



# KRISHNAMOORTHY

## MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

### 2023-24 Batch +2 Achievers Of Mathematics



$\frac{99}{100}$

*Kavya Sri K*



$\frac{100}{100}$

*Nivendhan S*



$\frac{99}{100}$

*Harini S*



$\frac{99}{100}$

*Palanisamy R*



$\frac{99}{100}$

*Kowsik S*



$\frac{99}{100}$

*Poovainandhan P*



# Q - BANK

## +1 MATHS

Volume - 1&2

Name of the student: \_\_\_\_\_

# VOLUME -1

## CHAPTER 1 - SETS, RELATIONS AND FUNCTIONS

### 2 MARKS

#### SET 1

- Find the number of subsets of  $A$  if  $A = \{x: x = 4n + 1, 2 \leq n \leq 5, n \in \mathbb{N}\}$ . (EG 1.1)
- If  $X = \{1, 2, 3, \dots, 10\}$  and  $A = \{1, 2, 3, 4, 5\}$ , find the number of sets  $B \subseteq X$  such that  $A - B = \{4\}$ . (EG 1.4)
- If  $n(A) = 10$  and  $n(A \cap B) = 3$ , find  $n((A \cap B)' \cap A)$ . (EG 1.7)
- If  $\mathcal{P}(A)$  denotes the power set of  $A$ , then find  $n(\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))))$ . (EG 1.9) If  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6\}$ , find  $n((A \cup B) \times (A \cap B) \times (A \Delta B))$ . (EG 1.8)
- Write the following in roster form. (i)  $\{x \in \mathbb{N}: x^2 < 121 \text{ and } x \text{ is a prime}\}$ . (EX 1.1 - 1)

#### SET 2

- If  $n(\mathcal{P}(A)) = 1024$ ,  $n(A \cup B) = 15$  and  $n(\mathcal{P}(B)) = 32$ , then find  $n(A \cap B)$ . (EX. 1.1 - 6)
- If  $n(A \cap B) = 3$  and  $n(A \cup B) = 10$ , then find  $n(\mathcal{P}(A \Delta B))$ . (EX 1.1 - 7)
- For a set  $A$ ,  $A \times A$  contains 16 elements and two of its elements are  $(1, 3)$  and  $(0, 2)$ . Find the elements of  $A$ . (EX 1.1 - 8)
- Check whether the following functions are one-to-one and onto. (i)  $f: \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(n) = n + 2$ . (ii)  $f: \mathbb{N} \cup \{-1, 0\} \rightarrow \mathbb{N}$  defined by  $f(n) = n + 2$ . (EG 1.14)
- Check the following functions for one-to-oneness and onto. (i)  $f: \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(n) = n^2$ . (ii)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(n) = n^2$ . (EG 1.15)

#### SET 3

- Check whether the following for one-to-oneness and onto. (i)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{1}{x}$ . (ii)  $f: \mathbb{R} - \{0\} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{1}{x}$ . (EG 1.16)
- If  $f: [-2, 2] \rightarrow B$  is given by  $f(x) = 2x^3$  then find  $B$  so that  $f$  is onto. (EG 1.19)
- Find the domain of  $f(x) = \frac{1}{1 - 2\cos x}$ . (EG 1.22)
- Let  $f = \{(1, 2), (3, 4), (2, 2)\}$  and  $g = \{(2, 1), (3, 1), (4, 2)\}$ . Find  $g \circ f$  and  $f \circ g$ . (EG 1.25)
- Let  $f = \{(1, 4), (2, 5), (3, 5)\}$  and  $g = \{(4, 1), (5, 2), (6, 4)\}$ . Find  $g \circ f$ . Can you find  $f \circ g$ ? (EG 1.26)
- Let  $f$  and  $g$  be the two functions from  $\mathbb{R}$  to  $\mathbb{R}$  defined by  $f(x) = 3x - 4$  and  $g(x) = x^2 + 3$ . Find  $g \circ f$  and  $f \circ g$ . (EG 1.27)

#### SET 4

- Find the domain of  $\frac{1}{1 - 2\sin x}$ . (EX 1.3 - 6)
- The weight of the muscles of a man is a function of his body weight  $x$  and can be expressed as  $W(x) = 0.35x$ . Determine the domain of this function. (EX 1.3 - 13)
- For the curve  $y = x^3$  given in the Figure, draw (i)  $y = (x + 1)^3$  with the same scale. (EX 1.4 - 1)
- Graph the functions  $f(x) = x^3$  and  $g(x) = \sqrt[3]{x}$  on the same coordinate plane. Find  $f \circ g$  and graph it on the plane as well. Explain your results. (EX. 1.4 - 3)
- From the curve  $y = \sin x$ , draw  $y = \sin |x|$ . (EX. 1.4 - 8)

### 3 MARKS

#### SET 1

1. In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C, and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A. **(EG 1.2)**
2. If A and B are two sets so that  $n(B - A) = 2n(A - B) = 4n(A \cap B)$  and if  $n(A \cup B) = 14$ , then find  $n(\mathcal{P}(A))$ . **(EG 1.5)**
3. Two sets have  $m$  and  $k$  elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of  $m$  and  $k$ . **(EG 1.6)**
4. Check the relation  $R = \{(1,1), (2,2), (3,3), \dots, (n,n)\}$  defined on the set  $S = \{1,2,3, \dots, n\}$  for the three basic relations. **(EG 1.10)**
5. Discuss the following relations for reflexivity, symmetricity and transitivity: (v) On the set of natural numbers the relation R defined by " $xRy$  if  $x + 2y = 1$ ". **(EX 1.2 - 1)**
6. On the set of natural numbers let R be the relation defined by  $aRb$  if  $2a + 3b = 30$ . Write down the relation by listing all the pairs. Check whether it is (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence **(EX 1.2 - 5)**

#### SET 2

7. On the set of natural numbers let R be the relation defined by  $aRb$  if  $a + b \leq 6$ . Write down the relation by listing all the pairs. Check whether it is (i) reflexive (ii) symmetric (iii) transitive (iv) equivalence **(EX 1.2 - 7)**
8. Let  $A = \{a, b, c\}$ . What is the equivalence relation of smallest cardinality on A? What is the equivalence relation of largest cardinality on A? **(EX 1.2 - 8)**
9. If  $f: \mathbb{R} - \{-1,1\} \rightarrow \mathbb{R}$  is defined by  $f(x) = \frac{x}{x^2-1}$ , verify whether  $f$  is one-to-one or not. **(EG 1.17)**
10. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = 2x^2 - 1$ , find the pre-images of 17, 4 and -2. **(EG 1.18)**
11. Find the largest possible domain for the real valued function  $f$  defined by  $f(x) = \sqrt{x^2 - 5x + 6}$ . **(EG 1.21)**
12. Find the range of the function  $f(x) = \frac{1}{1-3\cos x}$ . **(EG 1.23)**

#### SET 3

13. Find the largest possible domain for the real valued function given by  $f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$ . **(EG 1.24)**
14. Let  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = 2x - |x|$  and  $g(x) = 2x + |x|$ . Find  $f \circ g$ . **(EG 1.29)**
15. Write the values of  $f$  at  $-4, 1, -2, 7, 0$  if  $f(x) = \begin{cases} -x + 4 & \text{if } -\infty < x \leq -3 \\ x + 4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \leq x < 1 \\ x - x^2 & \text{if } 1 \leq x < 7 \\ 0 & \text{otherwise} \end{cases}$ . **(EX 1.3 - 2)**
16. Write the values of  $f$  at  $-3, 5, 2, -1, 0$  if  $f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$ . **(EX 1.3 - 3)**
17. Find the range of the function  $\frac{1}{2\cos x - 1}$ . **(EX 1.3 - 8)**
18. Show that the relation  $xy = -2$  is a function for a suitable domain. Find the domain and the range of the function. **(EX 1.3 - 9)**

**SET 4**

19. If  $f, g, h$  are real valued functions defined on  $\mathbb{R}$ , then prove that  $(f + g) \circ h = f \circ h + g \circ h$ . What can you say about  $f \circ (g + h)$ ? Justify your answer. (EX 1.3 - 11)
20. A salesperson whose annual earnings can be represented by the function  $A(x) = 30,000 + 0.04x$ , where  $x$  is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function  $S(x) = 25,000 + 0.05x$ . Find  $(A + S)(x)$  and determine the total family income if they each sell ₹ 1,50,00,000 worth of merchandise. (EX 1.3 - 16)
21. Graph the functions  $f(x) = x^3$  and  $g(x) = \sqrt[3]{x}$  on the same coordinate plane. Find  $f \circ g$  and graph it on the plane as well. Explain your results. (EX 1.4 - 3)
22. Write the steps to obtain the graph of the function  $y = 3(x - 1)^2 + 5$  from the graph  $y = x^2$ . (EX 1.4 - 4)
23. From the curve  $y = \sin x$ , graph the functions (iii)  $y = \sin\left(\frac{\pi}{2} + x\right)$  which is  $\cos x$  (iv)  $y = \sin\left(\frac{\pi}{2} - x\right)$  which is also  $\cos x$  (EX. 1.4 - 5)
24. From the curve  $y = |x|$ , draw (i)  $y = |x - 1| + 1$  (ii)  $y = |x + 1| - 1$  (iii)  $y = |x + 2| - 3$ . (EX 1.4 - 7)

**5 MARKS****SET 1**

1. In the set  $\mathbb{Z}$  of integers, define  $mRn$  if  $m - n$  is a multiple of 12. Prove that  $R$  is an equivalence relation. (Eg. 1.13)
2. In the set  $\mathbb{Z}$  of integers, define  $mRn$  if  $m - n$  is divisible by 7. Prove that  $R$  is an equivalence relation. (EX. 1.2 - 9)
3. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = 2x - 3$  prove that  $f$  is a bijection and find its inverse. (Eg. 1.30)
4. If  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = |x| + x$  and  $g(x) = |x| - x$ , find  $g \circ f$  and  $f \circ g$ . (EX 1.3 - 10)
5. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = 3x - 5$ , prove that  $f$  is a bijection and find its inverse. (EX 1.3 - 12)

**SET 2**

6. The formula for converting from Fahrenheit to Celsius temperatures is  $y = \frac{5x}{9} - \frac{160}{9}$ . Find the inverse of this function and determine whether the inverse is also a function. (EX. 1.3 - 19)
7. A simple cipher takes a number and codes it, using the function  $f(x) = 3x - 4$ . Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line  $y = x$ . (by drawing the lines). (EX 1.3 - 20)
8. Draw the graph of  $y = 2 \sin(x - 1) + 3$ . (Ill. 4)
9. For the curve  $y = x^3$  given in the Figure, draw (i)  $y = -x^3$  (ii)  $y = x^3 + 1$  (iii)  $y = x^3 - 1$  (iv)  $y = (x + 1)^3$  with the same scale. (EX. 1.4 - 1)
10. For the curve  $y = x^{\left(\frac{1}{3}\right)}$  given in the Figure, draw (i)  $y = -x^{\left(\frac{1}{3}\right)}$  (ii)  $y = x^{\left(\frac{1}{3}\right)} + 1$  (iii)  $y = x^{\left(\frac{1}{3}\right)} - 1$  (iv)  $y = (x + 1)^{\left(\frac{1}{3}\right)}$  (EX. 1.4 - 2)

## CHAPTER 2 - BASIC ALGEBRA

## 2 MARKS

## SET 1

- Are there two distinct irrational numbers such that their difference is a rational number? Justify. (EX 2.1 - 3)
- Find a positive number smaller than  $\frac{1}{2^{1000}}$ . Justify. (EX 2.1 - 5)
- Solve  $3|x - 2| + 7 = 19$  for  $x$ . (Eg. 2.2)
- Solve  $|2x - 3| = |x - 5|$ . (EG 2.3)
- Solve  $\left|\frac{2}{x-4}\right| > 1, x \neq 4$ . (EG 2.5)
- Solve for  $x$ : (iv)  $|x| - 10 < -3$ . (EX 2.2 - 1)
- Solve  $|5x - 12| < -2$ . (EX 2.2 - 6)
- A girl A is reading a book having 446 pages and she has already finished reading 271 pages. She wants to finish reading this book within a week. What is the minimum number of pages she should read per day to complete reading the book within a week? (EG 2.9)
- Solve  $23x < 100$  when (i)  $x$  is a natural number, (ii)  $x$  is an integer. (EX 2.3 - 2)
- Solve  $-2x \geq 9$  when (i)  $x$  is a real number, (ii)  $x$  is an integer, (iii)  $x$  is a natural number. (EX 2.3 - 2)

## SET 2

- If  $a$  and  $b$  are the roots of the equation  $x^2 - px + q = 0$ , find the value of  $\frac{1}{a} + \frac{1}{b}$ . (EG 2.10)
- Find the complete set of values of  $a$  for which the quadratic  $x^2 - ax + a + 2 = 0$  has equal roots. (EG 2.11)
- Construct the quadratic equation with roots 7 and -3 (EX 2.4 - 1)
- Discuss the nature of roots of, (iii)  $9x^2 + 5x = 0$ . (EX 2.4 - 8 iii)
- Write  $f(x) = x^2 + 5x + 4$  in completed square form. (EX 2.4 - 10)
- Solve the equation  $\sqrt{6 - 4x - x^2} = x + 4$ . (EG 2.15)
- Find the real roots of  $x^4 = 16$ . (EX 2.6 - 3)
- Find a quadratic polynomial  $f(x)$  such that,  $f(0) = 1, f(-2) = 0$  and  $f(1) = 0$ . (EG 2.16)
- Find the roots of the polynomial equation  $(x - 1)^3(x + 1)^2(x + 5) = 0$  and state their multiplicity. (EG 2.20)
- Factorize:  $x^4 + 1$ . (Hint: Try completing the square.) (EX 2.7 - 1)

## SET 3

- If  $x^2 + x + 1$  is a factor of the polynomial  $3x^3 + 8x^2 + 8x + a$ , then find the value of  $a$ . (EX 2.7 - 2)
- Resolve into partial fractions:  $\frac{1}{x^2 - a^2}$  (EX 2.9 - 1)
- Determine the region in the plane determined by the inequalities:  $x \leq 3y, x \geq y$ . (EX 2.10 - 1)
- Determine the region in the plane determined by the inequalities:  $2x + 3y \leq 6, x + 4y \leq 4, x \geq 0, y \geq 0$ . (EX 2.10 - 5)
- (i) Simplify:  $(x^{\frac{1}{2}}y^{-3})^{\frac{1}{2}}$ , where  $x, y \geq 0$ . (ii) Simplify:  $\sqrt{x^2 - 10x + 25}$ . (EG 2.31)
- Rationalize the denominator of  $\frac{\sqrt{5}}{\sqrt{6} + \sqrt{2}}$ . (EG 2.32)
- Find the square root of  $7 - 4\sqrt{3}$ . (EG 2.33)
- Simplify and hence find the value of  $n$ :  $\frac{3^{2n}9^{2}3^{-n}}{3^{3n}} = 27$ . (EX 2.11 - 4)
- Find the radius of the spherical tank whose volume is  $\frac{32\pi}{3}$  units. (EX 2.11 - 5)

30. Find the logarithm of 1728 to the base  $2\sqrt{3}$ . (EG 2.34)

#### SET 4

31. If the logarithm of 324 to base  $a$  is 4, then find  $a$ . (EG 2.35)

32. Solve  $x^{\log_3 x} = 9$ . (EG 2.38)

33. Compute  $\log_3 5 \log_{25} 27$ . (EG 2.39)

34. Let  $b > 0$  and  $b \neq 1$ . Express  $y = b^x$  in logarithmic form. Also state the domain and range of the logarithmic function. (EX 2.12 - 1)

35. Prove  $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$ . (EX 2.12 - 6)

36. Prove  $\log a + \log a^2 + \log a^3 + \dots + \log a^n = \frac{n(n+1)}{2} \log a$ . (EX 2.12 - 9)

37. Solve  $\log_{5-x}(x^2 - 6x + 65) = 2$ . (EX 2.12 - 12)

### 3 MARKS

#### SET 1

1. Prove that  $\sqrt{3}$  is an irrational number. (EX. 2.1 - 2)

2. Solve the following system of linear inequalities.  $3x - 9 \geq 0, 4x - 10 \leq 6$ . (EG 2.8)

3. Solve: (i)  $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$  (ii)  $\frac{5-x}{3} < \frac{x}{2} - 4$ . (EX 2.3 - 4)

4. To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84,87,95,91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A grade in the course? (EX 2.3 - 5)

5. Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40. (EX. 2.3 - 7)

#### SET 2

6. A model rocket is launched from the ground. The height  $h$  reached by the rocket after  $t$  seconds from lift off is given by  $h(t) = -5t^2 + 100t, 0 \leq t \leq 20$ . At what times the rocket is 495 feet above the ground? (EX. 2.3 - 8)

7. A Plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the second scheme he will be paid rupees 120 per hour. If he works  $x$  hours to complete the job, then for what value of  $x$  does the first scheme give better wages? (EX. 2.3 - 9)

8. A quadratic polynomial has one of its zeros  $1 + \sqrt{5}$  and it satisfies  $p(1) = 2$ . Find the quadratic polynomial. (EX 2.4 - 2)

9. If the equations  $x^2 - ax + b = 0$  and  $x^2 - ex + f = 0$  have one root in common and if the second equation has equal roots, then prove that  $ae = 2(b + f)$ . (EX 2.4 - 7)

10. Solve  $-x^2 + 3x - 2 \geq 0$ . (EX 2.5 - 2)

#### SET 3

11. Find a quadratic polynomial  $f(x)$  such that,  $f(0) = 1, f(-2) = 0$  and  $f(1) = 0$ . (EG 2.16)

12. Construct a cubic polynomial function having zeros at  $x = \frac{2}{5}, 1 + \sqrt{3}$  such that  $f(0) = -8$ . (EG 2.17)

13. Prove that  $ap + q = 0$  if  $f(x) = x^3 - 3px + 2q$  is divisible by  $g(x) = x^2 + 2ax + a^2$ . (EG 2.18)

14. Find the values of  $p$  for which the difference between the roots of the equation  $x^2 + px + 8 = 0$  is 2. (EG 2.23)

15. If  $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 = \frac{9}{2}$ , then find the value of  $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$  for  $x > 1$ . (EX 2.11 - 3)

**SET 4**

16. Simplify  $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$  (EX 2.11 - 7)
17. If  $x = \sqrt{2} + \sqrt{3}$  find  $\frac{x^2+1}{x^2-2}$  (EX 2.11 - 8)
18. Prove  $\log_{16} \frac{75}{16} - 2 \log_{9} \frac{5}{9} + \log_{243} \frac{32}{243} = \log 2$ . (EG 2.36)
19. If  $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$ , find the value of  $x$ . (EG 2.37)
20. Solve  $x^{\log_3 x} = 9$ . (EG 2.38)

**SET 5**

21. Given that  $\log_{10} 2 = 0.30103$ ,  $\log_{10} 3 = 0.47712$  (approximately), find the number of digits in  $2^8 3^{12}$ . (EG 2.40)
22. Compute  $\log_9 27 - \log_{27} 9$ . (EX 2.12 - 2)
23. Solve  $\log_4 2^{8x} = 2^{\log_2 8}$ . (EX 2.12 - 4)
24. If  $a^2 + b^2 = 7ab$ , show that  $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$ . (EX 2.12 - 5)
25. Prove  $\log_{a^2} a \log_{b^2} b \log_{c^2} c = \frac{1}{8}$ . (EX 2.12 - 8)

**5 MARKS****SET 1**

1. A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent? (EX. 2.3 - 6)
2. If one root of  $k(x-1)^2 = 5x-7$  is double the other root, show that  $k = 2, -25$ . (EX. 2.4 - 4)
3. If the difference of the roots of the equation  $2x^2 - (a+1)x + a - 1 = 0$  is equal to their product, then prove that  $a = 2$ . (EX 2.4 - 5)
4. Find the condition that one of the roots of  $ax^2 + bx + c$  may be (i) negative of the other, (ii) thrice the other, (iii) reciprocal of the other. (EX 2.4 - 6)
5. Use the method of undetermined coefficients to find the sum of  $1 + 2 + 3 + \dots + (n-1) + n, n \in N$ . (Eg. 2.19)

**SET 2**

6. The equations  $x^2 - 6x + a = 0$  and  $x^2 - bx + 6 = 0$  have one root in common. The other root of the first and the second equations are integers in the ratio 4:3. Find the common root. (EG 2.22)
7. Find all values of  $x$  for which  $\frac{x^3(x-1)}{x-2} > 0$ . (EX 2.8 - 1)
8. Solve  $\frac{x^2-4}{x^2-2x-15} \leq 0$ . (EX 2.8 - 3)
9. Resolve into partial fractions:  $\frac{2x}{(x^2+1)(x-1)}$ . (EG 2.26)
10. Resolve into partial fractions:  $\frac{x+1}{x^2(x-1)}$ . (EG 2.27)

**SET 3**

11. Resolve into partial fractions:  $\frac{3x+1}{(x-2)(x+1)}$  (EX 2.9 - 2)
12. Resolve into partial fractions:  $\frac{x}{(x-1)^3}$  (EX 2.9 - 4)
13. Resolve into partial fractions:  $\frac{1}{x^4-1}$  (EX 2.9 - 5)
14. Resolve into partial fractions:  $\frac{(x-1)^2}{x^3+x}$  (EX 2.9 - 6)

15. Resolve into partial fractions:  $\frac{x^2+x+1}{x^2-5x+6}$  (EX 2.9 - 7)

**SET 4**

16. Resolve into partial fractions:  $\frac{x^3+2x+1}{x^2+5x+6}$  (EX 2.9 - 8)

17. Resolve into partial fractions:  $\frac{x+12}{(x+1)^2(x-2)}$  (EX 2.9 - 9)

18. Resolve into partial fractions:  $\frac{6x^2-x+1}{x^3+x^2+x+1}$  (EX 2.9 - 10)

19. Resolve into partial fractions:  $\frac{2x^2+5x-11}{x^2+2x-3}$  (EX 2.9 - 11)

20. Resolve into partial fractions:  $\frac{7+x}{(1+x)(1+x^2)}$  (EX 2.9 - 12)

**SET 5**

21. Solve the linear inequalities and exhibit the solution set graphically:  $x + y \geq 3, 2x - y \leq 5, -x + 2y \leq 3$ . (EG 2.30)

22. Solve  $\log_8 x + \log_4 x + \log_2 x = 11$ . (EX 2.12 - 3)

23. Prove that  $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1$ . (EX 2.12 - 7)

24. If  $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ , then prove that  $xyz = 1$ . (EX 2.12 - 10)

25. Solve  $\log_2 x - 3 \log_{\frac{1}{2}} x = 6$ . (EX 2.12 - 11)

**CHAPTER 3 - TRIGONOMETRY****2 MARKS****SET 1**

1. Eliminate  $\theta$  from  $a \cos \theta = b$  and  $c \sin \theta = d$ , where  $a, b, c, d$  are constants. (EG 3.3)

2. For each given angle, find a coterminal angle with measure of  $\theta$  such that  $0^\circ \leq \theta < 360^\circ$  (i)  $395^\circ$  (iv)  $-270^\circ$  (v)  $-450^\circ$  (EX 3.1 - 2)

3. Convert (i)  $18^\circ$  to radians (ii)  $-108^\circ$  to radians. (Eg. 3.4)

4. Convert (i)  $\frac{\pi}{5}$  radians to degrees (ii) 6 radians to degrees. (Eg. 3.5)

5. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring  $15^\circ$ . (EG 3.6)

**SET 2**

6. Find all the angles between  $0^\circ$  and  $360^\circ$  which satisfy the equation  $\sin^2 \theta = \frac{3}{4}$ . (EX 3.3 - 5)

7. Show that  $\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} = 2$ . (EX 3.3 - 6)

8. An airplane propeller rotates 1000 times per minute. Find the number of degrees that a point on the edge of the propeller will rotate in 1 second. (EX 3.2 - 9)

9. A train is moving on a circular track of 1500 m radius at the rate of 66 km/hr. What angle will it turn in 20 seconds? (EX 3.2 - 10)

10. Find the value of: (i)  $\sin 765^\circ$  (ii)  $\operatorname{cosec} (-1410^\circ)$  (iii)  $\cot \left( \frac{-15\pi}{4} \right)$  (EG 3.12)

**SET 3**

11. Determine whether the following functions are even, odd or neither. (i)  $\sin^2 x - 2 \cos^2 x - \cos x$  (iii)  $\cos(\sin(x))$  (iv)  $\sin x + \cos x$ . (EG 3.14)

12. Find all the angles between  $0^\circ$  and  $360^\circ$  which satisfy the equation  $\sin^2 \theta = \frac{3}{4}$ . (EX 3.3 - 5)

13. Find the value of (i)  $\cos 105^\circ$  (ii)  $\sin 105^\circ$  (iii)  $\tan \frac{7\pi}{12}$ . (EX 3.4 - 5)

14. Prove that (i)  $\sin(45^\circ + \theta) - \sin(45^\circ - \theta) = \sqrt{2} \sin \theta$ . (EX 3.4 - 9)

15. Express each of the following as a sum or difference (v)  $\sin 5\theta \sin 4\theta$ . (EX 3.6 - 1)



## SET 4

16. Express each of the following as a product (ii)  $\cos 65^\circ + \cos 15^\circ$  (iii)  $\sin 50^\circ + \sin 40^\circ$  (EX 3.6 - 2)
17. Find the principal solution of (i)  $\sin \theta = \frac{1}{2}$  (ii)  $\sin \theta = \frac{-\sqrt{3}}{2}$   
(iii)  $\operatorname{cosec} \theta = -2$  (iv)  $\cos \theta = \frac{1}{2}$  (EG 3.42)
18. Find the general solution of  $\sin \theta = \frac{-\sqrt{3}}{2}$  (EG 3.43)
19. In a  $\Delta ABC$ ,  $a = 3$ ,  $b = 5$  and  $c = 7$ . Find the values of  $\cos A$ ,  $\cos B$  and  $\cos C$ . (EG 3.64)
20. Find the area of the triangle whose sides are 13 cm, 14 cm and 15 cm. (EG 3.67)

## 3 MARKS

## SET 1

1. Prove that  $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$ . (EG 3.1)
2. If  $a \cos \theta - b \sin \theta = c$  show that  $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$ . (EX 3.1 - 3)
3. If  $y = \frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha}$  then prove that  $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = y$ . (EX 3.1 - 6)
4. If  $\operatorname{cosec} \theta - \sin \theta = a^3$  and  $\sec \theta - \cos \theta = b^3$  then prove that  $a^2 b^2 (a^2 + b^2) = 1$ . (EX 3.1 - 11)
5. Eliminate  $\theta$  from the equations  $a \sec \theta - c \tan \theta = b$  and  $b \sec \theta + d \tan \theta = c$ . (EX 3.1 - 12)
6. If the arcs of same lengths in two circles subtend central angles  $30^\circ$  and  $80^\circ$  find the ratio of their radii. (EG 3.7)

## SET 2

7. In a circle of diameter 40 cm, a chord is of length 20 cm. Find the length of the minor arc of the chord. (EX 3.2 - 4)
8. A circular metallic plate of radius 8 cm and thickness 6 mm is melted and molded into a pie (a sector of the circle with thickness) of radius 16 cm and thickness 4 mm. Find the angle of the sector. (EX 3.2 - 11)
9. The terminal side of an angle  $\theta$  in standard position passes through the point (3, -4). Find the six trigonometric function values at an angle  $\theta$ . (EG 3.8)
10. If  $\sin \theta = \frac{3}{5}$  and the angle  $\theta$  is in the second quadrant, then find the values of other five trigonometric functions. (EG 3.9)
11. Prove that  $\tan 315^\circ \cot(-405^\circ) + \cot 495^\circ \tan(-585^\circ) = 2$ . (EG 3.13)
12.  $(\frac{5}{7}, \frac{2\sqrt{6}}{7})$  is a point on the terminal side of an angle  $\theta$  in standard position. Determine the trigonometric function values of angle  $\theta$ . (EX 3.3 - 2)

## SET 3

13. Prove that  $\frac{\cot(180^\circ + \theta) \sin(90^\circ - \theta) \cos(-\theta)}{\sin(270^\circ + \theta) \tan(-\theta) \operatorname{cosec}(360^\circ + \theta)} = \cos^2 \theta \tan \theta$ . (EX 3.3 - 4)
14. Show that  $\sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} = 2$ . (EX 3.3 - 6)
15. If  $\sin x = \frac{4}{5}$  (in I quadrant) and  $\cos y = \frac{-12}{13}$  (in II quadrant), then find (i)  $\sin(x - y)$ ,  
(ii)  $\cos(x - y)$ . (EG 3.16)
16. Prove that  $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$ . (EX 3.4 - 11)
17. Prove that  $\sin(n + 1)\theta \sin(n - 1)\theta + \cos(n + 1)\theta \cos(n - 1)\theta = \cos 2\theta$ ,  $n \in Z$ . (EX 3.4 - 15)

## SET 4

18. Find the value of  $\sin(22\frac{1}{2}^\circ)$ . (EG 3.22)

19. Prove that  $\sin x = 2^{10} \sin\left(\frac{x}{2^{10}}\right) \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{2^2}\right) \dots \dots \cos\left(\frac{x}{2^{10}}\right)$  (EG 3.25)
20. Prove that  $\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta$ . (EG 3.26)
21. Prove that  $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2 \tan 2\theta$ . (EX 3.5 - 8)
22. Show that  $\sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}$ . (EX 3.6 - 3)

**SET 5**

23. Show that  $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$ . (EX 3.6 - 4)
24. Prove that  $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \tan 3x$ . (EX 3.6 - 8)
25. Show that  $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$ . (EG 3.38)
26. In a  $\Delta ABC$ , if  $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$ , prove that  $a^2, b^2, c^2$  are in Arithmetic Progression. (EX 3.9 - 1)

**5 MARKS****SET 1**

1. If  $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$ ,  $y = \sum_{n=0}^{\infty} \sin^{2n} \theta$  and  $z = \sum_{n=0}^{\infty} \sin^{2n} \theta \cos^{2n} \theta$ ,  $0 < \theta < \frac{\pi}{2}$  then show that  $xyz = x + y + z$ . (EX. 3.1 - 7)
2. If  $\tan^2 \theta = 1 - k^2$  show that  $\sec \theta + \tan^3 \theta \operatorname{cosec} \theta = (2 - k^2)^{\frac{3}{2}}$ . Also, find the values of  $k$  for which this result holds. (EX. 3.1 - 8)
3. If  $\cot \theta (1 + \sin \theta) = 4m$  and  $\cot \theta (1 - \sin \theta) = 4n$ , then prove that  $(m^2 - n^2)^2 = mn$ . (EX. 3.1 - 10)
4. Find  $\sin(x - y)$ , given that  $\sin x = \frac{8}{17}$  with  $0 < x < \frac{\pi}{2}$  and  $\cos y = \frac{-24}{25}$  with  $\pi < y < \frac{3\pi}{2}$ . (EX. 3.4 - 4)

**SET 1**

5. If  $\theta + \phi = \alpha$  and  $\tan \theta = k \tan \phi$ , then prove that  $\sin(\theta - \phi) = \frac{k-1}{k+1} \sin \alpha$ . (EX 3.4 - 25)
6. Prove that  $32(\sqrt{3}) \sin \frac{\pi}{48} \cos \frac{\pi}{48} \cos \frac{\pi}{24} \cos \frac{\pi}{12} \cos \frac{\pi}{6} = 3$ . (EX 3.5 - 11)
7. If  $A + B + C = \pi$ , prove that  $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$ . (EG 3.41)
8. Show that  $\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$ . (EX. 3.6 - 4)

**SET 2**

9. If  $A + B + C = 180^\circ$ , P.T  $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$ . (EX. 3.7 - 1 11)
10. If  $A + B + C = 180^\circ$ , prove that (v)  $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$  (EX. 3.7 - 1 V)
11. If  $A + B + C = 180^\circ$ , prove that (vi)  $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$  (EX. 3.7 - 1 VI)
12. If  $x + y + z = xyz$ , then prove that  $\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \frac{2x}{1-x^2} \frac{2y}{1-y^2} \frac{2z}{1-z^2}$ . (EX. 3.7 - 3)

**SET 3**

13. If  $A + B + C = \frac{\pi}{2}$ , prove the  $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C$  (EX. 3.7 - 4(i))
14. Derive Projection formula from (i) Law of sines, (ii) Law of cosines. (EX 3.9 - 11)
15. In a  $\Delta ABC$ , if  $a = 18 \text{ cm}$ ,  $b = 24 \text{ cm}$  and  $c = 30 \text{ cm}$ , then show that its area is  $216 \text{ sq. cm}$ . (EX 3.10 - 6)
16. In  $\Delta ABC$ , we have (i)  $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$  (ii)  $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$  (iii)  $\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$ . (TH. 3.2) Napier's Formula

## CHAPTER 4 - COMBINATORICS AND MATHEMATICAL INDUCTION

### 2 MARKS

#### SET 1

1. A School library has 75 books on Mathematics, 35 books on Physics. A student can choose only one book. In how many ways a student can choose a book on Mathematics or Physics? (EG 4.3)
2. Find the total number of outcomes when 5 coins are tossed once. (EG 4.3)
3. Find the value of (v)  $\frac{12!}{9! \times 3!}$  (vi)  $\frac{(n+3)!}{(n+1)!}$  (EX4.1 - 14)
4. Find the value of  $n$  (i)  $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$  (EX4.1 - 16)

#### SET 2

5. If  $\frac{6!}{n!} = 6$ , then find the value of  $n$ . (EG 4.20)
6. If  $n! + (n - 1)! = 30$ , then find the value of  $n$ . (EG 4.21)
7. If  $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$  then find the value of  $A$ . (EG 4.23)
8. Evaluate: (i)  ${}^4P_4$  (ii)  ${}^5P_3$  (iii)  ${}^8P_4$  (iv)  ${}^6P_5$ . (EG 4.25)
9. Find the number of ways of arranging the letters of the word **BANANA**. (EG 4.36)

#### SET 3

10. Find the number of ways of arranging the letters of the word **RAMANUJAN** so that the relative positions of vowels and consonants are not changed. (EG 4.37)
11. If the different permutations of all letters of the word **BHASKARA** are listed as in a dictionary, how many strings are there in this list before the first word starting with **B**? (EG 4.41)
12. Find the distinct permutations of the letters of the word **MISSISSIPPI**? (EX 4.2 - 9)
13. If  ${}^{n+2}P_4 = 42 \times {}^nP_2$ , find  $n$ . (EG 4.26)
14. If  ${}^nP_r = 11880$  and  ${}^nC_r = 495$ , Find  $n$  and  $r$ . (EG 4.47)

#### SET 4

15. If  ${}^nC_{12} = {}^nC_9$  find  ${}^{21}C_n$ . (EX4.3 - 1)
16. If  ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$  find  $r$ . (EX4.3 - 2)
17. If  ${}^nP_r = 720$ , and  ${}^nC_r = 120$  find  $n, r$ . (EX 4.3 - 3)
18. How many diagonals are there in a polygon with  $n$  sides? (EG 4.60)
19. A polygon has 90 diagonals. Find the number of its sides? (EX 4.3 - 25)

### 3 MARKS

#### SET 1

- 1 (i) Find the number of strings of length 4, which can be formed using the letters of the word **BIRD**, without repetition of the letters.  
(ii) How many strings of length 5 can be formed out of the letters of the word **PRIME** taking all the letters at a time without repetition. (EG 4.8)
- 2 How many strings of length 6 can be formed using letters of the word **FLOWER** if (i) either starts with **F** or ends with **R**? (ii) neither starts with **F** nor ends with **R**? (EG 4.9)
- 3 Prove that  $\frac{(2n)!}{n!} = 2^n (1.3.5 \dots (2n - 1))$ . (EG 4.24)

- 4 If  $10P_r = 7P_{r+2}$  find  $r$ . (EG 4.27)  
 5 In how many ways 5 boys and 4 girls can be seated in a row so that no two girls are together. (EG 4.32)

**SET 3**

- 6 If the letters of the word **IITJEE** are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word **IITJEE** (EG 4.42)  
 7 Prove that  ${}^{24}C_4 + \sum_{r=0}^4 {}^{28-r}C_3 = {}^{29}C_4$ . (EG 4.48)  
 8 If  ${}^{n+2}C_7 : {}^{n-1}P_4 = 13 : 24$  find  $n$ . (EG 4.50)  
 9 If  ${}^nP_r = 720$ , and  ${}^nC_r = 120$  find  $n, r$ . (EX 4.3 - 3)  
 10 Prove that  ${}^{15}C_3 + 2 \times {}^{15}C_4 + {}^{15}C_5 = {}^{17}C_5$ . (EX 4.3 - 4)

**SET 5**

- 11 Prove that  ${}^{35}C_5 + \sum_{r=0}^4 {}^{39-r}C_4 = {}^{40}C_5$ . (EX 4.3 - 5)  
 12 Prove that  ${}^{2n}C_n = \frac{2^n \times 1 \times 3 \times \dots \times (2n-1)}{n!}$  (EX 4.3 - 7)  
 13 Prove that if  $1 \leq r \leq n$  then  $n \times {}^{n-1}C_{r-1} = (n-r+1) \times {}^nC_{r-1}$ . (EX 4.3 - 8)  
 14 Prove that the sum of first  $n$  positive odd numbers is  $n^2$ . (EG 4.62)  
 15 Prove that the sum of the first  $n$  non-zero even numbers is  $n^2 + n$ . (EX 4.4 - 3)

**5 MARKS****SET 1**

1. If the letters of the word **TABLE** are permuted in all possible ways and the words thus formed are arranged in the dictionary order (alphabetical order), find the ranks of the words (i) **TABLE**, (ii) **BLEAT** (EG 4.35)  
 2. Find the sum of all 4 - digit numbers that can be formed using the digits 1, 2, 4, 6, 8. (Eg. 4.43)  
 3. If the letters of the word **GARDEN** are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the ranks of the words (i) **GARDEN** (ii) **DANGER**. (EX. 4.2 - 16)  
 4. By the principle of mathematical induction, prove that, for all integers  $n \geq 1$ ,  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ . (EG 4.61)  
 5. By the principle of mathematical induction, prove that, for all integers  $n \geq 1$ ,  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ . (Eg. 4.63)

**SET 3**

6. Using the Mathematical induction, show that for any natural number  $n$ ,  $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ . (EG 4.64)  
 7. Prove that for any natural number  $n$ ,  $a^n - b^n$  is divisible by  $a - b$ , where  $a > b$ . (Eg. 4.65)  
 8. By the principle of mathematical induction, prove that, for  $n \geq 1$ ,  $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ . (EX. 4.4 - 1)  
 9. By the principle of mathematical induction, prove that, for  $n \geq 1$ ,  $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ . (EX. 4.4 - 2)  
 10. By the principle of Mathematical induction, prove that, for  $n \geq 1$   $1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \frac{n(n+1)(n+2)}{3}$ . (EX 4.4 - 4)

**SET 4**

11. Using the Mathematical induction, show that for any natural number  $n \geq 2$ ,  $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \dots \dots \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$ . (EX 4.4 - 5)
12. Using the Mathematical induction, show that for any natural number  $n \geq 2$ ,  $\frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots \dots \dots + \frac{1}{1+2+3+\dots+n} = \frac{n-1}{n+1}$ . (EX. 4.4 - 6)
13. Using the Mathematical induction, show that for any natural number  $n$ ,  $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots \dots \dots + \frac{1}{n.(n+1).(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$ . (EX. 4.4 - 7)
14. Using the Mathematical induction, show that for any natural number  $n$ ,  $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots \dots \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$ . (EX. 4.4 - 8)
15. Prove by Mathematical Induction that  $1! + (2 \times 2!) + (3 \times 3!) + \dots \dots \dots + (n \times n!) = (n+1)! - 1$ . (EX. 4.4 - 9)

**SET 5**

16. Using the Mathematical induction, show that for any natural number  $n$ ,  $x^{2n} - y^{2n}$  is divisible by  $x + y$ . (EX. 4.4 - 10)
17. Prove that  $3^{2n+2} - 8n - 9$  is divisible by 8 for all  $n > 1$ . (EG 4.66)
18. Use induction to prove that  $n^3 - 7n + 3$  is divisible by 3, for all natural numbers  $n$ . (EX 4.4 - 12)
19. Use induction to prove that  $5^{n+1} + 4 \times 6^n$  when divided by 20 leaves a remainder 9, for all natural numbers  $n$ . (EX 4.4 - 13)
20. Use induction to prove that  $10^n + 3 \times 4^{n+2} + 5$  is divisible by 9, for all natural numbers  $n$ . (EX. 4.4 - 14)

**CHAPTER 5 - BINOMIAL THEOREM, SEQUENCES AND SERIES****2 MARKS****SET - 1**

1. Find the expansion of  $(2x + 3)^5$ . (EG 5.1)
2. Evaluate  $98^4$ . (EG 5.2)
3. Find the middle term in the expansion of  $(x + y)^6$ . (EG 5.3)
4. Find the middle terms in the expansion of  $(x + y)^7$ . (EG 5.4)
5. Find the coefficient of  $x^6$  in the expansion of  $(3 + 2x)^{10}$ . (EG 5.5)

**SET - 2**

6. Expand  $\left(2x - \frac{1}{2x}\right)^4$ . (EG 5.8)
7. Using binomial theorem, indicate which of the following two number is larger:  $(1.01)^{1000000}$ , 10000. (EX 5.1 - 3)
8. Find the coefficient of  $x^6$  and the coefficient of  $x^2$  in  $\left(x^2 - \frac{1}{x^3}\right)^6$ . (EX 5.1 - 5)
9. Find the coefficient of  $x^4$  in expansion of  $(1 + x^3)^{50} \left(x^2 + \frac{1}{x}\right)^5$ . (EX 5.1 - 6)
10. Find the constant term of  $\left(2x^3 - \frac{1}{3x^2}\right)^5$ . (EX 5.1 - 7)

**SET - 3**

11. Find the coefficient of  $x^3$  in the expansion of  $(2 - 3x)^7$ . (Eg. 5.6)
12. Find the last two digits of the number  $3^{600}$ . (EX 5.1 - 8)
13. Prove that if  $a, b, c$  are in HP, if and only if  $\frac{a}{c} = \frac{a-b}{b-c}$ . (Eg. 5.12)

14. Find  $\sum_{k=1}^n \frac{1}{k(k+1)}$ . (EG 5.18)
15. Write the first 6 terms of the sequences whose  $n^{\text{th}}$  term  $a_n$  is given below.
- (ii)  $a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$  (EX 5.2 - 2)

## SET - 4

16. Find the sum:  $1 + \frac{4}{5} + \frac{7}{25} + \frac{10}{125} + \dots$  (Eg. 5.19)
17. Write the first 6 terms of the exponential series (ii)  $e^{-2x}$  (iii)  $e^{\frac{x}{2}}$ . (EX. 5.4 - 5)
18. Write the first 4 terms of the logarithmic series (iii)  $\log\left(\frac{1+3x}{1-3x}\right)$  (iv)  $\log\left(\frac{1-2x}{1+2x}\right)$ . Find the intervals on which the expansions are valid. (EX. 5.4 - 6)
19. Find the coefficient of  $x^4$  in the expansion of  $\frac{3-4x+x^2}{e^{2x}}$ . (EX. 5.4 - 9)

## 3 MARKS

## SET - 1

1. Expand  $(x^2 + \sqrt{1-x^2})^5 + (x^2 - \sqrt{1-x^2})^5$ . (EG 5.9)
2. Expand  $(2x^2 - 3\sqrt{1-x^2})^4 + (2x^2 + 3\sqrt{1-x^2})^4$ . (EX. 5.1 - 1(ii))
3. If  $t_k$  is the  $k^{\text{th}}$  term of a GP, then show that  $t_{n-k}, t_k, t_{n+k}$  also form a GP for any positive integer  $k$ . (EX. 5.2 - 6)
4. If  $a, b, c$  are in geometric progression, and if  $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ , then prove that  $x, y, z$  are in arithmetic progression. (EX. 5.2 - 7)
5. Find  $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$ . (EG 5.20)

## SET - 3

6. Compute the sum of first  $n$  terms of  $1 + (1+4) + (1+4+4^2) + (1+4+4^2+4^3) + \dots$  (EX 5.3 - 4)
7. Find the value of  $n$ , if the sum to  $n$  terms of the series  $\sqrt{3} + \sqrt{75} + \sqrt{243} + \dots$  is  $435\sqrt{3}$ . (EX 5.3 - 6)
8. Expand  $(1+x)^{\frac{2}{3}}$  up to four terms for  $|x| < 1$ . (EG 5.21)
9. Find  $\sqrt[3]{65}$ . (Eg. 5.24)
10. Find  $\sqrt[3]{1001}$  approximately (two decimal places). (EX. 5.4 - 2)
11. If  $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$ , then show that  $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \dots$ , (EX. 5.4 - 7)

