

11 - STD

Time : 1.30 Hrs

# SECOND MID TERM TEST - 2024

## MATHEMATICS

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Marks : 45

 $10 \times 1 = 10$ **I Answer all the questions.**

1. If  $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$  then for what value of  $\lambda$ ,  $A^2 = 0$ ?
- 1) 0      2)  $\pm 1$       3) -1      4) 1
2. If  $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$  then  $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$  is      1)  $\Delta$       2)  $K\Delta$       3)  $3K\Delta$       4)  $K^3\Delta$
3. Let A and B be two symmetric, matrices of small order. Then which one of the following statement is not true?      1)  $A + B$  is a symmetric matrix  
2) AB is a symmetric matrix      3)  $AB = (BA)^T$       4)  $A^T B = AB^T$
4. The value of  $\theta \in \left(0, \frac{\pi}{2}\right)$  for which the vectors  $\vec{a} = (\sin \theta)\hat{i} + (\cos \theta)\hat{j}$  and  $\vec{b} = \hat{i} - \sqrt{3}\hat{j} + 2\hat{k}$  are perpendicular, is equal to      1)  $\frac{\pi}{3}$       2)  $\frac{\pi}{6}$       3)  $\frac{\pi}{4}$       4)  $\frac{\pi}{2}$
5. If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} + x\hat{j} + \hat{k}$ ,  $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$  and  $\vec{a} \cdot (\vec{b} \times \vec{c})$  then x is equal to      1) 5      2) 7      3) 26      4) 10
6. If A is a square matrix, then which of the following is not symmetric?      1)  $A + A^T$       2)  $AA^T$       3)  $A^TA$       4)  $A-A^T$
7. If the points  $(x, -2)$ ,  $(5, 2)$ ,  $(8, 8)$  are collinear, then x is equal to      1) -3      2)      3) 1      4) 3
8. If  $\lambda\hat{i} + 2\lambda\hat{j} + 2\lambda\hat{k}$  is a unit vector, then the value of  $\lambda$  is      1)  $\frac{1}{3}$       2)  $\frac{1}{4}$       3)  $\frac{1}{9}$       4)  $\frac{1}{2}$
9. If  $A = \begin{bmatrix} 6 & 3 \\ 2 & a \end{bmatrix}$  is a singular matrix, then the value of a.      1) 6      2) 3      3) 1      4) 2
10. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a}|=4$ ,  $|\vec{b}|=3$  and  $\vec{a} \cdot \vec{b}=6$ , find the angle between  $\vec{a}$  and  $\vec{b}$       1)  $\pi$       2)  $\frac{\pi}{2}$       3)  $\frac{\pi}{4}$       4)  $\frac{\pi}{3}$

**II Answer any 3 questions. Q. no.'15 is compulsory.** $3 \times 2 = 6$ 

11. Determine  $3B + 4C - D$  if B, C and D are given by

$$B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}, C = \begin{bmatrix} -1 & -2 & 3 \\ -1 & 0 & 2 \end{bmatrix}, D = \begin{bmatrix} 0 & 4 & -1 \\ 5 & 6 & -5 \end{bmatrix}$$

12. If  $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$  Prove that the points P, Q, R are collinear.

13. Compute  $|A|$  using sarrus rule if  $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$
14. Find the value  $\lambda$  for which the vectors  $\vec{a}$  and  $\vec{b}$  are perpendicular, where  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ .
15. If  $(k, 2)$ ,  $(2, 4)$  and  $(3, 2)$  are vertices of the triangle of area 4 square units then determine the value of  $k$ .
- III Answer any 3 questions. Q. No. 20 is compulsory.**  $3 \times 3 = 9$
16. Find the area of the parallelogram whose adjacent sides are  $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ .
17. If two rows (columns) of a matrix are identical, then prove that, its determinant is zero.

18. Prove that  $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$

$$\sqrt{4 \times 54} = \sqrt{24}$$

19. Verify that  $|AB| = |A| |B|$  if  $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$  and  $B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ .
20. For any vector  $\vec{a}$  Prove that  $|\vec{a} x \hat{i}|^2 + |\vec{a} x \hat{j}|^2 + |\vec{a} x \hat{k}|^2 = 2|\vec{a}|^2$ .

**IV Answer all the questions.**

$4 \times 5 = 20$

21. a) Express the matrix  $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$  as the sum of a symmetric and a skew symmetric matrix. (OR) b) Prove that the medians of a triangle are concurrent.
22. a) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that  $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$ . (OR)

b) Prove that  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$ ,

23. a) Show that  $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$  (OR)

b) Show that the points whose position vectors  $4\hat{i} + 5\hat{j} + \hat{k}$ ,  $-\hat{j} - \hat{k}$ ,  $3\hat{i} + 9\hat{j} + 4\hat{k}$  and  $-4\hat{i} + 4\hat{j} + 4\hat{k}$  are coplanar.

24. a) Prove that  $\begin{vmatrix} 1 & x & x^2 \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2 - 2x \\ -x^2 & x^2 - 2x & -1 \end{vmatrix}$ . (OR)

b) Find the unit vectors perpendicular to each of the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  where  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ .