\ cm$. **Ex. 7.8 – 9**
46. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if $r + h = 6$. **(EX 7.8 – 11)**
47. Find the asymptotes of the function $f(x) = \frac{1}{x}$. **(EG 7.66)**
48. Sketch the curve $= f(x) = x^2 - x - 6$. . **(EG 7.69)**

CHAPTER 8
DIFFERENTIALS AND PARTIAL DERIVATIVES

1. Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate $f(3.2)$. **(EG 8.1)**
2. Use the linear approximation to find approximate values of $(123)^{\frac{2}{3}}$**(EX 8.1 – 2) March '23**
3. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator. **(EG 8.2)**
4. Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error. **(EG 8.3)**
5. A right circular cylinder has radius $r = 10\ cm$. and height $h = 20\ cm$. Suppose that the radius of the cylinder is increased from10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error. **(EG 8.4)**
6. The time T , taken for a complete oscillation of a single pendulum with length l , is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l. **(EX 8.1 – 6)**
7. Find Δf and df for the function f for the indicated values of $x, \Delta x$ and compare
   (i) $f(x) = x^3 - 2x^2$; $x = 2$, $\Delta x = dx = 0.5$ **(EX 8.2 – 3)**
8. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$. **Ex. 8. 2 – 4**
9. The trunk of a tree has diameter 30 cm. During the following year, circumference grew 6 cm.
   (i) Approximately, how much did the tree's diameter grow?
   (ii) What is the percentage increase in area of the tree's cross-section? **Ex. 8. 2 – 5**

10. Consider $f(x,y) = \frac{xy}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is not continuous at $(0,0)$ and continuous at all other points of $\mathbb{R}^2$. **(EG 8.9)**
11. Let $g(x,y) = \frac{x^2y}{x^4+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$.
 (i) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = 0$ along every line $y = mx, m \in \mathbb{R}$.
 (ii) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = \frac{k}{1+k^2}$ along every parabola $y = kx^2, k \in \mathbb{R}\setminus\{0\}$. **(EX 8.3 – 5)**
12. Let $g(x,y) = \frac{e^y \sin x}{x}$, for $x \neq 0$ and $g(0,0) = 1$. Show that g is continuous at $(0,0)$. **(EX 8.3 – 7)**
13. Let for all $w(x,y) = xy + \frac{e^y}{y^2+1}$. for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$. **(EG 8.14)**
14. Let $u(x,y) = e^{-2y}\cos(2x)$ for all $(x,y) \in \mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$. **(EG 8.15)**
15. If $V(x,y) = e^x(x\cos y - y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$. **(EX 8.4 – 7)**
16. If $w(x,y) = xy + sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$ **(EX 8.4 – 8)**
17. A firm produces two types of calculators each week, x number of type A and y number of type B . The weekly revenue and cost functions (in rupees) are $R(x,y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$ and $C(x,y) = 8x + 6y + 2000$ respectively. (i) Find the profit function $P(x,y)$, (ii) Find $\frac{\partial P}{\partial x}(1200,1800)$ and $\frac{\partial P}{\partial y}(1200,1800)$ and interpret these results. **(EX 8.4 – 10)**
18. Let $U(x,y,z) = x^2 - xy + \sin z, x,y,z \in \mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$. **(EG 8.17)**
19. Let $U(x,y,z) = xyz, x = e^{-t}, y = e^{-t}\cos t, z = \sin t, t \in \mathbb{R}$. Find $\frac{dU}{dt}$. **(EX 8.6 – 4)**
20. If $z(x,y) = x\tan^{-1}(xy), x = t^2, y = se^t, s,t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. at $s = t = 1$. **(EX 8.6 – 6)**
21. Let $U(x,y) = e^x \sin y$, where $x = st^2, y = s^2t, s,t \in \mathbb{R}$. Find $\frac{\partial U}{\partial s}, \frac{\partial U}{\partial t}$ and evaluate them at $s = t = 1$. **(EX 8.6 – 7)**
22. $W(x,y,z) = xy + yz + zx,\ x = u - v, y = uv, z = u + v, u,v \in \mathbb{R}$. Find $\frac{\partial W}{\partial u}, \frac{\partial W}{\partial v}$ and evaluate them at $\left(\frac{1}{2}, 1\right)$. **(EX 8.6 – 9)**
23. Prove that $f(x,y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous. What is the degree? Verify Euler's Theorem for f. **(EX 8.7 – 2)**
24. P.T $g(x,y) = x\log\left(\frac{y}{x}\right)$ is homogeneous. What is the degree? Verify Euler's Theorem for g. **(EX 8.7 – 3)**

