Tirupattur Dt

Class : 11

SECOND MID TERM TEST - 2024

MATHEMATICS

Time Allowed : 1.30 Hours] [Max. Marks : 50

Part-I

Answer all the Questions: **10X1=10**

1. What must be the matrix X, if $2X+\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}3&8\\7&2\end{bmatrix}$?

1) $\begin{bmatrix}1&3\\2&-1\end{bmatrix}$ 2) $\begin{bmatrix}1&-3\\2&-1\end{bmatrix}$ 3) $\begin{bmatrix}2&6\\4&-2\end{bmatrix}$ 4) $\begin{bmatrix}2&-6\\4&-2\end{bmatrix}$

2. If the points $(x,-2)$,(5,2), (8,8) are collinear , then x is equal to

1) -3 2) $\frac{1}{3}$ 3) 1 4) 3

3. The value of the determinant of $A=\begin{bmatrix}0&a&-b\\-a&0&c\\b&-c&0\end{bmatrix}$ is

1) $-2abc$ 2) abc 3) 0 4) $a^2+b^2+c^2$

4. If $\vec{a}+2\vec{b}$ and $3\vec{a}+m\vec{b}$ are parallel, then the value of m is

1) 3 2) $\frac{1}{3}$ 3) 6 4) $\frac{1}{6}$

5. A vector makes equal angle with the positive direction of the coordinate axes. Then each angle is equal to

1) $\cos^{-1}\left(\frac{1}{3}\right)$ 2) $\cos^{-1}\left(\frac{2}{3}\right)$ 3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 4) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

6. If $\lambda\hat{\imath}+2\lambda\hat{\jmath}+2\lambda\hat{k}$ is a unit vector , then the value of λ is

1) $\frac{1}{3}$ 2) $\frac{1}{4}$ 3) $\frac{1}{9}$ 4) $\frac{1}{2}$

7. $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=$ 1) 1 2) 0 3) ∞ 4) $-\infty$

8. $\lim\limits_{x\to 3}\lfloor x\rfloor=$ 1) 2 2) 3 3) does not exist 4) 0

9. $\lim\limits_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots\ldots+\frac{n}{n^2}\right)$ is 1) $\frac{1}{2}$ 2) 0 3) 1 4) ∞

10. At $x=\frac{3}{2}$ the function $f(x)=\frac{|2x-3|}{2x-3}$ is

1) continuous 2) discontinuous 3) differentiable 4) non – zero

Part-II

Answer any four Questions, Q.no:16 is compulsory. **4X2=8**

11. If $A=\begin{bmatrix}1&a\\0&1\end{bmatrix}$, then compute A^4. 7.1 → (5)

12. Find the value of x if $\begin{vmatrix}x-1&x&x-2\\0&x-2&x-3\\0&0&x-3\end{vmatrix}=0$ Eg: 7.21

13. Find a unit vector along the direction of the vector $5\hat{\imath}-3\hat{\jmath}+4\hat{k}$. Eg : 8.4

14. Find $|\vec{a}\times\vec{b}|$, where $\vec{a}=3\hat{\imath}+4\hat{\jmath}$ and $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$ Eg: 8.20

15. Prove that $f(x) = 2x^2 + 3x - 5$ is continuous at all points in R. 9.5 → (1)

16. Evaluate : $\lim_{x \to 0} \frac{\tan 2x}{x}$ 9.4 → 14

Part-III

Answer any four Questions, Q.no:22 is compulsory. **4X3=12**

17. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ab & bc & a^2+b^2 \end{vmatrix}$ Eg: 7.28

18. Show that the points $(a, b+c)$, $(b, c+a)$, and $(c, a+b)$ are collinear.. Eg: 7.34

19. If $|\vec{a}| = 5, |\vec{b}| = 6, |\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$. 8.3 → (8)

20. Find the angle between the vectors $5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $6\hat{\imath} - 8\hat{\jmath} - \hat{k}$ Eg: 8.16

21. Find the points of discontinuity of the function f, where $f(x) = \begin{cases} 4x+5 & if x \leq 3 \\ 4x-5 & if x > 3 \end{cases}$ 9.5 → 3(i)

22. Evaluate : $\lim_{x \to \infty} \left(\frac{x+2}{x-1}\right)^x$ (Creative)

Part-IV

Answer all the Questions: **4X5=20**

23. (a) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$ (OR) Eg: 7.29

(b) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$ 8.1 → (12)

24. (a) Show that the points whose position vectors $4\hat{\imath} + 5\hat{\jmath} + \hat{k}, -\hat{\jmath} - \hat{k}, 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ are coplanar. (OR) 8.2 → 10

(b) Prove that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

25. (a) Evaluate : $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$ (OR) 9.4 → 28

(b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ 7.8 → 4

26. (a) Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$ (OR) Eg: 7.25

(b) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3|\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a}.\vec{b} + 3\vec{b}.\vec{c} + 3\vec{c}.\vec{a}$ 8.3 → 14

1) 1) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

2) 4) 3

3) 3) 0

4) 3) 6

5) 3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

6) 1) $\frac{1}{3}$

7) 1) 1

8) 3) does not exist

9) 1) $\frac{1}{2}$

10) 2) discontinuous

11) $A^2 = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

12) $(x-1)(x-2)(x-3) = 0$

$\Rightarrow x = 1, 2, 3$

13) $\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

$= \frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{50}}$

14) $\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= 4\hat{i} - 3\hat{j} - \hat{k}$

$|\bar{a} \times \bar{b}| = \sqrt{26}$

15) $\lim_{x \to x_0} f(x) = 2x_0^2 + 3x - 5$

$= f(x_0)$

$\therefore f(x)$ is cont. at all points in $\mathbb{R}$.