## 5 MARKS

## SET 1

1. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers. (EX. 5.2 - 8)
2. If the roots of the equation  $(q-r)x^2 + (r-p)x + p-q = 0$  are equal, then show that  $p, q$  and  $r$  are in AP. (EX. 5.2 - 9)
3. If  $a, b, c$  are respectively the  $p^{\text{th}}, q^{\text{th}}$  and  $r^{\text{th}}$  terms of a GP, show that  $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$ . (EX. 5.2 - 10)
4. Prove that  $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$  is approximately equal to  $\frac{1}{x^2}$  when  $x$  is large. (Eg. 5.25)

**SET 2**

5. Prove that  $\sqrt[3]{x^3 + 6} - \sqrt[3]{x^3 + 3}$  is approximately equal to  $\frac{1}{x^2}$  when  $x$  is sufficiently large. (EX. 5.4 - 3)
6. Prove that  $\sqrt{\frac{1-x}{1+x}}$  is approximately equal to  $1 - x + \frac{x^2}{2}$  when  $x$  is very small. (EX. 5.4 - 4)
7. If  $p - q$  is small compared to either  $p$  or  $q$ , then show that  $\sqrt[n]{\frac{p}{q}} \approx \frac{(n+1)p + (n-1)q}{(n-1)p + (n+1)q}$ . Hence find  $\sqrt[8]{\frac{15}{16}}$ . (EX. 5.4 - 8)
8. Find the value of  $\sum_{n=1}^{\infty} \frac{1}{2n-1} \left( \frac{1}{9^{n-1}} + \frac{1}{9^{2n-1}} \right)$ . (EX. 5.4 - 10)

**CHAPTER 6 - TWO DIMENSIONAL ANALYTICAL GEOMETRY****2 MARKS****SET - 1**

1. Find the path traced out by the point  $\left( ct, \frac{c}{t} \right)$  here  $t \neq 0$  is the parameter and  $c$  is a constant. (Eg. 6.2)
2. Find the locus of a point  $P$  moves such that its distances from two fixed points  $A(1,0)$  and  $B(5,0)$  are always equal. (EG 6.3)
3. If  $\theta$  is a parameter, find the equation of the locus of a moving point, whose coordinates are  $(a \sec \theta, b \tan \theta)$ . (EG 6.4)
4. Find the locus of  $P$ , if for all values of  $\alpha$ , the co-ordinates of a moving point  $P$  is  
(i)  $(9 \cos \alpha, 9 \sin \alpha)$  (ii)  $(9 \cos \alpha, 6 \sin \alpha)$ . (EX. 6.1 - 1)
5. If  $\theta$  is a parameter, find the equation of the locus of a moving point, whose coordinates are  $x = a \cos^3 \theta, y = a \sin^3 \theta$ . (EX. 6.1 - 3)

**SET - 2**

6. A straight rod of length 8 units slides with its ends  $A$  and  $B$  always on the  $x$  and  $y$  axes respectively. Find the locus of the midpoint of the line segment  $AB$ . (EX. 6.1 - 5)
7. Show the points  $\left( 0, \frac{-3}{2} \right), (1, -1), \left( 2, \frac{-1}{2} \right)$  are collinear. (EG 6.9)
8. Find the equations of the straight lines, making the  $y$  - intercept of 7 and angle between the line and the  $y$  - axis is  $30^\circ$ . (Eg. 6.11)
9. Find the equation of the straight line passing through  $(-1, 1)$  and cutting off equal intercepts, but opposite in signs with the two coordinate axes. (EG 6.14)
10. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle  $150^\circ$  with positive direction of the  $x$  - axis. Find the equation of the line. (Eg. 6.16)

**SET - 3**

11. Rewrite  $\sqrt{3}x + y + 4 = 0$  in to normal form. (Eg. 6.20)
12. If  $P(r, c)$  is midpoint of a line segment between the axes, then show that  $\frac{x}{r} + \frac{y}{c} = 2$ . (EX. 6.2 - 2)
13. If  $p$  is length of perpendicular from origin to the line whose intercepts on the axes are  $a$  and  $b$ , then show that  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ . (EX. 6.2 - 4)
14. Find the equation of the line, if the perpendicular drawn from the origin makes an angle  $30^\circ$  with  $x$  - axis and its length is 12. (EX. 6.2 - 8)

15. Find the equations of a parallel line and a perpendicular line passing through the point (1, 2) to the line  $3x + 4y = 7$ . (EG 6.22)

## SET - 4

16. Find the nearest point on the line  $2x + y = 5$  from the origin. (EG 6.24)  
 17. Show that the lines are  $3x + 2y + 9 = 0$  and  $12x + 8y - 15 = 0$  are parallel lines. (EX 6.3 - 1)  
 18. Find the equation of the straight line parallel to  $5x - 4y + 3 = 0$  and having  $x$ -intercept 3. (EX 6.3 - 2)  
 19. Find the distance between the line  $4x + 3y + 4 = 0$  and a point  $(-2, 4)$  (EX 6.3 - 3)  
 20. Write the equation of the lines through the point (1, -1) (i) Parallel to  $x + 3y - 4 = 0$  (ii) perpendicular to  $3x + 4y = 6$  (EX 6.3 - 4)

## SET - 5

21. If  $(-4, 7)$  is one vertex of a rhombus and if the equation of one diagonal is  $5x - y + 7 = 0$ , then find the equation of another diagonal. (EX 6.3 - 5)  
 22. Find the distance between the parallel lines  $3x - 4y + 5 = 0$  and  $6x - 8y - 15 = 0$ . (EX. 6.3 - 12 (ii))  
 23. Find the family of straight lines (i) Perpendicular (ii) Parallel to  $3x + 4y - 12 = 0$ . (EX 6.3 - 13)  
 24. Separate the equations  $5x^2 + 6xy + y^2 = 0$ . (Eg. 6.33)  
 25. Find the combined equation of the straight lines whose separate equations are  $x - 2y - 3 = 0$  and  $x + y + 5 = 0$ . (EX. 6.4 - 1)  
 26. Find the separate equation of the following pair of straight lines (i)  $3x^2 + 2xy - y^2 = 0$  (EX 6.4 - 7)

## 3 MARKS

## SET - 1

1. Straight rod of the length 6 units, slides with its ends  $A$  and  $B$  always on the  $x$  and  $y$  axes respectively. If  $O$  is the origin, then find the locus of the centroid of  $\Delta OAB$ . (EG 6.5)  
 2. If  $\theta$  is a parameter, find the equation of the locus of a moving point, whose coordinates are  $(a(\theta - \sin \theta), a(1 - \cos \theta))$ . (EG 6.6)  
 3.  
 4. Find the value of  $k$  and  $b$ , if the points  $P(-3, 1)$  and  $Q(2, b)$  lie on the locus of  $x^2 - 5x + ky = 0$ . (EX 6.1 - 4)  
 5. If  $O$  is origin and  $R$  is a variable point on  $y^2 = 4x$ , then find the equation of the locus of the midpoint of the line segment  $OR$ . (EX 6.1 - 8)

## SET - 2

6. The coordinates of a moving point  $P$  are  $\left(\frac{a}{2}(\operatorname{cosec} \theta + \sin \theta), \frac{b}{2}(\operatorname{cosec} \theta - \sin \theta)\right)$ , where  $\theta$  is a variable parameter. Show that the equation of the locus  $P$  is  $b^2x^2 - a^2y^2 = a^2b^2$ . (EX 6.1 - 9)  
 7. If  $Q$  is a point on the locus of  $x^2 + y^2 + 4x - 3y + 7 = 0$ , then find the equation of locus of  $P$  which divides segment  $OQ$  externally in the ratio 3: 4, where  $O$  is origin. (EX. 6.1 - 13)  
 8. Find the equations of the straight lines, making the  $y$ -intercept of 7 and angle between the line and the  $y$ -axis is  $30^\circ$ . (EG 6.11)  
 9. Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of  $45^\circ$  with positive the  $x$ -axis. (EG 6.17)

## SET - 3



10. Find the equation of the lines make an angle  $60^\circ$  with positive  $x$  -axis and at a distance  $5\sqrt{2}$  units measured from the point  $(4,7)$  along the line  $x - y + 3 = 0$ . (EG 6.18)
11. Express the equation  $\sqrt{3}x - y + 4 = 0$  in the following equivalent form: (i) Slope and Intercept form (ii) Intercept form (iii) Normal form (EG 6.19)
12. Find the equation of the line passing through the point  $(1,5)$  and also divides the co-ordinate axes in the ratio 3: 10. (EX 6.2 - 3)
13. If  $p$  is length of perpendicular from origin to the line whose intercepts on the axes are  $a$  and  $b$ , then show that  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ . (EX 6.2 - 4)
14. Find the equation of the straight lines passing through  $(8, 3)$  and having intercepts whose sum is 1. (EX 6.2 - 9)

## SET - 4

15. A straight line is passing through the point  $A(1,2)$  with slope  $\frac{5}{12}$ . Find points on the line which are 13 units away from  $A$ . (EX 6.2 - 11)
16. Find the distance
  - (i) Between two points  $(5, 4)$  and  $(2, 0)$
  - (ii) From a point  $(1, 2)$  to the line  $5x + 12y - 3 = 0$
  - (iii) Between two parallel lines  $3x + 4y = 12$  and  $6x + 8y + 1 = 0$ . (EG 6.23)
17. Find the equation of the bisector of the acute angle between the lines  $3x + 4y + 2 = 0$  and  $5x + 12y - 5 = 0$ . (EG 6.25)
18. Find the equations of the straight lines in the family of the lines  $y = mx + 2$ , for which  $m$  and the  $x$  -coordinate of the point of intersection of the lines with  $2x + 3y = 10$  are integers. (EG 6.28)

## SET - 5

19. Find the equation of the line through the intersection of the lines  $3x + 2y + 5 = 0$  and  $3x - 4y + 6 = 0$  and the point  $(1,1)$ . (EG 6.29)
20. If a line joining two points  $(3, 0)$  and  $(5, 2)$  is rotated about the point  $(3, 0)$  in counter clockwise direction through an angle  $15^\circ$ , then find the equation of the line in the new position (EG 6.32)
21. Find the distance (i) between two points  $(5, 4)$  and  $(2, 0)$ (ii) from a point  $(1, 2)$  to the line  $5x + 12y - 3 = 0$  (iii) between two parallel lines  $3x + 4y = 12$  and  $6x + 8y + 1 = 0$ . (Eg. 6.23)
22. Find the equation of the straight line parallel to  $5x - 4y + 3 = 0$  and having  $x$  -intercept 3. (EX. 6.3 - 2)

## SET - 6

23. Find the equations of two straight lines which are parallel to the line  $12x + 5y + 2 = 0$  and at a unit distance from the point  $(1, -1)$ . (EX. 6.3 - 7)
24. Find the equations of straight lines which are perpendicular to the line  $3x + 4y - 6 = 0$  and are at a distance of 4 units from  $(2, 1)$ . (EX. 6.3 - 8)
25. Find the equation of a straight line parallel to  $2x + 3y = 10$  and which is such that the sum of its intercepts on the axes is 15. (EX 6.3 - 9)
26. Find the equation of the pair of lines through the origin and perpendicular to the pair of lines  $ax^2 + 2hxy + by^2 = 0$ . (Eg. 6.35)
27. If the pair of lines represented by  $x^2 - 2cxy - y^2 = 0$  and  $x^2 - 2dxy - y^2 = 0$  be such that each pair bisects the angle between the other pair, prove that  $cd = -1$ . (Eg. 6.37)

