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UNIT – I (SETS, RELATIONS AND FUNCTIONS)

1. In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A; 25% know Language B; 10% know Language C; 5% know Languages A and B; 4% know Languages B and C; and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A: [Eg:1.2]
2. If A and B are two sets so that n(B - A) =2n(A - B) =4n(A∩B) and if n(A∪B) =14, then find n(P(A)). [Eg:1.5]
3. Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k. [Eg:1.6]
4. In the set Z of integers, define mRn if m-n is a multiple of 12. Prove that R is an equivalence relation. [Eg:1.13]
5. Let X = $\{a, b, c, d\}$ and R = $\{(a, a,), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it (i) reflexive (ii)symmetric (iii)transitive (iv)equivalence [Ex:1.2-(2)]
6. Let A = $\{a, b, c\}$ and R = $\{(a, a,), (b, b), (a, c)\}$. Write down the minimum number of ordered pairs to be included to R to make it (i) reflexive (ii)symmetric (iii)transitive (iv)equivalence [Ex:1.2-(3)]
7. On the set of natural numbers let R be the relation defined by aRb if 2a + 3b = 30. Write down the relation by listing all the pairs. Check whether it is (i) reflexive(ii)symmetric(iii)transitive(iv)equivalence [Ex:1.2-(5)]
8. On the set of natural numbers let R be the relation defined by aRb if a + b ≤ 6. Write down the relation by listing all the pairs. Check whether it is (i) reflexive(ii)symmetric(iii)transitive(iv)equivalence [Ex:1.2-(7)]
9. In the set Z of integers, define mRn if m - n is divisible by 7. Prove that R is an equivalence relation. [Ex:1.2-(9)]
10. Find the range of the function f(x) = $\frac{1}{1-3\cos x}$ [Eg:1.23]
11. Let f, g : R →R be defined as f(x) = 2x - $|x|$ and g(x) = 2x + $|x|$. Find f∘g. [Eg:1.29]
12. If f : R →R is defined by f(x) =2x-3 prove that f is a bijection and find its inverse. [Eg:1.30]
13. Write the values of f at - 4, 1, - 2, 7, 0 if $f(x) = \begin{cases} -x+4; & -\infty < x \le -3 \\ x+4; & -3 < x < -2 \\ x^2 - x; & -2 \le x < 1 \\ x - x^2; & 1 \le x < 7 \\ 0; & \text{otherwise} \end{cases}$ [Ex:1.3-(2)]
14. Write the values of f at -3, 5, 2,-1, 0 if f(x) == $\begin{cases} x^2+x-5; & x \in (-\infty, 0) \\ x^2+3x-2; & x \in (3, \infty) \\ x^2; & x \in (0,2) \\ x^2-3; & \text{otherwise} \end{cases}$ [Ex:1.3-(3)]
15. If f, g : R → R are defined by f(x) = $|x|$ + x and g(x) = $|x|$- x, find g∘f and f∘g. [Ex:1.3-(10)]
16. If f : R → R is defined by f(x) =3x - 5, prove that f is a bijection and find its inverse. [Ex:1.3-(12)]
17. The formula for converting from Fahrenheit to Celsius temperatures is y = $\frac{5x}{9}$ - $\frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function. [Ex:1.3-(19)]
18. Write the steps to obtain the graph of the function y = $3(x-1)^2$+ 5 from the graph y = x^2. [Ex:1.4-(4)]
19. From the curve y = sin x, graph the functions (i) y = sin(-x) (ii) y = - sin(-x) (iii) y = sin $\left(\frac{\pi}{2}+x\right)$ which is cos x (iv) y = sin $\left(\frac{\pi}{2}-x\right)$ which is also cos x (refer trigonometry) [Ex:1.4-(5)]
20. From the curve y = x, draw (i) y = -x (ii) y = 2x (iii) y = x + 1 (iv) y = $\frac{1}{2}$x+1 (v) 2x + y + 3=0. [Ex:1.4-(6)]
21. From the curve y = $|x|$, draw(i) y = $|x-1|+1$ (ii) y = $|x+1| - 1$ (iii) y = $|x+2| + 3$. [Ex:1.4-(7)]

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UNIT – II (BASIC ALGEBRA)

1. A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent? [Ex:2.3-(6)]

2. Find the number of solutions of $X^2 + |X - 1| = 1$ [Eg:2.12]

3. A quadratic polynomial has one of its zeros $1 + \sqrt{5}$ and it satisfies p(1) =2.
   Find the quadratic polynomial. [Ex:2.4-(2)]

4. If one root of $k(x - 1)^2 = 5x - 7$ is double the other root, show that k = 2 or -25. [Ex:2.4-(4)]

5. Solve $\frac{x+1}{x+3} < 3$. [Eg:2.24]

6. Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$ [Ex:2.8-(1)]

7. Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$. [Ex:2.8-(2)]

8. Solve: $\frac{x^2-4}{x^2-2x-15} \le 0$ [Ex:2.8-(3)]

9. Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$ [Eg:2.26]