16) $\lim_{x \to 0} \frac{\tan 2x}{x} = \lim_{x \to 0} \frac{2 \frac{\sin 2x}{\cos 2x}}{2x}$

$= \lim_{x \to 0} \frac{\sin 2x}{2x} \lim_{x \to 0} \frac{2}{\cos 2x}$

$= (1) \frac{2}{\cos(0)} = 2$

17) LHS $= \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix} \times \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}$

$= \begin{vmatrix} 0+c^2+b^2 & 0+0+ab & 0+ac+0 \\ 0+0+ab & c^2+0+a^2 & bc+0+0 \\ 0+ac+0 & bc+0+0 & b^2+a^2+0 \end{vmatrix}$

$= \begin{vmatrix} c^2+b^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & b^2+a^2 \end{vmatrix} = \text{RHS}$

18) Let $|A| = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

$= \begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix} \quad C_1 \to C_1 + C_2$

$= (a+b+c) \begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & b+a & 1 \end{vmatrix}$

$= 0$

$\therefore$ The points are collinear.

19) $(\bar{a} + \bar{b} + \bar{c})^2 = \bar{0}^2$

$|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}) = \bar{0}^2$

$\Rightarrow \bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a} = -55$

20) $\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$

$\cos\theta = \frac{\sqrt{2}}{\sqrt{50}\sqrt{101}} = \frac{\sqrt{2}}{5\sqrt{101}}$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{5\sqrt{101}}\right)$

21) $\lim_{x \to 3^-} (4x+5) = 17$

$\lim_{x \to 3^+} (4x-5) = 7$

$\Rightarrow \lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$

$\therefore f(x)$ is discont. at $x = 3$.

22) $\lim_{x \to \infty} \left(\frac{x+2}{x-1}\right)^x = \lim_{x \to \infty} \left(\frac{x-1+3}{x-1}\right)^{x-1+1} = \lim_{x \to \infty} \left(1 + \frac{3}{x-1}\right)^{x-1+1}$

Let $x - 1 = y$; $y \to \infty$ as $x \to \infty$

$\lim_{x \to \infty} \left(\frac{x+2}{x-1}\right)^x = \lim_{y \to \infty} \left(1 + \frac{3}{y}\right)^{y+1}$

$= \lim_{y \to \infty} \left(1 + \frac{3}{y}\right)^y \cdot \lim_{y \to \infty} \left(1 + \frac{3}{y}\right)$

$= e^3 (1) = e^3$

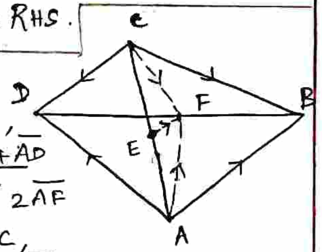
23a) LHS $= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times (-1) \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & -b & -c \\ c & a & b \\ b & c & a \end{vmatrix}$

$= \text{RHS}$.

23b)



In ΔABD,

$\overline{AF} = \frac{\overline{AB} + \overline{AD}}{2}$

$\overline{AB} + \overline{AD} = 2\overline{AF}$

In ΔCBC,

$\overline{CF} = \frac{\overline{CB} + \overline{CD}}{2} \Rightarrow \overline{CB} + \overline{CD} = 2\overline{CF}$

In ΔACF,

$\overline{EF} = \frac{\overline{CF} + \overline{AF}}{2} \Rightarrow \overline{CF} + \overline{AF} = 2\overline{EF}$

$\therefore \overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = 2(\overline{AF} + \overline{CF})$

$= 2(2\overline{EF}) = 4\overline{EF}$

24a) $\overline{AB} = s\,\overline{BC} + t\,\overline{CD}$

$3s - 7t = -4 \to (1)$

$10s - 5t = -6 \to (2)$

$5s = -2 \to (3)$

Solving (2), (3),

$\boxed{s = -2/5,\ t = 2/5}$

In (1) $\Rightarrow -4 = -4$

24b) (Proof)

25a) $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$

$= \lim_{x \to 0} \frac{\sin x (1 - \cos x)}{x^3 \cos x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1}{2\cos x} \lim_{x \to 0} \left(\frac{\sin x/2}{x/2}\right)^2$

$= (1) \frac{1}{2(1)} (1)^2 = \frac{1}{2}$

25b) LHS $= \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix} \quad R_1 \to R_1 - \quad R_2 \to R_2 -$

$= a[b(1+c) + c] + bc$

$= abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \text{RHS}$

26a) Taking $p = 0, q = 0, r = 0$

$|A| = 0$ [in separate case]

$\Rightarrow pqr$ is a factor of $|A|$

Taking $p + q + r = 0$,

$|A| = \begin{vmatrix} p^2 & p^2 & p^2 \\ q^2 & q^2 & q^2 \\ r^2 & r^2 & r^2 \end{vmatrix} = 0$

$\Rightarrow pqr(p+q+r)^2$ is a factor

$m = 6 - 5 = 1$

$|A| = K(p+q+r)pqr(p+q+r)^2$

Solving $\boxed{K = 2}$

LHS = RHS.

26b) $\bar{a}\cdot\bar{b} = \frac{3}{2}$; $\bar{b}\cdot\bar{c} = \frac{-21}{2}$

$\bar{c}\cdot\bar{a} = \frac{-11}{2}$

$4\bar{a}\cdot\bar{b} + 3\bar{b}\cdot\bar{c} + 3\bar{c}\cdot\bar{a} = -42$