## SET - 7

28. Show that  $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$  represents a pair of parallel lines. (EX. 6.4 - 2)
29. Show that  $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$  represents a pair of perpendicular lines. (EX. 6.4 - 3)

30. Find the equation of the pair of straight lines passing through the point  $(1, 3)$  and perpendicular to the lines  $2x - 3y + 1 = 0$  and  $5x + y - 3 = 0$ . (EX. 6.4 - 6)
31. Find the separate equation of the following pair of straight lines  $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ . (EX. 6.4 - 7(iii))
32. The slope of one of the straight lines  $ax^2 + 2hxy + by^2 = 0$  is twice that of the other, show that  $8h^2 = 9ab$ . (EX. 6.4 - 8)
33. The slope of one of the straight lines  $ax^2 + 2hxy + by^2 = 0$  is three times the other, show that  $3h^2 = 4ab$ . (EX 6.4 - 9)

### 5 MARKS

#### SET 1

1. The sum of the distance of a moving point from the points  $(4,0)$  and  $(-4,0)$  is always 10 units. Find the equation of the locus of the moving point. (EX. 6.1 - 15)
2. The Pamban Sea Bridge is a railway bridge of length about 2065 m constructed on the Palk Strait, which connects the Island town of Rameswaram to Mandapam, the main land of India. The Bridge is restricted to a uniform speed of only 12.5 m/s. If a train of length 560m starts at the entry point of the bridge from Mandapam, then (i) Find an equation of the motion of the train. (ii) When does the engine touch island (iii) When does the last coach cross the entry point of the bridge? (iv) What is the time taken by a train to cross the bridge? (Eg. 6.10)
3. The quantity demanded of a certain type of Compact Disk is 22,000 units when a unit price is ₹ 8. The customer will not buy the disk, at a unit price of ₹ 30 or higher. On the other side the manufacturer will not market any disk if the price is ₹ 6 or lower. However, if the price ₹ 14 the manufacturer can supply 24,000 units. Assume that the quantity demanded and quantity supplied are linearly proportional to the price. Find (i) the demand equation (ii) supply equation (iii) the market equilibrium quantity and price. (iv) The quantity of demand and supply when the price is ₹ 10. (Eg. 6.13)
4. A straight line  $L$  with negative slope passes through the point  $(9,4)$  cuts the positive coordinate axes at the points  $P$  and  $Q$ . As  $L$  varies, find the minimum value of  $|OP| + |OQ|$ , where  $O$  is the origin. (Eg. 6.15)
5. The normal boiling point of water is  $100^\circ\text{C}$  or  $212^\circ\text{F}$  and the freezing point of water is  $0^\circ\text{C}$  or  $32^\circ\text{F}$ . (i) Find the linear relationship between  $C$  and  $F$  Find (ii) the value of  $C$  for  $98.6^\circ\text{F}$  and (iii) the value of  $F$  for  $38^\circ\text{C}$  (EX. 6.2 - 5)

#### SET 2

6. An object was launched from a place  $P$  in constant speed to hit a target. At the  $15^{\text{th}}$  second it was 1400m away from the target and at the  $18^{\text{th}}$  second 800m away. Find (i) the distance between the place and the target (ii) the distance covered by it in 15 seconds. (iii) time taken to hit the target. (EX. 6.2 - 6)
7. If  $p_1$  and  $p_2$  are the lengths of the perpendiculars from the origin to the straight lines  $x \sec \theta + y \operatorname{cosec} \theta = 2a$  and  $x \cos \theta - y \sin \theta = a \cos 2\theta$ , then prove that  $p_1^2 + p_2^2 = a^2$ . (EX. 6.3 - 11)
8. Find the image of the point  $(-2,3)$  about the line  $x + 2y - 9 = 0$ . (EX. 6.3 - 17)
9. Find atleast two equations of the straight lines in the family of the lines  $y = 5x + b$ , for which  $b$  and the  $x$ -coordinate of the point of intersection of the lines with  $3x - 4y = 6$  are integers. (EX. 6.3 - 19)
10. Show that the straight lines  $x^2 - 4xy + y^2 = 0$  and  $x + y = 3$  form an equilateral triangle. (Eg. 6.36)

#### SET 3

11. If the equation  $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$  represents a pair of straight lines, find (i) the value of  $\lambda$  and the separate equations of the lines (ii) point of intersection of the lines (iii) angle between the lines **(Eg. 6.38)**
12. Show that the straight lines joining the origin to the points of intersection of  $3x - 2y + 2 = 0$  and  $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$  are at right angles. **(Eg. 6.41)**
13. Prove that the equation to the straight lines through the origin, each of which makes an angle  $\alpha$  with the straight line  $y = x$  is  $x^2 - 2xy \sec 2\alpha + y^2 = 0$ . **(EX. 6.4 - 5)**
14. A  $\Delta OPQ$  is formed by the pair of straight lines  $x^2 - 4xy + y^2 = 0$  and the line  $PQ$ . The equation of  $PQ$  is  $x + y - 2 = 0$ . Find the equation of the median of the triangle  $\Delta OPQ$  drawn from the origin  $O$ . **(EX 6.4 - 10)**
15. Find  $p$  and  $q$ , if the following equation represents a pair of perpendicular lines  $6x^2 + 5xy - py^2 + 7x + qy - 5 = 0$ . **(EX 6.4 - 11)**

**SET 4**

16. Find the value of  $k$ , if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting,  $12x^2 + 7xy - 12y^2 - x + 7y + k = 0$ . **(EX 6.4 - 12)**
17. For what value of  $k$  does the equation  $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$  represent two straight lines. **(EX 6.4 - 13)**
18. Show that the equation  $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$  represents a pair of parallel lines. Find the distance between them. **(EX 6.4 - 14)**
19. Show that the equation  $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$  represents a pair of parallel lines. Find the distance between them. **(EX 6.4 - 15)**
20. Prove that one of the straight lines given by  $ax^2 + 2hxy + by^2 = 0$  will bisect the angle between the co-ordinate axes if  $(a + b)^2 = 4h^2$ . **(EX 6.4 - 16)**

**SET 5**

21. If the pair of straight lines  $x^2 - 2kxy - y^2 = 0$  bisect the angle between the pair of straight lines  $x^2 - 2lxy - y^2 = 0$ , Show that the later pair also bisects the angle between the former. **(EX 6.4 - 17)**
22. 18. Prove that the straight lines joining the origin to the points of intersection of
23.  $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$  and  $3x - 2y - 1 = 0$  are at right angles. **(EX 6.4 - 18)**

# VOLUME -2

## CHAPTER 7 MATRICES AND DETERMINANTS

### 2 MARKS

#### SET 1

1. Suppose that a matrix has 12 elements. What are the possible orders it can have? What if it has 7 elements?(EG 7.1)
2. Construct a  $2 \times 3$  matrix whose  $(i, j)^{th}$  element is given by  $a_{ij} = \frac{\sqrt{3}}{2} |2i - 3j|$ .(EG 7.2)
3. Find  $x, y, a$ , and  $b$  if  $\begin{bmatrix} 3x + 4y & 6 & x - 2y \\ a + b & 2a - b & -3 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 5 & -5 & -3 \end{bmatrix}$ .(EG 7.3)
4. Compute  $A + B$  and  $A - B$  if  $A = \begin{bmatrix} 4 & \sqrt{5} & 7 \\ -1 & 0 & 0.5 \end{bmatrix}$ ,  $B = \begin{bmatrix} \sqrt{3} & \sqrt{5} & 7.3 \\ 1 & \frac{1}{3} & \frac{1}{4} \end{bmatrix}$ .(EG 7.4)
5. Simplify :  $\sec \theta \begin{bmatrix} \sec \theta & \tan \theta \\ \tan \theta & \sec \theta \end{bmatrix} - \tan \theta \begin{bmatrix} \tan \theta & \sec \theta \\ \sec \theta & \tan \theta \end{bmatrix}$ .(EG 7.7)

#### SET 2

6. Solve for  $x$  if  $\begin{bmatrix} x & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = [0]$ (EG 7.9)
7. If  $A = \begin{bmatrix} 1 & a \\ 1 & 0 \end{bmatrix}$ , then compute  $A^4$ .(EX 7.1 - 5)
8. Consider the matrix  $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$  (i) Show that  $A_\alpha A_\beta = A_{(\alpha+\beta)}$  (EX 7.1 - 6)
9. If  $A$  is a square matrix such that  $A^2 = A$ , find the value of  $7A - (I + A)^3$ .(EX 7.1 - 12)
10. If  $A$  is a  $3 \times 4$  matrix and  $B$  is a matrix such that both  $A^T B$  and  $BA^T$  are defined, what is the order of the matrix  $B$ ?(EX 7.1 - 16)
11. Let  $A$  and  $B$  be two symmetric matrices. Prove that  $AB = BA$  if and only if  $AB$  is a symmetric matrix.(EX 7.1 - 22)

#### SET 3

12. Compute  $|A|$  using Sarrus rule if  $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$ .(EG 7.17)
13. If  $A$  is a square matrix and  $|A| = 2$ , find the value of  $|AA^T|$ .(EX 7.2 - 16)
14. If the area of the triangle with vertices  $(-3, 0)$ ,  $(3, 0)$  and  $(0, k)$  is 9 square units, find the values of  $k$ .(EG 7.32)
15. Find the area of the triangle whose vertices are  $(-2, -3)$ ,  $(3, 2)$ , and  $(-1, -8)$ .(EG 7.33)

### 3 MARKS

#### SET 1

1. Determine the value of  $x + y$  if  $\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 3 \\ y & x + 6 \end{bmatrix}$ .(EX 7.1 - 3)
2. If  $A = \begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$  and such that  $(A - 2I)(A - 3I) = O$ , find the value of  $x$ .(EX 7.1 - 7)

3. Find the matrix  $A$  which satisfies the matrix relation  $A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$ . (EX 7.1 - 14)

4. (i) For what value of  $x$ , the matrix  $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$  is skew-symmetric. (EX7.1 - 20)

5. Prove that  $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$ . (EG 7.22)

**SET 2**

6. Show that  $\begin{vmatrix} b + c & bc & b^2c^2 \\ c + a & ca & c^2a^2 \\ a + b & ab & a^2b^2 \end{vmatrix} = 0$ . (EX7.2 - 2)

7. Find the value of  $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$  if  $x, y, z \neq 1$ . (EX7.2 - 13)

8. If  $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$ , prove that  $\sum_{k=1}^n \det(A^k) = \frac{1}{3} \left( 1 - \frac{1}{4^n} \right)$ . (EX7.2 - 14)

9. Determine the roots of the equation  $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ . (EX7.2 - 19)

10. Solve  $\begin{vmatrix} x + a & b & c \\ a & x + b & c \\ a & b & x + c \end{vmatrix} = 0$ . (EX7.3 - 3)

**SET 3**

11. Solve  $\begin{vmatrix} 4 - x & 4 + x & 4 + x \\ 4 + x & 4 - x & 4 + x \\ 4 + x & 4 + x & 4 - x \end{vmatrix} = 0$ . (EX7.3 - 5)