10. Resolve into partial fractions: $\frac{x+1}{x^2(x-1)}$ [Eg:2.27]

11. Resolve into partial fractions: $\frac{3x+1}{(x-2)(x+1)}$ [Ex:2.9-(2)]

12. Resolve into partial fractions: $\frac{x^2+x+1}{x^2-5x+6}$ [Ex:2.9-(7)]

13. Resolve into partial fractions: $\frac{x^2+2x+1}{x^2+5x+6}$ [Ex:2.9-(8)]

14. Resolve into partial fractions: $\frac{x+12}{(x+1)^2(x-2)}$ [Ex:2.9-(9)]

15. Resolve into partial fractions: $\frac{2x^2+5x-11}{x^2+2x-3}$ [Ex:2.9-(11)]

16. Resolve into partial fractions: $\frac{7+x}{(1+x)(1+x^2)}$ [Ex:2.9-(12)]

17. Resolve into partial fractions: $\frac{1}{(x-1)(x^2-9)}$ [creative]

18. Simplify: $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$ [Ex:2.11-(7)]

19. Prove $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$ [Eg:2.36]

20. Solve $\log_8 x + \log_4 x + \log_2 x = 11$. [Ex:2.12-(3)]

21. If $a^2 + b^2 = 7ab$; show that $\log\frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$ [Ex:2.12-(5)]

22. Prove $\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab} = 0$ [Ex:2.12-(6)]

23. Prove that $\log 2 + 16\log\log\frac{16}{15} + 12\log\log\frac{25}{24} + 7\log\log\frac{81}{80} = 1$ [Ex:2.12-(7)]

24. Prove $\log_{a^2} a \log_{b^2} b \log_{c^2} c = \frac{1}{8}$ [Ex:2.12-(8)]

25. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that xyz = 1. [Ex:2.12-(10)]

26. Solve $\log_{5-x}(x^2 - 6x + 65) = 2$ [Ex:2.12-(12)]

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UNIT – III (TRIGONOMETRY)

1. If a cos θ - b sin θ = c, show that a sin θ + b cos θ = $\pm\sqrt{a^2+b^2-c^2}$ [Ex:3.1-(3)]

2. If $\frac{\cos^4\alpha}{\cos^2\beta}+\frac{\sin^4\alpha}{\sin^2\beta}=1$, prove that (i) $\sin^4\alpha+\sin^4\beta=2\sin^2\alpha\sin^2\beta$ (ii) $\frac{\cos^4\beta}{\cos^2\alpha}+\frac{\sin^4\beta}{\sin^2\alpha}=1$ [Ex:3.1-(5)]

3. If $\tan^2\theta=1-k^2$, show that sec θ + $\tan^3\theta$ cosec θ = $(2-k^2)^{3/2}$.
   Also, find the values of k for which this result holds. [Ex:3.1-(8)]

4. If cot θ (1 +sin θ) =4m and cot θ (1 - sin θ) =4n, then prove that $(m^2-n^2)^2$ = 4mn [Ex:3.1-(10)]

5. Prove that $\frac{\cot(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\tan(-\theta)\operatorname{cosec}(360°+\theta)}=\cos^2\theta\cot\theta$ [Ex:3.3-(4)]

6. Find the values of (i) cos 15° and (ii) tan 165 °. [Eg:3.15]

7. If sin x = $\frac{4}{5}$ (in I quadrant)and cos y = $-\frac{12}{13}$ (in II quadrant), then find (i) sin(x - y), (ii) cos(x - y). [Eg:3.16]

8. If sin x = $\frac{15}{17}$ and cos y = $\frac{12}{13}$, $0<x<\frac{\pi}{2}$, $0<y<\frac{\pi}{2}$, find the value of (i) sin(x + y) (ii) cos(x - y) (iii) tan(x + y) [Ex:3.4-(1)]

9. If sin A = $\frac{3}{5}$ and cos B = $\frac{9}{41}$, $0<A<\frac{\pi}{2}$, $0<B<\frac{\pi}{2}$, find the value of (i) sin(A + B) (ii) cos(A - B) [Ex:3.4-(2)]

10. If $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$,
    then prove that cos α + cos β + cos γ = sin α + sin β + sin γ = 0. [Ex:3.4-(19)]

11. Show that (i) tan(45° + A) = $\frac{1+\tan A}{1-\tan A}$ (ii) tan(45° - A) = $\frac{1-\tan A}{1+\tan A}$ [Ex:3.4-(20)]

12. If $\theta+\emptyset=\alpha$ and tan θ = k tan $\emptyset$, then prove that $\sin(\theta-\emptyset)=\frac{k-1}{k+1}\sin\alpha$ [Ex:3.4-(25)]