Chapter 9
Applications of Integration

1. Estimate the value of $\int_0^{0.5} x^2 dx$ using the Riemann sums corresponding to 5 subintervals of equal width and applying (i) left-end rule (ii) right-end rule (iii) the mid-point rule. **(EG 9.1)**
2. Evaluate $\int_1^4 (2x^2 + 3)dx$, as the limit of a sum. **(EG 9.4)**
3. Evaluate: $\int_0^1 \left(\frac{2x+7}{5x^2+9}\right) dx$. **(EG 9.6)**
4. Evaluate: $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x} dx$. **(EG 9.8)**
5. Evaluate: $\int_0^9 \frac{1}{x+\sqrt{x}} dx$. **(EG 9.9)**
6. Evaluate: $\int_1^2 \frac{x}{(x+1)(x+2)} dx$. **Eg. 9.10**
7. Evaluate: $\int_0^{1.5} [x^2]dx$, where $[x]$ is the greatest integer function. **(EG 9.14)**
8. Evaluate: $\int_{-4}^{4} |x+3| dx$ **Eg.9.15**

9. Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3}\log_e 2$. **(EG 9.16)**
10. Prove that $\int_0^{\frac{\pi}{4}} \frac{\sin 2x dx}{\sin^4 x+\cos^4 x} = \frac{\pi}{4}$. **(EG 9.17)**
11. Evaluate: $\int_0^{\pi} \frac{x dx}{1+\sin x}$. **(EG 9.21)**
12. Evaluate: $\int_0^{a} \frac{f(x)}{f(x)+f(a-x)} dx$. **(EG 9.26)**
13. Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx = \frac{\pi}{8}\log 2$. **(EG 9.27)**
14. Show that $\int_0^{1} (\tan^{-1} x + \tan^{-1}(1-x))dx$ **Eg. 9.28**
15. Evaluate: $\int_2^{3} \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ **Eg. 9.29**
16. Evaluate: $\int_0^{1} \sqrt{\frac{1-x}{1+x}}\, dx$**(EX 9.3 -1 iii)**
17. valuate the integrals using properties of $\int_0^{1} |5x-3| dx$ **(EX 9.3 – 2 vi)**
18. Evaluate the integrals using properties of $\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx$ **(EX 9.3 – 2 ix)**
19. Evaluate the integrals using properties of $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{dx}{1+\sqrt{\tan x}}$**(EX 9.3 – 2 x)**
20. Evaluate: $\int_{-1}^{1} e^{-\lambda x}(1-x^2)dx$. **(EG 9.34)**
21. Evaluate $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}} dx$ **(EX 9.4 – 3)**
22. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sin^2 x}$ **(EX 9.5 – 1)**
23. Evaluate: $\int_0^{2a} x^2\sqrt{2a-x^2}dx$. **(EG 9.40)**
24. Prove that $\int_0^{\infty} e^{-x}x^n dx = n!$, where n is a positive integer. **Eg. 9.43**
25. Show that $\Gamma(n) = 2\int_0^{\infty} e^{-x^2}x^{2n-1}dx$. **Eg. 9.45**
26. Evaluate: $\int_0^{\infty} \frac{x^n}{n^x} dx$, where n is a positive integer ≥ 2. **(EG 9.46)**
27. Find the area of the region bounded by the line $6x+5y=30$, $x-$ axis and the lines $x=-1$ and $x=3$. **Eg. 9.47**
28. Find the area of the region bounded by the line $7x-5y=35$, x $-$ axis and the lines $x=-2$ and $x=3$. **Eg. 9.48**
29. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **Eg. 9.49**
30. Find the area of the region bounded between the parabola $y^2=4ax$ and its latus rectum. **Eg. 9.50**
31. Find the area of the region bounded by x $-$ axis, the sine curve $y=\sin x$, the lines $x=0$ and $x=2\pi$. **Eg. 9.52**
32. Find the area of the region bounded by x $-$ axis, the curve $y=|\cos x|$, the lines $x=0$ and $x=\pi$. **Eg. 9.53**
33. Find the area of region bounded between the parabola $x^2=y$ and the curve $y=|x|$. **Eg. 9.55**
34. Find the volume of a sphere of radius a. **(EG 9.62)**
35. Find the volume of a right-circular cone of base radius r and height h. **(EG 9.63)**
36. Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ about the major axis. **(EG 9.66)**
37. Find, by integration, the volume of the solid generated by revolving about y $-$axis the region bounded by the curves $y=\log x$, $y=0, x=0$ and $y=2$. **(EG 9.69)**
38. Find, by integration, the volume of the solid generated by revolving about the x $-$axis, the region enclosed by $y=e^{-2x}, y=0, x=0$ and $x=1$. **(EX 9.9 – 2)**