12. In a triangle  $ABC$ , if  $\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A(1 + \sin A) & \sin B(1 + \sin B) & \sin C(1 + \sin C) \end{vmatrix} = 0$ ,  
prove that  $\Delta ABC$  is an isosceles triangle. (EG 7.26)

13. Show that  $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$ . (EG 7.28)

14. If  $\cos 2\theta = 0$ , determine  $\begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix}^2$ . (EX7.4 - 5)

15. Find the value of the product:  $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$ . (EX7.4 - 6)

**5 MARKS****SET 1**

1. Express the matrix  $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$  as the sum of a symmetric and a skew-symmetric matrices. (EG 7.13)

2. Verify the property  $A(B + C) = AB + AC$ , when the matrices  $A, B$ , and  $C$  are given by

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}, C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}. \text{(EX7.1 - 13)}$$

3. Prove that  $\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2. \text{(EX7.2 - 3)}$

4. Prove that  $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right). \text{(EX7.2 - 4)}$

### SET 2

5. If  $a, b, c$  are all positive, and are  $p^{\text{th}}, q^{\text{th}}$  and  $r^{\text{th}}$  terms of a G.P., show that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0. \text{(EX7.2 - 12)}$$

6. Using Factor Theorem, prove that  $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9). \text{(EG 7.23)}$

7. Prove that  $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx). \text{(EG 7.24)}$

8. Prove that  $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3. \text{(EG 7.25)}$

### SET 3

9. Show that using Factor Theorem  $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a). \text{(EX7.3 - 1)}$

10. Show that  $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc. \text{(EX7.3 - 2)}$

11. Show that  $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a). \text{(EX7.3 - 4)}$

12. Show that  $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x). \text{(EX7.3 - 6)}$

## CHAPTER 8 VECTOR ALGEBRA - I

### 2 MARKS

### SET 1

- Represent graphically the displacement of (i) 45 cm  $30^\circ$  north of east.  
(ii) 80 km  $60^\circ$  south of west. (EX8.1 - 1)
- Prove that the relation  $R$  defined on the set  $V$  of all vectors by ' $\vec{a}R\vec{b}$  if  $\vec{a} = \vec{b}$ ' is an equivalence relation on  $V$ . (EX8.1 - 2)

3. Let  $\vec{a}$  and  $\vec{b}$  be the position vectors of the points  $A$  and  $B$ . Prove that the position vectors of the points which trisect the line segment  $AB$  are  $\frac{\vec{a}+2\vec{b}}{3}$  and  $\frac{\vec{b}+2\vec{a}}{3}$ . (EX8.1 - 3)
4. Let  $A$  and  $B$  be two points with position vectors  $2\vec{a} + 4\vec{b}$  and  $2\vec{a} - 8\vec{b}$ . Find the position vectors of the points which divide the line segment joining  $A$  and  $B$  in the ratio 1: 3 internally and externally. (EG 8.3)
5. If  $D$  is the midpoint of the side  $BC$  of a triangle  $ABC$ , prove that  $\vec{AB} + \vec{AC} = 2\vec{AD}$ . (EX8.1 - 9)

**SET 2**

6. Find a unit vector along the direction of the vector  $5\vec{i} - 3\vec{j} + 4\vec{k}$ . (EG 8.4)
7. Find a direction ratio and direction cosines of the following vectors. (i)  $3\vec{i} + 4\vec{j} - 6\vec{k}$ , (ii)  $3\vec{i} - 4\vec{k}$  (EG 8.5)
8. (i) Find the direction cosines of a vector whose direction ratios are 2, 3, -6. (EG 8.6)
9. Can a vector have direction angles  $30^\circ, 45^\circ, 60^\circ$ ? (EG 8.6)
10. Find the direction cosines of  $\vec{AB}$  where  $A$  is (2, 3, 1) and  $B$  is (3, -1, 2). (EG 8.6)

**SET 3**

11. Find the direction cosines of the line joining (2, 3, 1) and (3, -1, 2). (EG 8.6)
12. The direction ratios of a vector are 2, 3, 6 and its magnitude is 5. Find the vector. (EG 8.6)
13. If  $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$  are the direction cosines of some vector, then find  $a$ . (EX8.2 - 5)
14. Show that the following vectors are coplanar  $\vec{i} - 2\vec{j} + 3\vec{k}$ ,  $-2\vec{i} + 3\vec{j} - 4\vec{k}$  and  $-\vec{j} + 2\vec{k}$  (EX8.2 - 9)
15. The position vectors  $\vec{a}, \vec{b}, \vec{c}$  of three points satisfy the relation  $2\vec{a} - 7\vec{b} + 5\vec{c} = \vec{0}$ . Are these points collinear? (EX8.2 - 14)

**SET 4**

16. Find the value or values of  $m$  for which  $m(\vec{i} + \vec{j} + \vec{k})$  is a unit vector. (EX8.2 - 16)
17. Find  $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$  if  $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$  and  $\vec{b} = 3\vec{i} + 2\vec{j} - \vec{k}$ . (EG 8.12)
18. If  $\vec{a} = 2\vec{i} + 2\vec{j} + 3\vec{k}$ ,  $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$  and  $\vec{c} = 3\vec{i} + \vec{j}$  be such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$  then find  $\lambda$ . (EG 8.13)
19. If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  prove that  $\vec{a}$  and  $\vec{b}$  are perpendicular. (EG 8.14)
20. For any vector  $\vec{r}$  prove that  $\vec{r} = (\vec{r} \cdot \vec{i})\vec{i} + (\vec{r} \cdot \vec{j})\vec{j} + (\vec{r} \cdot \vec{k})\vec{k}$ . (EG 8.15)

**SET 5**

21. Find the angle between the vectors  $5\vec{i} + 3\vec{j} + 4\vec{k}$  and  $6\vec{i} - 8\vec{j} - \vec{k}$ . (EG 8.16)
22. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a}| = 10, |\vec{b}| = 15$  and  $\vec{a} \cdot \vec{b} = 75\sqrt{2}$ . Find the angle between  $\vec{a}$  and  $\vec{b}$ . (EX8.3 - 31)
23. Show that the points (2, -1, 3), (4, 3, 1) and (3, 1, 2) are collinear. (EX8.3 - 9)
24. Find the projection of the vector  $\vec{i} + 3\vec{j} + 7\vec{k}$  on the vector  $2\vec{i} + 6\vec{j} + 3\vec{k}$ . (EX8.3 - 12)
25. Find  $\lambda$  when the projection of  $\vec{a} = \lambda\vec{i} + \vec{j} + 4\vec{k}$  on  $\vec{b} = 2\vec{i} + 6\vec{j} + 3\vec{k}$  is 4 units. (EX8.3 - 13)

**SET 6**

26. Find  $|\vec{a} \times \vec{b}|$  where  $\vec{a} = 3\vec{i} + 4\vec{j}$  and  $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ . (EG 8.20)
27. For any two vectors  $\vec{a}$  and  $\vec{b}$ , prove that  $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$ . (EG 8.25)

28. Find the magnitude of  $\vec{a} \times \vec{b}$  if  $\vec{a} = 2\vec{i} + \vec{j} + 3\vec{k}$  and  $\vec{b} = 3\vec{i} + 5\vec{j} - 2\vec{k}$ .(EX8.4 - 1)
29. Show that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$ .(EX8.4 - 2)
30. Find  $\vec{a} \cdot \vec{b}$  when (i)  $\vec{a} = \vec{i} - \vec{j} + 5\vec{k}$  and  $\vec{b} = 3\vec{i} - 2\vec{k}$

### 3 MARKS

#### SET 1

1. Vector addition is commutative.(RE8.5)
2. If  $\vec{a}$  and  $\vec{b}$  are vectors represented by two adjacent sides of a regular hexagon, then find the vectors represented by other sides.(EG 8.2)
3. A quadrilateral is a parallelogram if and only if its diagonals bisect each other.(TH8.4)
4. If  $D$  and  $E$  are the midpoints of the sides  $AB$  and  $AC$  of a triangle  $ABC$ , prove that  $\vec{BE} + \vec{DC} = \frac{3}{2}\vec{BC}$ .(EX8.1 - 4)
5. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.(EX8.1 - 5)

#### SET 2

6. Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.(EX8.1 - 6)
7. If  $\vec{PO} + \vec{OQ} = \vec{QO} + \vec{OR}$ , prove that the points  $P, Q, R$  are collinear.(EX8.1 - 8)
8. If  $G$  is the centroid of a triangle  $ABC$ , prove that  $\vec{GA} + \vec{GB} + \vec{GC} = \vec{0}$ .(EX8.1 - 10)
9. Let  $A, B$ , and  $C$  be the vertices of a triangle. Let  $D, E$ , and  $F$  be the midpoints of the sides  $BC, CA$  and  $AB$  respectively. Show that  $\vec{AD} + \vec{BE} + \vec{CF} = \vec{0}$ .(EX8.1 - 11)
10. Show that the points whose position vectors are  $2\vec{i} + 3\vec{j} - 5\vec{k}$ ,  $3\vec{i} + \vec{j} - 2\vec{k}$  and  $6\vec{i} - 5\vec{j} + 7\vec{k}$  are collinear.(EG 8.7)

#### SET 3

11. Show that the vectors  $5\vec{i} + 6\vec{j} + 7\vec{k}$ ,  $7\vec{i} - 8\vec{j} + 9\vec{k}$  and  $3\vec{i} + 20\vec{j} + 5\vec{k}$  are coplanar.(EG 8.10)
12. Show that the vectors  $2\vec{i} - \vec{j} + \vec{k}$ ,  $3\vec{i} - 4\vec{j} - 4\vec{k}$  and  $\vec{i} - 3\vec{j} - 5\vec{k}$  form a right angled triangle.(EX8.2 - 7)
13. Find the value of  $\lambda$  for which the vectors  $\vec{a} = 3\vec{i} + 2\vec{j} + 9\vec{k}$  and  $\vec{b} = \vec{i} + \lambda\vec{j} + 3\vec{k}$  are parallel.(EX8.2 - 8)
14. If  $\vec{a} = 2\vec{i} + 3\vec{j} - 4\vec{k}$ ,  $\vec{b} = 3\vec{i} - 4\vec{j} - 5\vec{k}$  and  $\vec{c} = -3\vec{i} + 2\vec{j} + 3\vec{k}$  find the magnitude and direction cosines of (ii)  $3\vec{a} - 2\vec{b} + 5\vec{c}$ .(EX8.2 - 11)
15. Find the unit vector parallel to  $3\vec{a} - 2\vec{b} + 4\vec{c}$  if  $\vec{a} = 3\vec{i} - \vec{j} - 4\vec{k}$ ,  $\vec{b} = -2\vec{i} + 4\vec{j} - 3\vec{k}$  and  $\vec{c} = \vec{i} + 2\vec{j} - \vec{k}$ .(EX8.2 - 13)

#### SET 4

16. Show that the points  $A(1, 1, 1)$ ,  $B(1, 2, 3)$  and  $C(2, -1, 1)$  are vertices of an isosceles triangle.(EX8.2 - 17)
17. Find the projection of  $\vec{AB}$  on  $\vec{CD}$  where  $A, B, C, D$  are the points  $(4, -3, 0)$ ,  $(7, -5, -1)$ ,  $(-2, 1, 3)$ ,  $(0, 2, 5)$ .(EG 8.17)