13. Find the values of (i) sin 18° (ii) cos 18° (iii) sin 72° (iv) cos 36° (v) sin 54° [Eg:3.29]

14. Prove that cos A cos 2A cos 2^2 A cos 2^3 A…cos 2^{n-1} A = $\frac{\sin 2^n A}{2^n\sin A}$ [Eg:3.32]

15. Prove that sin 4α = 4 tan α $\frac{1-\tan^2\alpha}{(1+\tan^2\alpha)^2}$ [Ex:3.5-(5)]

16. If A + B = 45 °, show that (1 + tan A) (1+tanB) =2. [Ex:3.5-(6)]

17. Prove that (1 +tan1°)(1 +tan2°)(1 +tan3°) … (1 +tan 44°) is a multiple of 4. [Ex:3.5-(7)]

18. Show that $\cot\left(7\frac{1}{2}°\right)=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$. [Ex:3.5-(9)]

19. Prove that $32\sqrt{3}\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6}=3$. [Ex:3.5-(11)]

20. Show that cos 10° cos 30° cos 50° cos 70° = $\frac{3}{16}$. [Eg:3.38]

21. Prove that $\frac{\sin 4x+\sin 2x}{\cos 4x+\cos 2x}$ = tan3x. [Ex:3.6-(8)]

22. If A + B + C = π, prove the following (i) cosA + cosB + cosC = 1+ 4 sin $\left(\frac{A}{2}\right)$ sin $\left(\frac{B}{2}\right)$ sin $\left(\frac{C}{2}\right)$ [Eg:3.39]

23. Prove that sin $\left(\frac{A}{2}\right)$ + sin $\left(\frac{B}{2}\right)$ + sin $\left(\frac{C}{2}\right)$= 1+ 4sin$\left(\frac{\pi-A}{4}\right)$ sin$\left(\frac{\pi-B}{4}\right)$ sin$\left(\frac{\pi-C}{4}\right)$ if A + B + C = π [Eg:3.40]

24. If A + B + C = π, prove that $\cos^2$ A + $\cos^2$ B + $\cos^2$ C = 1 - 2 cosA cosB cosC [Eg:3.41]

25. If A + B + C = 180°, prove that sin 2A + sin2B + sin2C = 4sinA sinB sinC [Ex:3.7-(1) -i]

26. If A + B + C = 180° , then tan $\left(\frac{A}{2}\right)$ tan $\left(\frac{B}{2}\right)$ + tan $\left(\frac{B}{2}\right)$ tan $\left(\frac{C}{2}\right)$ + tan $\left(\frac{C}{2}\right)$ tan $\left(\frac{A}{2}\right)$ = 1 [Ex:3.7-(1) -v]

27. If A + B + C = $\frac{\pi}{2}$, prove the following (i) sin 2A + sin2B + sin2C = 4cosAcosB cosC
(ii) cos 2A + cos2B + cos2C = 1+4sinAsinB sinC. [Ex:3.7-(4)]

28. Solve cos x + sin x = cos2x + sin2x [Eg:3.47]

29. Solve $\sqrt{3}$ sin θ - cos θ = $\sqrt{2}$ [Eg:3.54]

30. Solve $\sqrt{3}$ $\tan^2\theta$ + ($\sqrt{3}$ – 1) tan θ - 1 =0 [Eg:3.55]

31. Solve the following equation: sin θ + $\sqrt{3}$ cos θ = 1 [Ex:3.8-(3)-vii]

32. In a Δ ABC, prove that sin $\left(\frac{B-C}{2}\right)$ = $\frac{b-c}{a}$ cos $\frac{A}{2}$ [Eg:3.58]

33. Using Heron's formula, show that the equilateral triangle has the maximum area for any fixed perimeter.
[Hint: In xyz ≤ k; maximum occurs when x = y = z]. [Eg:3.63]

34. Find the principal value of (i) $\sin^{-1}\frac{1}{\sqrt{2}}$ (ii) $\cos^{-1}\frac{\sqrt{3}}{2}$ (iii) $\text{cosec}^{-1}(-1)$ (iv) $\sec^{-1}(-\sqrt{2})$ (v) $\tan^{-1}(\sqrt{3})$.

35. State and prove the law of Sines.

36. Prove Napier's Formula

37. prove the law of Cosines

38. prove the Projection Formula

39. Prove the Heron's Formula (Area of a triangle)

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UNIT – IV (COMBINATORICS AND MATHEMATICAL INDUCTION)

1. Prove that $\frac{(2n)!}{n!} = 2^n(1.3.5 \ldots (2n-1))$. [Eg:4.24]

2. How many strings can be formed using the letters of the word LOTUS if the word

   (i) either starts with L or ends with S? (ii) neither starts with L nor ends with S? [Ex:4.1-(12)]

3. If the letters of the word TABLE are permuted in all possible ways and the words thus formed are arranged in the dictionary order (alphabetical order), find the ranks of the words (i) TABLE, (ii) BLEAT [Eg:4.35]

4. If the letters of the word IITJEE are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word IITJEE [Eg:4.42]

5. In how many ways 4 mathematics books, 3 physics books, 2 chemistry books and 1 biology book can be arranged on a shelf so that all books of the same subjects are together. [Ex:4.2-(11)]

6. How many strings are there using the letters of the word INTERMEDIATE, if [Ex:4.2-(14)]

   (i) The vowels and consonants are alternative (ii) All the vowels are together

   (iii) Vowels are never together (iv) No two vowels are together.

7. If the letters of the word FUNNY are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word FUNNY. [Ex:4.2-(18)]

8. If nPr = 11880 and nCr = 495, Find n and r. [Eg:4.47]

9. If (n+2)C7 :(n-1) P4 = 13:24 find n. [Eg:4.50]

10. An exam paper contains 8 questions, 4 in Part A and 4 in Part B. Examiners are required to answer 5 questions. In how many ways can this be done if (i) There are no restrictions of choosing a number of questions in either parts.