Chapter 10
ORDINARY DIFFFERNTIAL EQUAIONS

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1. Find the differential equation of the family of circles passing through thepoints $(a,0)$ and $(-a,0)$. **(EG 10.4)**
2. Find the differential equation of the family of all ellipses having foci on the x −axis and centre at the origin. **(EG 10.6)**
3. Find the differential equation of the family of circles passing through the origin and having their centres on the x −axis. **(EX 10.3 – 3)**
4. Find the differential equation of the family of parabolas with vertex at $(0,-1)$ and having axis along the y −axis. **(EX 10.3 – 5)**
5. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants. **(EX 10.3 – 7)**
6. Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$. **(EX 10.4 – 6)**
7. Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$. **(EG 10.11)** **June '23 (5 m)**
8. The velocity v, of a parachute falling vertically satisfies the equation $v\frac{dv}{dx} = g\left(1 - \frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x. **(EX 10.5 – 2)**
9. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1,0)$. **(EX 10.5 – 3)**
10. Solve the following differential (ii) $ydx + (1+x^2)\tan^{-1} x\, dy = 0$ **(EX 10.5 – 4 ii)**
11. Solve the following differential (iii) $\sin\frac{dy}{dx} = a, y(0) = 1$ **(EX 10.5 – 4 iii)**
12. Solve: $\frac{dy}{dx} + 2y = e^{-x}$. **(EG 10.22)**
13. Solve: $\frac{dy}{dx} + 2y\cot x = 3x^2 cosec^2 x$. **(EG 10.24)**
14. Solve: $\cos x\frac{dy}{dx} + y\sin x = 1$ **(EX 10.7 – 1)**
15. $(1-x^2)\frac{dy}{dx} - xy = 1$ **(EX 10.7 – 2)**
16. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. **(EX 10.8 – 2)** **June '23**
17. The engine of a motor boat moving at $10\ m/s$ is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine. **(EX 10.8 – 4)**
18. Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later? **(EX 10.8 – 5)**

CHAPTER 11
PROBABILITY DISTRIBUTIONS LAPLACE

12. Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10, and (iv) the number of elements in inverse image of X. **(EG 11.2)**
13. An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images. **(EG 11.3)**
14. Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ` 30 for each black ball selected and we lose ` 20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images. **(EG 11.4)**
15. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ` 15 for each red ball selected and we lose ` 10 for each black ball selected. X denotes the winning amount, then find the values of X and number of points in its inverse images. **(EX 11.1 – 4)**

16. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images. **(EX 11.1 – 5)**
17. If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find (ii) $P(X \leq 3)$ and, (iii) $P(X \geq 2)$ **(EG 11.7)**
18. Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0, & -\infty < x < 2 \\ 0.25, & -2 \leq x < -1 \\ 0.60, & -1 \leq x < 0 \\ 0.90, & 0 \leq x < 1 \\ 1, & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$. **(EG 11.9)**
19. Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by

$f(x) = \begin{cases} \frac{x^2+1}{k}, & for\ x = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$. Find (i) the value of k (ii) cumulative distribution function (iii) $P(X \geq 1)$. **(EX 11.2 – 4)**
20. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \leq X < 5)$ (iii) $P(3 < X)$ **(EX 11.2 – 6)**

21. The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}, & 0 \leq x < 1 \\ \frac{3}{5}, & 1 \leq x < 2 \\ \frac{4}{5}, & 2 \leq x < 3 \\ \frac{9}{10}, & 3 \leq x < 4 \\ 1, & 4 \leq x < \infty \end{cases}$$

Find (i) the probability mass function (ii) $P(X < 3)$ and (iii) $P(X \geq 2)$. **(EX 11.2 – 7)**
22. Find the constant C such that the function $f(x) = \begin{cases} Cx^2, & 1 < x < 4 \\ 0, & \text{Otherwise} \end{cases}$ is a density function, and compute (i) $P(1.5 < X < 3.5)$ (ii) $P(X \leq 2)$ (iii) $P(3 < X)$. **(EG 11.11)**
23. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 1, & 1 \leq x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2 \leq X \leq 0.7)$. **(EG 11.13)**
24. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k, & 200 \leq x \leq 600 \\ 0, & \text{Otherwise} \end{cases}$

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Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres? **(EX 11.3 – 3)**

25. The probability density function of X is given by

$f(x) = \begin{cases} ke^{\frac{-x}{3}}, & for\ x > 0 \\ 0, & for\ x \leq 0 \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv) $P(5 \leq X)$ (v) $P(X \leq 4)$. **(EX 11.3 – 4)**

26. If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x+1, & -1 \leq x < 0 \\ -x+1, & 0 \leq x < 1 \\ 0, & Otherwise \end{cases}$$

then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \leq X \leq 0.5)$ **(EX 11.3 – 5)**

27. Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c (ii) Mean and variance. **(EG 11.16)**

28. Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance. **(EG 11.17)**

29. find the mean and variance.

(i) $f(x) = \begin{cases} \frac{1}{10}, & x = 2,5 \\ \frac{1}{5}, & x = 0,1,3,4 \end{cases}$ **(EX 11.4 – 1)**

30. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X. **(EX 11.4 – 2)**

31. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3) = 10$ and $E(X+3)^2 = 116$, find μ and σ^2. **(EX 11.4 – 3)**

32. A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers(iii) the probability of getting at least one correct answer. **(EG 11.20)**

33. The mean and variance of a binomial variate X are respectively 2 and 1.5. Find (i) $P(X = 0)$ (ii) $P(X = 1)$ (iii) $P(X \geq 1)$ **(EG 11.21)** **June '23**

34. The probability that $Mr. Q$ hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time. **(EX 11.5 – 2)**

35. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive. **(EX 11.5 – 4)**

36. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items. **(EX 11.5 – 5)**

37. The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X = 3)$ (iii) $P(X \geq 2)$. **(EX 11.5 – 7)**

38. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution. **(EX 11.5 – 9)**

CHAPTER 12
DISCRETE MATHEMATICS

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1. Verify the (i) closure property, (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}$. **(EG 12.2)**
2. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}_e$ = the set of all even integers. **(EG 12.4)**
3. Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b; \forall\, a, b \in \mathbb{N}$. (exponentiation property) **(EG 12.6)**
4. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set. $m * n = m + n - mn;\ m, n \in \mathbb{Z}$. **(EG 12.7)**
5. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$. **(EX 12.1 – 3)**
6. Write down the (i) conditional statement (ii) converse statement (iii) inverse statement, and (iv) contrapositive statement for the two statements p and q given below. p: The number of primes is infinite. q: Ooty is in Kerala. **(EG 12.15)**
7. Construct the truth table for $(p \,\bar{\vee}\, q) \wedge (p \,\bar{\vee}\, \neg q)$. **(EG 12.16)**
8. Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$ **(EG 12.17)**
9. Establish the equivalence property connecting the bi-conditional with conditional: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$. **(EG 12.18) June '23**
10. Using the equivalence property, S.T. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$. **(EG 12.19)**
11. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent **(EX 12.2 – 10)**
12. Check whether the statement $p \rightarrow (q \rightarrow p)$ is a tautology or a contradiction without using the truth table. **(EX 12.2 – 12)**
13. Prove $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ without using truth table. **(EX 12.2 – 14)**

