11 R

Register No.

Second Mid-Term Test - 2024
MATHEMATICS

Time : 1.30 Hrs. Marks : 50

PART - A

I. **Choose the correct answer** 10 x 1 = 10

1. What must be the matrix x, if $2x + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

2. If the points (x, –2) (5, –2) (8, 8) are collinear, then x is equal to a) –3 b) $\frac{1}{3}$ c) 1 d) 3

3. Let A and B be two symmetric matrices of same order. Then which one of the following statement is not time?
a) A + B is a symmetric matrix b) AB is a symmetric matrix c) $AB = (BA)^T$ d) $A^TB = AB^T$

4. The value of $\vec{AB}+\vec{BC}+\vec{DA}+\vec{CD}$ is a) $\vec{AD}$ b) $\vec{CA}$ c) $\vec{O}$ d) $-\vec{AD}$

5. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of three collinear points, then which of the following is true?
a) $\vec{a} = \vec{b}+\vec{c}$ b) $2\vec{a} = \vec{b}+\vec{c}$ c) $\vec{b} = \vec{c}+\vec{a}$ d) $4\vec{a}+\vec{b}+\vec{c} = 0$

6. If $\vec{a}$ and $\vec{b}$ having same magnitude and angle between them is 60^0 and their scalar product is $\frac{1}{2}$ then $|\vec{a}|$ is a) 2 b) 3 c) 7 d) 1

7. If the projection of 5i – j – 3k on the vector i + 3j + λk is same as the projection i + 3i + λk on 5i – j – 3k, then λ is equal to a) ±4 b) ±3 c) ±5 d) ±1

8. $\lim_{x \to \infty} \frac{\sin x}{x}$ a) 1 b) 0 c) ∞ d) –∞

9. If $\lim_{x \to 0} \frac{\sin px}{\tan 3x} = 4$ then value of p is a) 6 b) 9 c) 12 d) 4

10. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is a) continuous b) discontinuous c) differentiable d) non-zero

PART - B

Answer any four questions. Q.No.17 is compulsory. 4 x 2 = 8

11. Compute |A| using sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$
12. Find the area of the triangle whose vertices are (0, 0), (1, 2), (4, 3)
13. Find a unit vector along the direction of the vector 5i – 3j + 4k.
14. Find the projection of the vector i + 3j + 7k on the vector 2i + 6j + 3k.
15. Let $f(x) = \begin{cases} x+1, & x > 0 \\ x-1, & x < 0 \end{cases}$ verify the existence of limit as $x \to 0$.
16. Prove that $f(x) = 2x^2 + 3x - 5$ is continuous at all points in R.
17. Show that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{o}$

PART - C

Answer any four questions. Q.No.24 is compulsory. 4 x 3 = 12

18. Solve for x if, $[x \;\; 2 \;\; -1] \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = 0$

19. Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ a^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

20. State and prove, 'section formula' for internal division.

21. Prove that the points whose position vectors, $2i + 4j + 3k$, $4i + j + 9k$ and $10i - j + 6k$ form a right angled triangle.

22. Find the relation between a and b if $\lim_{x \to 3} f(x)$ exists where $f(x) = \begin{cases} ax + b & \text{if } x > 3 \\ 3ax - 4b + 1 & \text{if } x < 3 \end{cases}$

23. A tomato wholesaler finds that the price of a newly harvested tomatoes is ₹0.16 per kg, if he purchases fewer than 100 kgs each day. However, if he purchases atleast 100 kgs daily, the price drops to ₹0.14 per kg. Find the total cost function and discuss the cost when the purchase is 100 kgs.

24. Prove that, $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

PART - D

Answer all the questions. **4 x 5 = 20**

25. a) Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of symmetric and a skew symmetric matrices.

(OR)

25. b) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

26. a) If ABCD is a quadrilateral, E and F are the midpoints of AC and BD respectively. Prove that, $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$ **(OR)**

26. b) Show that the following vectors are coplanar, $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{j} + 2\hat{k}$

27. a) Show that, $\lim_{x \to 0^+} x\left[\frac{1}{x} + \frac{2}{x} + \dots + \frac{15}{x}\right] = 120$ **(OR)** b) Prove that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

28. a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ **(OR)**

b) If $\vec{a}$, $\vec{b}$ are unit vectors and θ is the angle between then, show that

(i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (ii) $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$