    (ii) At least two questions from Part A must be answered. [Eg:4.56]

11. Find the number of strings of 5 letters that can be formed with the letters of the word PROPOSITION. [Eg:4.58]

12. In an examination a student has to answer 5 questions, out of 9 questions in which 2 are compulsory. In how many ways a student can answer the questions? [Ex:4.3-(15)]

13. A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of (i) exactly 3 women? (ii) at least 3 women? (iii) at most 3 women? [Ex:4.3-(17)]

14. 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife' s relatives? [Ex:4.3-(18)]

15. By the principle of mathematical induction, prove that, for all integers $n \geq 1$, [Eg:4.61]

$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.

16. By the principle of mathematical induction, prove that, for all integers $n \geq 1$, [Eg:4.63]

$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

17. Using the Mathematical induction, show that for any natural number n, [Eg:4.64]

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

18. Prove that for any natural number n, $a^n - b^n$ is divisible by a - b, where a >b. [Eg:4.65]

19. Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$. [Eg:4.66]

20. By the principle of mathematical induction, prove that, for $n \geq 1$ [Ex:4.4-(1)]

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

21. By the principle of mathematical induction, prove that, for $n \geq 1$ [Ex:4.4-(2)]

$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.

22. Prove that the sum of the first n non-zero even numbers is $n^2 + n$. [Ex:4.4-(3)]

23. Using the Mathematical induction, show that for any natural number n, [Ex:4.4-(7)]

$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$.

24. Prove by Mathematical Induction that $1! + (2 \times 2!) + (3 \times 3!) + \ldots + (n \times n!) = (n+1)! - 1$. [Ex:4.4-(9)]

25. Use induction to prove that $n^3 - 7n + 3$, is divisibleby3,for all natural numbers n. [Ex:4.4-(12)]

26. Prove that using the Mathematical induction

$\sin(\alpha) + \sin\left(\alpha + \frac{\pi}{6}\right) + \sin\left(\alpha + \frac{2\pi}{6}\right) + \ldots + \sin\left(\alpha + \frac{(n-1)\pi}{6}\right) = \frac{\sin\left(\alpha + \frac{(n-1)\pi}{12}\right) \times \sin\frac{n\pi}{12}}{\sin\frac{\pi}{12}}$ [Ex:4.4-(15)]

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UNIT – V (BINOMIAL THEOREM, SEQUENCES AND SERIES)

1. If a and b are distinct integers, prove that a - b is a factor of $a^n - b^n$, whenever n is a positive integer. [Ex:5.1-(12)]
2. In the binomial expansion of $(a + b)^n$, if the coefficients of the 4^{th} and 13^{th} terms are equal then, find n. [Ex:5.1-(13)]
3. In the binomial expansion of $(1+x)^n$, the coefficients of the 5^{th}, 6^{th} and 7^{th} terms are in AP. Find all values of n. [Ex:5.1-(15)]
4. Prove that $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^n = \frac{(2n)!}{(n!)^2}$. [Ex:5.1-(16)]
5. If the product of the 4^{th}, 5^{th} and 6^{th} terms of a geometric progression is 4096 and if the product of the 5^{th}, 6^{th} and 7^{th} terms of it is 32768, find the sum of first 8 terms of the geometric progression. [Eg:5.15]
6. If t_k is the k^{th} term of a GP, then show that t_{n-k}, t_n, t_{n+k} also form a GP for any positive integer k. [Ex:5.2-(6)]
7. If a, b, c are in geometric progression, and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then prove that x, y, z are in arithmetic progression. [Ex:5.2-(7)]
8. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers. [Ex:5.2-(8)]
9. If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of a GP, show that (q - r) log a + (r - p) log b + (p - q) log c = 0. [Ex:5.2-(10)]
10. Compute the sum of first n terms of the following series: 8 + 88 + 888 + 8888 + … [Ex:5.3-(3) (i)]
11. Find the sum: $1 + \frac{4}{5} + \frac{7}{25} + \frac{10}{125} + \dots$ [Eg:5.19]
12. Find $\sqrt[3]{65}$. [Eg:5.24]
13. Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ .when x is large. [Eg:5.25]
14. Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large. [Ex:5.4-(3)]
15. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small. [Ex:5.4-(4)]
16. If p - q is small compared to either p or q, then show that $\sqrt[n]{\frac{p}{q}} \simeq \frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$. Hence find $\sqrt[8]{\frac{15}{16}}$. [Ex:5.4-(8)]

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UNIT – VI (TWO DIMENSIONAL ANALYTICAL GEOMETRY)

