SECOND MID TERM TEST- NOVEMBER 2024

11 TH STANDARD

REG NO.

TIME:1.30Hrs **MATHEMATICS** **Marks:45**

PART-I

Choose the correct answer

10X1=10

1. If A is a square matrix, then which of the following is not symmetric?
   (1) $A+A^T$ (2) AA^T (3) $A^T A$ (4) $A-A^T$

2. If the square of the matrix $\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$ is the unit matrix of order 2, then α, β and γ should satisfy the relation.
   (1) $1+\alpha^2+\beta\gamma=0$ (2) $1-\alpha^2-\beta\gamma=0$ (3) $1-\alpha^2+\beta\gamma=0$ (4) $1+\alpha^2-\beta\gamma=0$

3. If $(k, 2)$, $(2, 4)$ and $(3, 2)$ are vertices of the triangle of area 4 square units then determine the value of k.
   (1) $k=-1,3$ (2) $k=\square 1,7$ (3) $k=\square 1,-7$ (4) $k=-1,7$

4. If $\lambda\hat{i}+2\lambda\hat{j}+2\lambda\hat{k}$ is a unit vector, then the value of λ is
   (1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{9}$ (4) $\frac{1}{2}$

5. If $|\vec{a}+\vec{b}|=60, |\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$, then $|\vec{a}|$ is
   (1) 42 (2) 12 (3) 22 (4) 32

6. Find λ, when the projection of $\vec{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.
   (1) 3 (2) 6 (3) 5 (4) 4

7. The value of $\theta \in \left(0, \frac{\pi}{2}\right)$ for which the vectors $\vec{a}=(\sin\theta)\hat{i}+(\cos\theta)\hat{j}$ and $\vec{b}=\hat{i}-\sqrt{3}\hat{j}+2\hat{k}$ are perpendicular, is equal to
   (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

8. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A)=0.6$, then $P(\bar{A}\cap B)$ is (1) 0.96 (2) 0.24 (3) 0.56 (4) 0.66

9. A bag contains 5 white and 3 black balls. Five balls are drawn successively without replacement. The probability that they are alternately of different colours is
   (1) $\frac{3}{14}$ (2) $\frac{5}{14}$ (3) $\frac{1}{14}$ (4) $\frac{9}{14}$

10. A number is selected from the set $\{1,2,3,...,20\}$. The probability that the selected number is divisible by 3 or 4 is
   (1) $\frac{2}{5}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

PART – II

Answer any 3 questions (Question No. 15 is Compulsory) **3×2=6**

11. Find the value of the product ; $\begin{vmatrix}\log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9\end{vmatrix} \times \begin{vmatrix}\log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4\end{vmatrix}$.

12. If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$ are the direction cosines of some vector, then find a.

13. If A and B are two independent events such that $P(A) = 0.4$ and $P(A\cup B)=0.9$. Find $P(B)$.

14. For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k}$.

15. If A and B are symmetric matrices of same order, prove that $AB - BA$ is a skew-symmetric matrix.

PART - III

Answer any 3 questions (Question No. 20 is Compulsory) **3×3=9**

16. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A^T = \begin{bmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.

17 Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ if $x, y, z \neq 1$.

18. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.

19. A die is rolled. If it shows an odd number, then find the probability of getting 5.

20. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2\,|\vec{b}|^2$

PART - IV

Answer all the questions **4×5=20**

21. a) If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$.

(OR)

b) Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$.

22. a) If $ABCD$ is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$.

(OR)

b) Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

23. a) A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

(OR)

b) Find the cosine and sine angle between the vectors $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 2\hat{k}$.

24. a) A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II.

(OR)

b) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$.

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