Thiruvallur . DT.

SECOND MID TERM TEST - 2024

Standard XI

Reg.No. | | | | | | |

MATHEMATICS

Time : 1.30 hrs *Marks : 50*

Part - I

I. Choose the correct answer: $10 \times 1 = 10$

1. What must be the matrix X, if $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?

   a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

2. If A is a square matrix, then which of the following is not symmetric?

   a) $A + A^T$ b) AA^T c) A^TA d) $A - A^T$

3. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is

   a) Δ b) $k\Delta$ c) $3k\Delta$ d) $k^3\Delta$

4. If A and B are symmetric matrices of order n where $(A \neq B)$, then

   a) A + B is skew symmetric b) A + B is symmetric

   c) A + B is a diagonal matrix d) A + B is a zero matrix

5. A vector makes equal angle with the positive direction of the coordinate axes. Then each angle is equal to

   a) $\cos^{-1}\left(\frac{1}{3}\right)$ b) $\cos^{-1}\left(\frac{2}{3}\right)$ c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ d) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

6. If ABCD is a parallelogram, then $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD}$ is equal to

   a) $2(\overline{AB} + \overline{AD})$ b) $4\overline{AC}$ c) $4\overline{BD}$ d) $\overline{O}$

7. The value of $\overline{AB} + \overline{BC} + \overline{DA} + \overline{CD}$ is

   a) $\overrightarrow{AD}$ b) $\overrightarrow{CA}$ c) $\vec{O}$ d) $-\overrightarrow{AD}$

8. If $|\vec{a}+\vec{b}|=60$, $|\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$, then $|\vec{a}|$ is

a) 42 b) 12 c) 22 d) 32

9. $\lim_{x\to\infty} \frac{\sin x}{x}$

a) 1 b) 0 c) ∞ d) $-\infty$

10. $\lim_{x\to\pi/2} \frac{2x-\pi}{\cos x}$

a) 2 b) 1 c) -2 d) 0

Part - II

II. **Answer any 4 questions. (Q.No.16 is compulsory)** 4 x 2 = 8

11. If $A=\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}$, then compute A^4

12. Prove that $\begin{vmatrix}\sec^2\theta & \tan^2\theta & 1\\ \tan^2\theta & \sec^2\theta & -1\\ 38 & 36 & 2\end{vmatrix}=0$

13. Find a direction ratio and direction cosines of $3\hat{i}-4\hat{k}$.

14. Find the angle between the vectors $5\hat{i}+3\hat{j}+4\hat{k}$ and $6\hat{i}-8\hat{j}-\hat{k}$

15. Compute : $\lim_{x\to1} \frac{x^3-1}{x-1}$

16. Determine the values of a and b so that the matrix $\begin{bmatrix}7 & 3\\ -2 & a\end{bmatrix}$ is singular.

Part - III

III. **Answer any 4 questions. (Q.No.22 is compulsory)** 4 x 3 = 12

17. Show that f(x) f(y) = f(x + y), where $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

18. If a, b, c and x are positive real numbers, then show that $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1 \\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1 \end{vmatrix}$ is zero.

19. If D and E are the midpoints of the sides AB and AC of a triangle ABC. Prove that $\overrightarrow{BE}+\overrightarrow{DC}=\frac{3}{2}\overrightarrow{BC}$

20. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, prove that $\vec{a}$ and $\vec{b}$ are perpendicular.

21. Prove that $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

22. Find the value of the product $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$

Part - IV

IV. Answer all the questions. $4\times5=20$

23. a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(OR)

b) Prove that the points whose position vector $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$ and $10\hat{i}-\hat{j}+6\hat{k}$ form a right angled triangle.

24. a) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}=4\overrightarrow{EF}$

(OR)

b) Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$ by using Factor Theorem.

25. a) Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, prove that $\vec{a}=\pm\frac{2}{\sqrt{3}}\left(\vec{b}\times\vec{c}\right)$

(OR)

b) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

26. a) Find the relation between a and b if $\lim_{x\to 3} f(x)$ exists where

$$f(x)=\begin{cases} ax+b & \text{if } x>3 \\ 3ax-4b+1 & \text{if } x<3 \end{cases}$$

(OR)

b) Express the matrix $A=\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrices.

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