1. Find the locus of a point P moves such that its distances from two fixed points A(1, 0) and B (5, 0) are always equal. [Eg:6.3]
2. Find the equation of the locus of a point such that the sum of the squares of the distance from the points (3, 5), (1,-1) is equal to 20 [Ex:6.1-(6)]
3. If R is any point on the x-axis and Q is any point on the y-axis and P is a variable point on RQ with RP = b, PQ = a. then find the equation of locus of P. [Ex:6.1-(11)]
4. The sum of the distance of a moving point from the points (4, 0) and (-4, 0) is always10 units. Find the equation of the locus of the moving point. [Ex:6.1-(15)]
5. A straight line L with negative slope passes through the point (9; 4) cuts the positive coordinate axes at the points P and Q. As L varies, find the minimum value of $|OP|+|OQ|$, where O is the origin. [Eg:6.15]
6. Find the equation of the lines make an angle 60°with positive x-axis and at a distance $5\sqrt{2}$ units measured from the point (4, 7) along the line x - y + 3=0. [Eg:6.18]
7. Express the equation $\sqrt{3}x - y + 4=0$ in the following equivalent form: (i) Slope and Intercept form (ii) Intercept form (iii) Normal form [Eg:6.19]
8. Rewrite $\sqrt{3}x + y + 4=0$ in to normal form. [Eg:6.20]
9. Show that the points(1,3),(2,1)and ($\frac{1}{2}$, 4) are collinear, by using (i)concept of slope (ii)using a straight line and (iii)any other method [Ex:6.2-(10)]
10. Find the equations of a parallel line and a perpendicular line passing through the point (1,2) to the line 3x+ 4y = 7. [Eg:6.22]
11. Find the distance (i) between two points(5,4)and(2,0) (ii) from a point(1,2) to the line 5x + 12y - 3 =0 [Eg:6.23] (iii) between two parallel lines 3x + 4y = 12 and 6x + 8y + 1=0.
12. Find the equation of the line through the intersection of the lines 3x+2y+5=0 and 3x - 4y + 6=0 and the point(1,1). [Eg:6.29]
13. If a line joining two points (3,0) and (5,2) is rotated about the point (3,0) in counter clockwise direction through an angle15°, then find the equation of the line in the new position. [Eg:6.32]
14. Find the equation of the lines passing through the point of intersection lines 4x - y + 3=0 and 5x+2y +7=0, and (i) through the point (-1, 2) (ii) Parallel to x-y +5=0 (iii) Perpendicular to x - 2y + 1=0 [Ex:6.3-(6)]
15. Find the equations of straight lines which are perpendicular to the line 3x + 4y - 6 =0 and are at a distance of 4 units from (2,1). [Ex:6.3-(8)]
16. Find the equation of a straight line parallel to 2x + 3y = 10 and which is such that the sum of its intercepts on the axes is15. [Ex:6.3-(9)]
17. If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines x sec θ + y cosec θ = 2a and x cos θ - y sin θ = a cos 2 θ, then prove that $P_1^2 + P_2^2 = a^2$. [Ex:6.3-(11)]
18. Separate the equations $5x^2 + 6xy + y^2 = 0$ [Eg:6.33]
19. Show that the straight lines $x^2 - 4xy + y^2 = 0$ and x + y = 3 form an equilateral triangle. [Eg:6.36]

20. If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, find (i) the value of λ and the separate equations of the lines (ii) point of intersection of the lines (iii) angle between the lines [Eg:6.38]
21. Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $tan^{-1}(5)$. [Ex:6.4-(4)]
22. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$. [Ex:6.4-(8)]
23. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$. [Ex:6.4-(9)]
24. A ΔOPQ is formed by the pair of straight lines $x^2 - 4xy + y^2 = 0$ and the line PQ. The equation of PQ is $x + y - 2 = 0$. Find the equation of the median of the triangle ΔOPQ drawn from the origin O. [Ex:6.4-(10)]
25. Find p and q, if the following equation represents a pair of perpendicular lines $6x^2 + 5xy - py^2 + 7x + qy - 5 = 0$ [Ex:6.4-(11)]
26. Find the value of k, if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting, $12x^2 + 7xy - 12y^2 - x + 7y + k = 0$. [Ex:6.4-(12)]
27. For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight lines. [Ex:6.4-(13)]
28. Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them. [Ex:6.4-(14)]
29. Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them. [Ex:6.4-(15)]
30. Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a + b)^2 = 4h^2$. [Ex:6.4-(16)]
31. If the pair of straight lines $x^2 - 2kxy - y^2 = 0$ bisect the angle between the pair of straight lines $x^2 - 2lxy - y^2 = 0$, Show that the later pair also bisects the angle between the former. [Ex:6.4-(17)]
32. Prove that the straight lines joining the origin to the points of intersection of $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and $3x - 2y - 1 = 0$ are at right angles. [Ex:6.4-(18)]

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UNIT – VII (MATRICES AND DETERMINANTS)

1. Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices. [Eg:7.13]

2. Express the matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix. [Ex:7.1-(17)-ii]

3. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$ = 4 $a^2b^2c^2$. [Ex:7.2-(3)]

4. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ = abc$\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. [Ex:7.2-(4)]

5. If $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$, prove that *a*, *b*, *c* are in *G.P.* or α is a root of $ax^2 + 2bx + c = 0$. [Ex:7.2-(8)]

6. If *A* = $\begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1-\frac{1}{4^n}\right)$. [Ex:7.2-(14)]

7. Verify that det(*AB*) = (det *A*) (det *B*) for A = $\begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$. [Ex:7.2-(20)]

8. Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$ = (x-y) (y-z) (z-x) (xy+yz+zx). [Eg:7.24]

9. Prove that |*A*| = $\begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix}$ = 2 pqr $(p+q+r)^2$. [Eg:7.25]

10. Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2\,(x+2a)$ [Ex:7.3-(1)]

11. Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix}$ = 8 abc [Ex:7.3-(2)]

12. Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$ = (a+b+c) (a-b) (b-c) (c-a). [Ex:7.3-(4)]

13. Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ = (x-y) (y-z) (z-x). [Ex:7.3-(6)]

14. Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$. [Eg:7.29]

15. Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$. [Eg:7.30]

16. If A_1 , B_1 , C_1 are the cofactors of a_i , b_i , c_i , respectively, i = 1 to 3 in $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, [Eg:7.31]

show that $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = |A|^2$

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UNIT – VIII (VECTOR ALGEBRA)

1. prove the Section Formula - Internal Division (Theorem 8.1)
2. Prove : The medians of a triangle are concurrent. (Theorem 8.3)
3. Prove : A quadrilateral is a parallelogram if and only if its diagonals bisect each other. (Theorem 8.4)
4. If *D* and *E* are the midpoints of the sides *AB* and *AC* of a triangle *ABC*, prove that $\overrightarrow{BE}+\overrightarrow{DC}=\frac{1}{2}\overrightarrow{BC}$. [Ex:8.1-(4)]
5. If *ABCD* is a quadrilateral and *E* and *F* are the midpoints of *AC* and *BD* respectively, then prove that $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}=4\overrightarrow{EF}$ [Ex:8.1-(12)]
6. Prove that the points whose position vectors $2\hat{\imath}+4\hat{\jmath}+3\hat{k}$, $4\hat{\imath}+\hat{\jmath}+9\hat{k}$ and $10\hat{\imath}-\hat{\jmath}+6\hat{k}$ form a right angled triangle. [Eg:8.9]
7. Show that the vectors $5\hat{\imath}+6\hat{\jmath}+7\hat{k}$, $7\hat{\imath}-8\hat{\jmath}+9\hat{k}$,$3\hat{\imath}+20\hat{\jmath}+5\hat{k}$ are coplanar. [Eg:8.10]
8. A triangle is formed by joining the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). Find the direction cosines of the medians. [Ex:8.2-(4)]
9. Show that the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$,$\hat{\imath}-3\hat{\jmath}-5\hat{k}$ form a right angled triangle. [Ex:8.2-(7)]
10. Show that the following vectors $\hat{\imath}-2\hat{\jmath}+3\hat{k}$, $-2\hat{\imath}+3\hat{\jmath}-4\hat{k}$,$-\hat{\jmath}+2\hat{k}$ are coplanar [Ex:8.2-(9)-i]
11. Show that the points whose position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}$, $-\hat{\jmath}-\hat{k}$,$3\hat{\imath}+9\hat{\jmath}+4\hat{k}$ and $-4\hat{\imath}+4\hat{\jmath}+4\hat{k}$ are coplanar. [Ex:8.2-(10)]
12. Show that the vectors $\vec{a}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$, $\vec{b}=6\hat{\imath}+2\hat{\jmath}-3\hat{k}$ and $\vec{c}=3\hat{\imath}-6\hat{\jmath}+2\hat{k}$are mutually orthogonal. [Ex:8.3-(6)]
13. Show that the vectors $-\hat{\imath}-2\hat{\jmath}-6\hat{k}$, $2\hat{\imath}-\hat{\jmath}+\hat{k}$ and$-\hat{\imath}+3\hat{\jmath}+5\hat{k}$ form a right angled triangle. [Ex:8.3-(7)]
14. If $\vec{a},\vec{b}$ are unit vectors and θ is the angle between them, show that $tan\frac{\theta}{2}=\frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$. [Ex:8.3-(10)-iii]
15. Three vectors $\vec{a},\vec{b}$ and $\vec{c}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=4$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Find $4\vec{a}.\vec{b}+3\vec{b}.\vec{c}+3\vec{c}.\vec{a}$ [Ex:8.3-(14)]
16. Find the cosine and sine angle between the vectors $\vec{a}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$ and $\vec{b}=4\hat{\imath}-2\hat{\jmath}+2\hat{k}$. [Eg:8.23]
17. Find the area of the triangle whose vertices are *A*(3, - 1, 2), *B*(1, - 1, - 3) and *C*(4, - 3, 1). [Ex:8.4-(6)]

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UNIT – IX (DIFFERENTIAL CALCULUS - LIMITS AND CONTINUITY)

1. Calculate $\lim_{x\to 0}|x|$ [Eg: 9.1] 2. $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$ [Theorem 9.4]

Evaluate the following limits :

3. $\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$ [Ex: 9.2-(8)] 4. $\lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{x}$ [Ex: 9.2-(12)] 5. $\lim_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$ (a>b) [Ex: 9.2-(15)]

6. Suppose that the diameter of an animal's pupils is given by $f(x) = \frac{160x^{-0.4}+90}{4x^{-0.4}+15}$, where *x* is the intensity of light and *f*(*x*) is in *mm.* Find the diameter of the pupils with (a) minimum light (b) maximum light. [Eg: 9.28]

