SECOND MID-TERM TEST - 2024

Reg. No.

XI - MATHEMATICS

Time Allowed : 1.30 Hrs. *Maximum Marks: 45*

I. Choose the correct answer: **10 x 1 = 10**

1. If $A=\begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if $xy = 1$, then $\det(AA^T)$ is equal to
   a) $(a-1)^2$ b) $(a^2+1)^2$ c) a^2-1 d) $(a^2-1)^2$
2. If the square of the matrix $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is the unit matrix of order 2, then α, β and γ should satisfy the relation
   a) $1+\alpha^2+\beta\gamma=0$ b) $1-\alpha^2-\beta\gamma=0$ c) $1-\alpha^2+\beta\gamma=0$ d) $1+\alpha^2-\beta\gamma=0$
3. A vector $\overrightarrow{OP}$ makes 60° and 45° with the positive direction of the x and y axis respectively. Then the angle between $\overrightarrow{OP}$ and the z-axis is
   a) 45° b) 60° c) 90° d) 30°
4. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+x\hat{j}+\hat{k}$, $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$ and $\vec{a}.(\vec{b}\times\vec{c})=70$, then x is equal to
   a) 5 b) 7 c) 26 d) 10
5. If $\lambda\hat{i}+2\lambda\hat{j}+2\lambda\hat{k}$ is a unit vector, then the value of λ is
   a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{2}$
6. $\lim\limits_{x\to 0}\frac{a^x-b^x}{x}=$ a) log ab b) $\log\left(\frac{a}{b}\right)$ c) $\log\left(\frac{b}{a}\right)$ d) $\frac{a}{b}$
7. $\lim\limits_{x\to\infty}\frac{\sin x}{x}$ a) 1 b) 0 c) ∞ d) $-\infty$
8. If $|\vec{a}|=13$, $|\vec{b}|=5$ and $\vec{a}.\vec{b}=60$ then $|\vec{a}\times\vec{b}|$ is
   a) 15 b) 35 c) 45 d) 25
9. If $\overrightarrow{BA}=3\hat{i}+2\hat{j}+\hat{k}$ and the position vector of B is $\hat{i}+3\hat{j}-\hat{k}$, then the position vector A is
   a) $4\hat{i}+2\hat{j}+\hat{k}$ b) $4\hat{i}+5\hat{j}$ c) $4\hat{i}$ d) $-4\hat{i}$
10. Let A and B be two symmetric matrices of same order. Then which one of the following statement is not true?
    a) A + B is a symmetric matrix b) AB is symmetric matrix
    c) $AB=(BA)^T$ d) $A^TB=AB^T$

II. Answer any 3 questions. (Q.No.15 is compulsory) **3 x 2 = 6**

11. If (k, 2), (2, 4) and (3, 2) are vertices of the triangle of area 4 square units, then determine the value of k.
12. Without expanding, evaluate the following determinants : $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$
13. Find the value λ for which the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, where $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$
14. Calculate $\lim\limits_{x\to 3}\left(x^3-2x+6\right)$
15. Find the angle between the vectors $2\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ using vector product.

III. Answer any 3 questions. (Q.No.20 is compulsory) **3 x 3 = 9**

16. Verify that $|AB| = |A|\,|B|$ if $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

17. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, show that A^2 is a unit matrix.

18. Show that the points whose position vectors are $2\hat{i}+3\hat{j}-5\hat{k}$, $3\hat{i}+\hat{j}-2\hat{k}$ and $6\hat{i}-5\hat{j}+7\hat{k}$ are collinear.

19. Show that the vectors $-\hat{i}-2\hat{j}-6\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $-\hat{i}+3\hat{j}+5\hat{k}$ form a right angled triangle.

20. Find the relation between a and b if $\lim_{x\to 3} f(x)$ exists where $f(x) = \begin{cases} ax+b & \text{if } x>3 \\ 3ax-4b+1 & \text{if } x<3 \end{cases}$.

IV. Answer all the questions. **4 x 5 = 20**

21. a) Prove that altitudes of the triangle are concurrent by vector method. ***(OR)***

b) If a, b, c are all positive and are p^{th}, q^{th} and r^{th} terms of a G.P., show that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

22. a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$ by using Factor Theorem.

(OR)

b) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

23. a) Show that the following vectors are coplanar : $\hat{i}-2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}-4\hat{k}$, $-\hat{j}+2\hat{k}$

(OR)

b) Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=4$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Find $4\vec{a}.\vec{b}+3\vec{b}.\vec{c}+3\vec{c}.\vec{a}$

24. a) If $\vec{a},\vec{b},\vec{c}$ are position vectors of the vertices A,B,C of a triangle ABC is $\frac{1}{2}\left[\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right]$. Also deduce the condition for collinearity of the points A, B and C. ***(OR)***

b) A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water after t minutes (in grams per litre) is $C(t) = \frac{30t}{200+t}$. What happens to the concentration as $t \to \infty$?