2 MARKS

CHAPTER 7
APPLICATION OF DIFFERENTIAL CALCULUS

1. For the function $f(x) = x^2, x \in [0,2]$ compute the average rate of changes
2. in the subintervals $[0,0.5], [0.5,1], [1,1.5], [1.5,2]$ and the instantaneous rate of changes at the points $x = 0.5, 1, 1.5, 2$. **(EG 7.1)**
3. The temperature in celsius in a long rod of length $10\ m$, insulated at both ends, is a function of length x given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero **(Eg.7.2).**
4. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t^2), 0 \le t \le 10$. What is the rate at which the person forgets the words 2 days after learning? **(Eg.7.3).**
5. A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively? **Example 7.4**
6. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units. **Ex 7.1- 4**
7. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres. **Ex 7.1- 5**
8. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point $(1, 2)$. **Eg 7.11 March '23**

9. For what value of x the tangent of the curve $y = x^3 - 5x^2 + x - 2$ is parallel to the line $y = x$. **Eg 7.12**
10. Find the angle of intersection of the curve $y = \sin x$ with the positive x –axis. **Eg 7.16**
11. Find the slope of the tangent to the curves at the respective given points. (i) $y = x^4 + x^2 - x$ at $x = 1$ **EX 7.2 – 1)**
12. Compute the value of $'c'$ satisfied by the Rolle's theorem for the function $f(x) = x^2(1 - x)^2$, $x \in [0,1]$. **(EG 7.19)**
13. Find the values in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$. **(EG 7.20)**
14. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals. (iii) $f(x) = x - 2\log x$, $x \in [2,7]$. **(EX 7.3 – 1)**
15. Write the Maclaurin series expansion of e^x
16. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of $x - 1$. **(EX 7.4 – 4)**
17. Evaluate: $\lim\limits_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$. **(EG 7.33)**
18. Compute the limit $\lim\limits_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)$. **Eg 7.34**
19. Evaluate the limit $\lim\limits_{x\to 0}\left(\frac{\sin mx}{x}\right)$. **(EG 7.35)**
20. Evaluate the limit $\lim\limits_{x\to 0}\left(\frac{\sin x}{x^2}\right)$. **(EG 7.36)**
21. Evaluate: $\lim\limits_{x\to 0^+}(x\log x)$. **Eg 7.40**
22. Evaluate: $\lim\limits_{x\to\infty}\left(\frac{e^x}{x^m}\right)$, $m \in \mathbb{N}$. **Eg 7.42**
23. $\lim\limits_{x\to\infty}\left(\frac{x}{\log x}\right)$ **(EX 7.5 – 3)**
24. $\lim\limits_{x\to\infty} e^{-x}\sqrt{x}$ **(EX 7.5 – 5)**
25. Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval $(2,7)$ and strictly decreasing in the interval $(-2,0)$. **Eg. 7.46**
26. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$. **(EG 7.47) June '23**
27. Find the absolute extrema on the given closed interval $f(x) = x^2 - 12x + 10$; $[1,2]$ **Ex. 7.6 – 1(i)**
28. Find the stationary point and point of inflection $f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}$; $[-1,1]$ **(EX 7.6 – 1 related)**
29. Find the local extremum of the function $f(x) = x^4 + 32x$. **Eg. 7.59**
30. Find intervals of concavity and points of inflexion for the $f(x) = x(x-4)^3$
31. Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2-6x+7}{x+5}$. **(EG 7.67)**

