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SECOND MIDTERM EXAMINATION NOVEMBER 2024
XI STANDARD – MATHEMATICS

Time : 1.30 hrs Max marks : 45

I. Choose the correct answer. 10 x 1 = 10

1. If the points $(x,-2),(5,2),(8,8)$ are collinear, then x is equal to
(1) - 3 (2) $\frac{1}{3}$ (3) 1 (4) 3

2. If A is a square matrix, then which of the following is not symmetric ?
(1) $A+A^T$ (2) AA^T (3) A^TA (4) $A-A^T$

3. If $A=\begin{bmatrix}\lambda & 1\\ -1 & -\lambda\end{bmatrix}$ then for what values of λ, $A^2=O$?
(1) 0 (2) ±1 (3) - 1 (4) 1.

4. If $A=\begin{bmatrix}a & x\\ y & a\end{bmatrix}$ and if $xy=1$ then $det(AA^T)$ is equal to
(1) $(a-1)^2$ (2) $(a^2+1)^2$ (3) a^2-1 (4) $(a^2-1)^2$

5. The value of x, for which the matrix $A=\begin{bmatrix}e^{x-2} & e^{7+x}\\ e^{2+x} & e^{2x+3}\end{bmatrix}$ is singular
(1) 9 (2) 8 (3) 7 (4) 6

6. The value of $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{DA}+\overrightarrow{CD}$ is
(1) $\overrightarrow{AD}$ (2) $\overrightarrow{CA}$ (3) $\vec{0}$ (4) $-\overrightarrow{AD}$

7. If $\vec{a}+2\vec{b}$ and $3\vec{a}+m\vec{b}$ are parallel, then the value of m is
(1) 3 (2) $\frac{1}{3}$ (3) 6 (4) $\frac{1}{6}$

8. If $ABCD$ is a parallelogram, then $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}$ is equal to
(1) $2(\overrightarrow{AB}+\overrightarrow{AD})$ (2) $4\overrightarrow{AC}$ (3) $4\overrightarrow{BD}$ (4) $\vec{0}$

9. If $\vec{a},\vec{b},\vec{c}$ are the position vectors of three collinear points, then which of the following is true ?
(1) $\vec{a}=\vec{b}+\vec{c}$ (2) $2\vec{a}=\vec{b}+\vec{c}$ (3) $\vec{b}=\vec{c}+\vec{a}$ (4) $4\vec{a}+\vec{b}+\vec{c}=\vec{0}$

10. If $|\vec{a}+\vec{b}|=60$, $|\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$ then $|\vec{a}|$ is
(1) 42 (2) 12 (3) 22 (4) 32

II. Answer any three of the following. (Question number 15 is compulsory) 3 x 2 = 6

11. If $A=\begin{bmatrix}1 & -2 & 3\\ 1 & 2 & 1\\ x & 2 & -3\end{bmatrix}$ is singular, find the value of x.

12. Construct an $m\times n$ matrix $A=[a_{ij}]$ where a_{ij} is given by $a_{ij}=\frac{(i-2j)^2}{2}$ with $m=2,n=3$

13. Compute $|A|$ if $A=\begin{bmatrix}3 & 4 & 1\\ 0 & -1 & 2\\ 5 & -2 & 6\end{bmatrix}$

14. If G is the centroid of a triangle ABC, Prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$.

15. Find $|\vec{a}\times\vec{b}|$, where $\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$

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III. Answer any three of the following. (Question number 20 is compulsory) 3 x 3 = 9

16. Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ if $x, y, z \neq 1$

17. Prove that any square matrix can be expressed as the sum of a symmetric matrix and a Skew symmetric matrix.

18. If $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ are perpendicular, then find the value of λ.

19. Find the angle between the vectors $5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $6\hat{\imath} - 8\hat{\jmath} - \hat{k}$

20. Find the area of the parallelogram whose adjacent sides are $\vec{a} = 3\vec{i} + \vec{j} + 4\vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$

IV. Answer all the questions. 4 x 5 = 20

21.(a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$.

OR

(b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

22. (a) Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

OR

(b). If $A = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and such that $(A-2I)(A-3I) = O$, find the value of x

23. (a) Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$.

OR

(b) Show that the points whose position vectors $4\hat{\imath} + 5\hat{\jmath} + \hat{k}$, $-\hat{\jmath} - \hat{k}$, $3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ are coplanar

24. (a) Prove that the points whose position vectors $2\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$, $4\hat{\imath} + \hat{\jmath} + 9\hat{k}$ and $10\hat{\imath} - \hat{\jmath} + 6\hat{k}$ form a right angled triangle

OR

(b) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD, respectively, Prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

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