7. Show that $\lim_{x\to\infty}\frac{1+2+3+\cdots+n}{3n^2+7n+2} = \frac{1}{6}$ [Ex: 9.3-8(i)] 8. Show that $\lim_{x\to 0^+}\left[\left\lfloor\frac{1}{x}\right\rfloor+\left\lfloor\frac{2}{x}\right\rfloor+\cdots+\left\lfloor\frac{15}{x}\right\rfloor\right]$ =120 [Eg: 9.31]

9. $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ [Result 9.1] 10. Evaluate : $\lim_{x\to\infty}\left(\frac{x+2}{x-2}\right)^x$ [Eg: 9.33]

Evaluate the following limits :

11. $\lim_{x\to 0}\frac{tan\,2x}{sin\,2x}$ [Ex: 9.4-(8)] 12. $\lim_{x\to 0}\frac{3^x-1}{\sqrt{x+1}-1}$ [Ex: 9.4-(16)]

13. $\lim_{x\to\frac{\pi}{2}}(1+sin\,x)^{2cosec\,x}$ [Ex: 9.4-(21)] 14. $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$ [Ex: 9.4-(23)]

15. $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ [Ex: 9.4-(24)] 16. $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$ [Ex: 9.4-(28)]

17. Describe the interval(s) on which each function is continuous.

(i) *f*(*x*) = tan *x* (ii) $g(x) = \begin{cases} sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ (iii) $h(x) = \begin{cases} x\,sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ [Eg: 9.36]

18. Determine if *f* defined by $f(x) = \begin{cases} x^2 sin\frac{1}{x}, & if\ x\neq 0\\ 0, & if\ x=0\end{cases}$ is continuous in R [Eg: 9.38]

19. Find the points of discontinuity of the function *f*, where $f(x) = \begin{cases}\sin x, & 0\le x\le\frac{\pi}{4}\\ \cos x, & \frac{\pi}{4}<x<\frac{\pi}{2}\end{cases}$ [Ex: 9.5-3(iv)]

20. Show that the function $\begin{cases}\frac{x^3-1}{x-1}, & if\ x\neq 1\\ 3, & if\ x=1\end{cases}$ is continuous on $(-\infty, \infty)$ [Ex: 9.5-(5)]

21. For what value of α is this function $f(x) = \begin{cases}\frac{x^4-1}{x-1}, & if\ x\neq 1\\ \alpha, & if\ x=1\end{cases}$ continuous at *x* = 1 ? [Ex: 9.5-(6)]

22. If *f* and *g* are continuous functions with *f*(3) = 5 and $\lim_{x\to 3}[2f(x)-g(x)] = 4$, find *g*(3). [Ex: 9.5-(8)]

23. A function *f* is defined as follows : $f(x) = \begin{cases} 0 & for\ x<0;\\ x & for\ 0\le x<1;\\ -x^2+4x-2, & for\ 1\le x<3;\\ 4-x & for\ x\ge 3\end{cases}$ Is the function continuous? [Ex: 9.5-(10)]

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UNIT – X (DIFFERENTIAL CALCULUS- DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION)

1. Differentiate $(2x + 1)^5 (x^3 - x + 1)^4$. [Eg: 10.13]

2. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y' [Eg: 10.16/ Eg: 10.24]

3. Find $\frac{dy}{dx}$ if $x = a(t - \sin t)$, $y = a(1 - \cos t)$. [Eg: 10.27]

4. Find the derivative of x^x with respect to $x \log x$. [Eg: 10.28]

5. Differentiate $\sin(ax^2+bx+c)$ with respect to $\cos(lx^2+mx+n)$. [Eg: 10.30]

6. Find y'' if $x^4 + y^4 = 16$. [Eg: 10.34]

7. Find the derivative. $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$ [Ex: 10.4(18)]

8. If $y = e^{\tan^{-1}x}$, show that $(1+x^2)\, y'' + (2x-1)\, y' = 0$ [Ex: 10.4(24)]

9. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\, y_2 - 3xy_1 - y = 0$ [Ex: 10.4(25)]

10. If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ then prove that at $\theta = \frac{\pi}{2}$, $y'' = \frac{1}{a}$ [Ex: 10.4(26)]

11. If $\sin y = x\sin(a+y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$, $a \neq n\pi$ [Ex: 10.4(27)]

12. If $y = (\cos^{-1}x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$ [Ex: 10.4(28)]

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UNIT – XI (INTEGRAL CALCULUS)

1. A ball is thrown vertically upward from the ground with an initial velocity of 39.2 m/sec. If the only force considered is that attributed to the acceleration due to gravity, find (i) how long will it take for the ball to strike the ground? (ii) the speed with which will it strike the ground? and (iii) how high the ball will rise? [Ex: 11.4(4)]

2. Evaluate : $\int (x-3)\sqrt{x+2}\, dx$ [Eg: 11.27]

3. Evaluate : $\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$ [Eg: 11.28]

4. Evaluate : $\int \frac{x+3}{(x+2)^2(x+1)} dx$ [Eg: 11.29 – (ii)]

Integrate the following with respect to x :

5. $\int \frac{1}{(x-1)(x+2)^2} dx$ [Ex: 11.5(18)]

6. $\int \frac{3x-9}{(x-1)(x+2)(x^2+1)} dx$ [Ex: 11.5(19)]

7. $\int \frac{x^3}{(x-1)(x-2)} dx$ [Ex: 11.5(20)]