CHAPTER 8
DIFFERENTIALS AND PARTIAL DERIVATIVES

1. Find a linear approximation for indicated points. (i) $f(x) = x^3 - 5x + 12, x_0 = 2$ (ii) $g(x) = \sqrt{x^2+9}$, $x_0 = -4$ **(EX 8.1 – 3)**
2. Let $f, g: (a,b) \to \mathbb{R}$ be differentiable functions. Show that $d(fg) = fdg + gdf$. **(EG 8.5)**
3. Let $g(x) = x^2 + \sin x$. Calculate the differential dg. **Eg. 8. 6**
4. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease? **Eg. 8. 7**
5. Find differential dy for (iii) $y = e^{x^2-5x+7}\cos(x^2-1)$ **(EX 8.2 – 1)**
6. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$ **Ex. 8. 2 – 2 (i)**
7. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following:(i) Change in the volume (ii) change in the surface area **(EX 8.1 – 5**

8. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number. **Ex. 8. 1 – 7 June'23**
9. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease? **(EG 8.7) March '23**
10. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately. **Ex. 8. 2 – 6**
11. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately? **Ex. 8. 2 – 7**
12. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area. **(EX 8.2 – 10)**
13. Let $f(x,y) = \frac{3x-5y+8}{x^2+y^2+1}$ for all $(x,y) \in \mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$. **(EG 8.8)**
14. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{e^x \sin y}{y}\right)$, if the limit exists. **(EX 8.3 – 4)**
15. Let $F(x,y) = x^3y + y^2x + 7$ for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial F}{\partial x}(-1,3)$ and $\frac{\partial F}{\partial y}(-2,1)$. **(EG 8.12)**
16. Find the partial derivatives of the following functions at the indicated points. $(i)\ f(x,y) = 3x^2 - 2xy + y^2 + 5x + 2, (2,-5)$ **(EX 8.4 – 1)**
17. If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$. **(EX 8.4 – 4) June '23**
18. If $w(x,y,z) = x^2y + y^2z + z^2x,\ x,y,z \in \mathbb{R}$, find the differential dw. **(EG 8.16)**
19. If $w(x,y) = x^3 - 3xy + 2y^2, x,y \in \mathbb{R}$, find linear approximation for w at $(1,-1)$. **(EX 8.5 – 1)**
20. If $v(x,y) = x^2 - xy + \frac{1}{4}y^2 + 7, x,y \in \mathbb{R}$, find the differential dv. **(EX 8.5 – 3)**
21. Let $V(x,y,z) = xy + yz + zx, x,y,z \in \mathbb{R}$. Find the differential dV. **(EX 8.5 – 5)**
22. Verify the above theorem for $F(x,y) = x^2 - 2y^2 + 2xy$ and $x(t) = \cos t, y(t) = \sin t, t \in [0,2\pi]$. **(EG 8.18)**
23. Let $g(x,y) = x^2 - yx + \sin(x+y), x(t) = e^{3t}, y(t) = t^2, t \in \mathbb{R}$. Find $\frac{dg}{dt}$. **(EG 8.19)**
24. If $u(x,y) = x^2y + 3xy^4, x = e^t$ and $y = \sin t$, find $\frac{du}{dt}$ and evaluate it at $t = 0$. **(EX 8.6 – 1)**
25. Show that $F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1. **(EG 8.21)**
26. Determine whether the homogeneous or not. If it is so, find the degree.$(iv)\ U(x,y,z) = xy + \sin\left(\frac{y^2-2z^2}{xy}\right)$. **(EX 8.7 – 1)**
27. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$. **(EX 8.7 – 4)**

Chapter 9
Applications of Integration

1. Find an approximate value of $\int_1^{1.5} x\,dx$ by applying the left-end rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$. **(EX 9.1 – 1)**
2. Find an approximate value of $\int_1^{1.5} x^2\,dx$by applying the right-end rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$. **(EX 9.1 – 2)**
3. Find an approximate value of $\int_1^{1.5} (2-x)\,dx$by applying the mid-point rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$. **(EX 9.1 – 3)**
4. Evaluate: $\int_0^1 [2x]\,dx$ where $[\cdot]$ is the greatest integer function. **Eg. 9. 7**
5. Show that $\int_0^{\pi} g(\sin x)\,dx = 2\int_0^{\frac{\pi}{2}} g(\sin x)\,dx$ where $g(\sin x)$ is a function of $\sin x$. **(EG 9.20)**
6. Show that $\int_0^{2\pi} g(\cos x)\,dx = 2\int_0^{\pi} g(\cos x)\,dx$, where $g(\cos x)$ is a function of $\cos x$. **(EG 9.22)**
7. If $f(x) = f(a+x)$, then $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$. **(EG 9.23)**

8. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx$. **Eg. 9.24**
9. Evaluate: $\int_{-\log 2}^{\log 2} e^{-|x|}dx$. **Eg. 9.25**
10. Evaluate: $\int_3^4 \frac{dx}{x^2-4}$ **(EX 9.3 – 1)**
11. Evaluate the integrals using properties of $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right)dx$ **(EX 9.3 – 2 i)**
12. Evaluate $\int_0^{\pi} x^2\cos nx\,dx$where n is a positive integer. **(EG 9.31)**
13. Evaluate: $\int_0^{2\pi} x^2\sin nx\,dx$, where n is a positive integer. **(EG 9.33)**
14. Evaluate: $\int_0^{\infty} \frac{1}{a^2+x^2}dx\,, a>0, b\in\mathbb{R}$. **Eg. 9.35**
15. Evaluate: $\int_0^{\frac{\pi}{2}}(\sin^2 x+\cos^4 x)dx$ **Eg. 9.37**
16. Evaluate: $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$. **Eg. 9.38**
17. Find the values of $\int_0^{\frac{\pi}{2}} \sin^5 x\cos^4 x\,dx$ **Eg. 9.39(i)**
18. Evaluate: $\int_0^1 x^3(1-x)^4dx$. **Eg. 9.42**
19. Evaluate $\int_0^{\frac{\pi}{4}} \sin^6 2x\,dx$**(EX 9.6 – 1 iii)**
20. Evaluate $\int_0^{2\pi} \sin^7\frac{x}{4}dx$**(EX 9.6 – 1 vi)**
21. Evaluate: $\int_0^{\infty} e^{-ax}x^n dx$, where $a>0$. **Eg. 9.44**
22. Evaluate: $\int_0^{\infty} x^5e^{-3x}dx$ **Ex. 9. 7 – 1(i)** **June '23**

Chapter 10
ORDINARY DIFFFERNTIAL EQUAIONS

1. Determine the order and degree (if exists) of

   $(iii)\ \frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ $(iv)\ 3\left(\frac{d^2y}{dx^2}\right) = \left(4+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}$

   $(v)\ dy+(xy-\cos x)dx = 0$ **(EG 10.1)**
2. Determine the order and degree (if exists) of

   $(ii)\ \left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}}+3\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4=0$ $(v)\ y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}$ $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1+\frac{dy}{dx}}$

   $(viii)\ \frac{d^2y}{dx^2} = xy+\cos\left(\frac{dy}{dx}\right)$ $(ix)\ \frac{d^2y}{dx^2}+5\frac{dy}{dx}+\int y\,dx = x^3$ $(x)\ x = e^{xy\left(\frac{dy}{dx}\right)}$ **(EX 10.1 – 1)**

3. Express each of the following physical statements in the form of differential equation.
   (i) Radium decays at a rate proportional to the amount Q present. **(EX 10.2 – 1(i))**
   (ii) The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population. **(EX 10.2 – 1(ii))**
   (iii) For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. **(EX 10.2 – 1(iii))**
   (iv)A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ` 400 per year. **(EX 10.2 – 1(iv))**
4. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop. **(EX 10.2 – 2)** **June '23**
5. Find the differential equation for the family of all straight lines passing through the origin. **(EG 10.2)**
6. Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x+B\sin x$. **(EG 10.3)**

7. Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant. **(EG 10.5)**
8. Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$. **(EG 10.8)**