8. $\int \frac{2x+4}{x^2+4x+6} dx$ [Eg: 11.32(i)]

Evaluate the following integrals

9. $\int x\cos x\, dx$ [Eg: 11.33(ii)]

10. $\int sin^{-1} x\, dx$ [Eg: 11.33(iv)]

11. Evaluate : $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$ [Eg: 11.34]

12. Integrate the following with respect to x : $\int \tan^{-1}\left(\frac{8x}{1-16x^2}\right)dx$ [Ex: 11.7-3-(iii)]

13. Integrate the following with respect to x : $\int e^{ax} \cos bx\, dx$ [Ex: 11.8-1-(i)]

14. Prove that $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{\sqrt{x^2+a^2}}\right| + c$ [Type – I Proof-(vi)]

15. Evaluate : $\int \frac{1}{(x-2)^2+1} dx$ [Eg: 11.38-(i)]

Evaluate the following integrals

16. $\int \frac{3x+5}{x^2+4x+7} dx$ [Eg: 11.40-(i)]

17. $\int \frac{x+1}{x^2-3x+1} dx$ [Eg: 11.40-(ii)]

18. $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$ [Eg: 11.40-(iii)]

19. $\int \frac{5x-7}{\sqrt{3x-x^2-2}} dx$ [Eg: 11.40-(iv)]

Integrate the following with respect to x :

20. $\int \frac{5x-2}{2+2x+x^2} dx$ [Ex: 11.11-1-(ii)]

21. $\int \frac{2x+1}{\sqrt{9+4x-x^2}} dx$ [Ex: 11.11-2-(i)]

22. $\int \frac{2x+3}{\sqrt{x^2+4x+1}} dx$ [Ex: 11.11-2-(iii)]

Evaluate the following :

23. $\int \sqrt{4-x^2}\, dx$ [Eg: 11.41-(i)]

24. $\int \sqrt{(x-3)(5-x)}\, dx$ [Eg: 11.41-(iv)]

Integrate the following with respect to x :

25. $\int \sqrt{x^2+2x+10}\, dx$ [Ex: 11.12-1-(i)]

26. $\int \sqrt{9-(2x+5)^2}\, dx$ [Ex: 11.12-2-(i)]

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UNIT – XII (INTRODUCTION TO PROBABILITY THEORY)

1. Eight coins are tossed once, find the probability of getting [Ex:12.1-(5)]
   (i) exactly two tails (ii) atleast two tails (iii) atmost two tails
2. Given that $P(A) = 0.52$, $P(B) = 0.43$, and $P(A\cap B) = 0.24$, [Eg:12.14]
   find (i) $P(A \cap \bar{B})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$
3. A coin is tossed twice. Events *E* and *F* are defined as follows *E*= Head on first toss, *F* = Head on second toss.
   Find (i) $P(E \cup F)$ (ii) $P(E / F)$ (iii) $P(\bar{E} / F)$ (iv) Are the events *E* and *F* independent? [Eg:12.19]
4. A main road in a City has 4 crossroads with traffic lights. Each traffic light opens or closes the traffic with the probability of 0.4 and 0.6 respectively. Determine the probability of (i) a car crossing the first crossroad without stopping (ii) a car crossing first two crossroads without stopping (iii) a car crossing all the crossroads, stopping at third cross. (iv) a car crossing all the crossroads, stopping at exactly one cross. [Eg:12.23]
5. One bag contains 5 white and 3 black balls. Another bag contains 4 white and 6 black balls. If one ball is drawn from each bag, find the probability that (i) both are white (ii) both are black (iii) one white and one black [Ex:12.3-(8)]
6. Suppose the chances of hitting a target by a person *X* is 3 times in 4 shots, by *Y* is 4 times in 5 shots, and by *Z* is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits? [Ex:12.3-(12)]
7. A factory has two machines I and II. Machine-I produces 40% of items of the output and Machine-II produces 60% of the items. Further 4% of items produced by Machine-I are defective and 5% produced by Machine-II are defective. If an item is drawn at random, find the probability that it is a defective item. [Eg:12.25]
8. A factory has two machines I and II. Machine I produces 40% of items of the output and Machine II produces 60% of the items. Further 4% of items produced by Machine I are defective and 5% produced by Machine II are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine II. [Eg:12.26]
9. A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work? [Eg:12.27]
10. The chances of *X*, *Y* and *Z* becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if *X*, *Y* and *Z* become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that *Z* was appointed as the manager? [Eg:12.28]
11. A consulting firm rents car from three agencies such that 50% from agency *L*, 30% from agency *M* and 20% from agency *N*. If 90% of the cars from *L*, 70% of cars from *M* and 60% of the cars from *N* are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency *N*? [Eg:12.29]

12. A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective? [Ex:12.4-(1)]
13. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn? [Ex:12.4-(2)]
14. A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,
    (i) find the probability that the selected pipe is a defective one.
    (ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y? [Ex:12.4-(3)]
15. The chances of A, B and C becoming manager of a certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B, and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager? [Ex:12.4-(4)]
16. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that
    (i) the husband is watching the television during the prime time of television
    (ii) if the husband is watching the television, the wife is also watching the television. [Ex:12.4-(5)]