CHAPTER 11
PROBABILITY DISTRIBUTIONS LAPLACE

1. Suppose two coins are tossed once. If X denotes the number of tails, (i)write down the sample space (ii)find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images. **(EG 11.1)**
2. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images. **(EX 11.1 – 1)**
3. In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images. **(EX 11.1 – 2)**
4. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred. **(EG 11.5)**
5. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours. **(EG 11.6) June '23**
6. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred. **(EX 11.2 – 1)**
7. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls. **(EX 11.2 – 3)**
8. The probability density function of X is given by
$$f(x) = \begin{cases} kxe^{-2x}, & for\ x > 0 \\ 0, & for\ x \leq 0 \end{cases}$$
Find the value of k . **(EX 11.3 – 1)**
9. Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & for\ x \geq 0 \\ 0, & Otherwise \end{cases}$ **(EG 11.18)**
10. Find the mean and variance. $f(x) = \begin{cases} \frac{4-x}{6}, x = 1,2,3 \end{cases}$**(EX 11.4 – 1)**
11. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. **(EX 11.4 – 4)**
12. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Find the expected life of this electronic equipment. **(EX 11.4 – 6)**
13. A lottery with 600 tickets gives one prize of `200, four prizes of `100, and six prizes of ` 50. If the ticket costs is ` 2, find the expected winning amount of a ticket. **(EX 11.4 – 8)**
14. Find the binomial distribution function for each of the following. (i) Five fair coins are tossed once and X denotes the number of heads. (ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared. **(EG 11.19)**
15. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where
(i) $n = 6, p = \frac{1}{3}, k = 3$ **(EX 11.5 – 1)**
16. If $X \sim B(n, p)$ such that $4P(X = 4) = P(x = 2)$ and $n = 6$. Find the distribution, mean and standard deviation. **(EX 11.5 – 8)**

CHAPTER 12
DISCRETE MATHEMATICS

1. Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary): $(i)\ a * b = a + 3ab - 5b^2, \forall\ a, b \in \mathbb{Z}$ $(ii) a * b = \left(\frac{a-1}{b-1}\right), \forall\ a, b \in \mathbb{Q}$ **(EG**
2. **Uniqueness of Identity):** In an algebraic structure the identity element (if exists) must be unique. **(TH 12.1)**
3. **(Uniqueness of Inverse):** In an algebraic structure the inverse of an element (if exists) must be unique. **(TH 12.2)**
4. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$. **(EG 12.8)** **June '23 , March '23**
5. Determine whether $*$ is a binary operation on the sets given below.
   $(i)\ a * b\ =\ a.|b|$ on $\mathbb{R}$ $(ii)\ a * b\ =\ min\ (a, b)$ on $A = \{1,2,3,4,5\}$
   $(iii)\ a * b = a\sqrt{b}$ is binary on $\mathbb{R}$. **(EX 12.1 – 1)**
6. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$? **(EX 12.1 – 2)**
7. Let $A = \{a + \sqrt{5}b : a, b \in \mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A. **(EX 12.1 – 4)**
8.
9. Fill in the following table so that the binary operation $*$ on $A = \{a, b, c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

(EX 12.1 – 6)

10. Consider the binary operation $*$ defined on the set $A\ =\ \{a, b, c, d\}$ by the following table: Is it commutative and associative?

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	a	a	c

(EX 12.1 – 7)

11. Write the statements in words corresponding to $\neg p, p \wedge q$, $p \vee q$ and $q \vee \neg p$, where p is 'It is cold' and q is 'It is raining.' **(EG 12.12)**
12. How many rows are needed for following statement formulae?
    $(i)\ p \vee \neg t \wedge (\ p \vee \neg s)$ $(ii)\big((\ p \wedge q) \vee (\neg r \vee \neg s)\big) \wedge (\neg t \wedge v)$ **(EG 12.13)**
13. Consider $p \rightarrow q$: If today is Monday, then $4 + 4 = 8$. **(EG 12.14)**
14. Construct the truth table for the following statements. $(i)\ \neg p \wedge \neg q$ $(ii)\ \neg(\ p \wedge \neg q)$ $(iii)\ (p \vee q) \vee \neg q$ $(iv)\ (\neg p \rightarrow r) \wedge (p \leftrightarrow q)$ **(EX 12.2 – 6)**

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- *Math is like going to the gym for your brain. It sharpens your mind.*

- *Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding.*

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