18. If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are three unit vectors satisfying  $\vec{a} - \sqrt{3}\vec{b} + \vec{c} = \vec{0}$  then find the angle between  $\vec{a}$  and  $\vec{c}$ . (EG 8.18)
19. Show that the vectors  $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ ,  $\vec{b} = 6\vec{i} + 2\vec{j} - 3\vec{k}$  and  $\vec{c} = 3\vec{i} - 6\vec{j} + 2\vec{k}$  are mutually orthogonal. (EX8.3 - 6)
20. If  $|\vec{a}| = 5$ ,  $|\vec{b}| = 6$ ,  $|\vec{c}| = 7$  and  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , find  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ . (EX8.3 - 8)

**SET 5**

21. Let  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  be three vectors such that  $|\vec{a}| = 3$ ,  $|\vec{b}| = 4$ ,  $|\vec{c}| = 5$  and each one of them being perpendicular to the sum of the other two, find  $|\vec{a} + \vec{b} + \vec{c}|$ . (EX8.3 - 11)
22. Find the vectors of magnitude 6 which are perpendicular to both vectors  $\vec{a} = 4\vec{i} - \vec{j} + 3\vec{k}$  and  $\vec{b} = -2\vec{i} + \vec{j} - 2\vec{k}$ . (EG 8.22)
23. Find the cosine and sine angle between the vectors  $\vec{a} = 2\vec{i} + \vec{j} + 3\vec{k}$  and  $\vec{b} = 4\vec{i} - 2\vec{j} + 2\vec{k}$ . (EG 8.23)
24. Find the area of the parallelogram whose adjacent sides are  $\vec{a} = 3\vec{i} + \vec{j} + 4\vec{k}$  and  $\vec{b} = \vec{i} - \vec{j} + \vec{k}$ . (EG 8.24)
25. Find the area of a triangle having the points  $A(1,0,0)$ ,  $B(0,1,0)$ , and  $C(0,0,1)$  as its vertices. (EG 8.26)

**SET 6**

26. Find the vectors of magnitude  $10\sqrt{3}$  that are perpendicular to the plane which contains  $\vec{i} + 2\vec{j} + \vec{k}$  and  $\vec{i} + 3\vec{j} + 4\vec{k}$ . (EX8.4 - 3)
27. Find the unit vectors perpendicular to each of the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ , where  $\vec{a} = \vec{i} + \vec{j} + \vec{k}$  and  $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ . (EX8.4 - 4)
28. Find the area of the parallelogram whose two adjacent sides are determined by the vectors  $\vec{i} + 2\vec{j} + 3\vec{k}$  and  $3\vec{i} - 2\vec{j} + \vec{k}$ . (EX8.4 - 5)
29. For any vector  $\vec{a}$  prove that  $|\vec{a} \times \vec{i}|^2 + |\vec{a} \times \vec{j}|^2 + |\vec{a} \times \vec{k}|^2 = 2|\vec{a}|^2$ . (EX8.4 - 8)
30. Find the angle between the vectors  $2\vec{i} + \vec{j} - \vec{k}$  and  $\vec{i} + 2\vec{j} + \vec{k}$  by using cross product. (EX8.4 - 10)

**5 MARKS****SET 1**

1. The medians of a triangle are concurrent. (TH8.3)
2. If  $ABCD$  is a quadrilateral and  $E$  and  $F$  are the mid-points of  $AC$  and  $BD$  respectively, then prove that  $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$ . (EX8.1 - 12)
3. Find a point whose position vector has magnitude 5 and parallel to the vector  $4\vec{i} - 3\vec{j} + 10\vec{k}$ . (EG 8.8)
4. Prove that the points whose position vectors  $2\vec{i} + 4\vec{j} + 3\vec{k}$ ,  $4\vec{i} + \vec{j} + 9\vec{k}$  and  $10\vec{i} - \vec{j} + 6\vec{k}$  form a right angled triangle. (EG 8.9)
5. A triangle is formed by joining the points  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$ . Find the direction cosines of the medians. (EX8.2 - 4)

**SET 2**

6. Show that the points whose position vectors  $4\vec{i} + 5\vec{j} + \vec{k}$ ,  $-\vec{j} - \vec{k}$ ,  $3\vec{i} + 9\vec{j} + 4\vec{k}$  and  $-4\vec{i} + 4\vec{j} + 4\vec{k}$  are coplanar. (EX8.2 - 10)
7. The position vectors of the vertices of a triangle are  $\vec{i} + 2\vec{j} + 3\vec{k}$ ,  $3\vec{i} - 4\vec{j} + 5\vec{k}$  and  $-2\vec{i} + 3\vec{j} - 7\vec{k}$ . Find the perimeter of the triangle. (EX8.2 - 12)

8. Three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are such that  $|\vec{a}| = 2$ ,  $|\vec{b}| = 3$ ,  $|\vec{c}| = 4$  and  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ . Find  $4\vec{a} \cdot \vec{b} + 3\vec{b} \cdot \vec{c} + 3\vec{c} \cdot \vec{a}$ . (EX8.3 - 14)
9. The position vectors of the points  $P, Q, R, S$  are  $\vec{i} + \vec{j} + \vec{k}$ ,  $2\vec{i} + 5\vec{j}$ ,  $3\vec{i} + 2\vec{j} - 3\vec{k}$  and  $\vec{i} - 6\vec{j} - \vec{k}$  respectively. Prove that the line  $PQ$  and  $RS$  are parallel. (EX8.2 - 15)
10. Show that the points  $(4, -3, 1)$ ,  $(2, -4, 5)$  and  $(1, -1, 0)$  form a right angled triangle. (EG 8.19)

**SET 3**

11. If  $\vec{a}, \vec{b}$  are unit vectors and  $\theta$  is the angle between them, show that (i)  $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$   
(ii)  $\cos \frac{\theta}{2} = \frac{1}{2} |\vec{a} + \vec{b}|$  (ii)  $\tan \frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$ . (EX8.3 - 10)
12. If  $\vec{a} = -3\vec{i} + 4\vec{j} - 7\vec{k}$  and  $\vec{b} = 6\vec{i} + 2\vec{j} - 3\vec{k}$ , verify (i)  $\vec{a}$  and  $\vec{a} \times \vec{b}$  are perpendicular to each other. (ii)  $\vec{b}$  and  $\vec{a} \times \vec{b}$  are perpendicular to each other. (EG 8.21)
13. Find the area of the triangle whose vertices are  $A(3, -1, 2)$ ,  $B(1, -1, -3)$  and  $C(4, -3, 1)$ . (EX8.4 - 6)
14. If  $\vec{a}, \vec{b}, \vec{c}$  are the position vectors of the vertices  $A, B, C$  of a triangle  $ABC$ , then prove that the area of triangle  $ABC$  is  $\frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ . Also deduce the condition for collinearity of the points  $A, B$  and  $C$ . (EX8.4 - 7)
15. Let  $\vec{a}, \vec{b}, \vec{c}$  be unit vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$  and the angle between  $\vec{b}$  and  $\vec{c}$  is  $\frac{\pi}{3}$ . Prove that  $\vec{a} = \pm \frac{2}{\sqrt{3}} (\vec{b} \times \vec{c})$ . (EX8.4 - 9)

**CHAPTER 9 LIMITS AND CONTINUITY****2 MARKS****SET 1**

1. Evaluate  $\lim_{x \rightarrow 2^-} [x]$  and  $\lim_{x \rightarrow 2^+} [x]$ . (EG 9.3)
2. Check if  $\lim_{x \rightarrow -5} f(x)$  exists or not, where  $f(x) = \begin{cases} \frac{|x+5|}{x+5}, & \text{for } x \neq -5 \\ 0, & \text{for } x = -5 \end{cases}$ . (EG 9.5)
3. Test the existence of the limit,  $\lim_{x \rightarrow 1} \frac{4|x-1|+x-1}{|x-1|}$ ,  $x \neq 1$ . (EG 9.6)
4. Write a brief description of the meaning of the notation  $\lim_{x \rightarrow 8} f(x) = 25$ . (EX9.1 - 19)
5. If  $f(2) = 4$ , can you conclude anything about the limit of  $f(x)$  as  $x$  approaches 2? (EX9.1 - 20)

**SET 2**

6. Evaluate:  $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$  if it exists by finding  $f(3^-)$  and  $f(3^+)$ . (EX9.1 - 22)
7. Verify the existence of  $\lim_{x \rightarrow 1} f(x)$ , where  $f(x) = \begin{cases} \frac{|x-1|}{x-1}, & \text{for } x \neq 1 \\ 0, & \text{for } x = 1 \end{cases}$ . (EX9.1 - 23)
8. Compute  $\lim_{x \rightarrow 0} \left( \frac{x^2+x}{x} + 4x^3 + 3 \right)$ . (EG 9.10)
9. Compute  $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$ . (EG 9.14)
10. Find  $\lim_{t \rightarrow 1} \frac{\sqrt{t}-1}{t-1}$ . (EG 9.17)

**SET 3**

11. Find  $\lim_{x \rightarrow 0} \frac{(2+x)^5 - 2^5}{x}$  (EG 9.18)
12. Find the positive integer  $n$  so that  $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 27$ . (EG 9.19)
13. Find the relation between  $a$  and  $b$  if  $\lim_{x \rightarrow 3} f(x)$  exists where  $f(x) =$   
 $\begin{cases} ax + b & , \text{ if } x > 3 \\ 3ax - 4b + 1 & , \text{ if } x < 3 \end{cases}$  (EG 9.20)
14.  $\lim_{x \rightarrow 3} \frac{x^4 - 16}{x - 2}$ . (EX9.2 - 1)
15.  $\lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}$ ,  $m$  and  $n$  are integers. (EX9.2 - 2)

**SET 4**

16.  $\lim_{\sqrt{x} \rightarrow 3} \frac{x^2 - 81}{\sqrt{x} - 3}$ . (EX9.2 - 3)
17.  $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ ,  $x > 0$ . (EX9.2 - 4)
18.  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$ . (EX9.2 - 6)
19.  $\lim_{x \rightarrow \infty} \frac{x^3 + x}{x^4 - 3x^2 + 1}$ . (EX9.3 - 4)
20. A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water after  $t$  minutes (in grams per litre) is  $C(t) = \frac{30t}{200+t}$ . What happens to the concentration as  $t \rightarrow \infty$ ? (EX9.3 - 10)

**SET 4**

21. Prove that  $\lim_{x \rightarrow 0} \sin x = 0$ . (EG 9.30)
22. Evaluate  $\lim_{x \rightarrow 0} (1 + \sin x)^{2 \operatorname{cosec} x}$ . (EG 9.32)
23. Do the limits of following functions exist as  $x \rightarrow 0$ ? State reasons for your answer. (i)  $\frac{\sin |x|}{x}$ . (EG 9.35)
24.  $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{x+2}$ . (EX9.4 - 5)
25. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin \alpha x}{\sin \beta x}$ . (EX9.4 - 7)

**SET 4**

26.  $\lim_{x \rightarrow 0} \frac{\tan 2x}{\sin 5x}$ . (EX9.4 - 8)
27.  $\lim_{\alpha \rightarrow 0} \frac{\sin(\alpha^n)}{(\sin \alpha)^m}$ . (EX9.4 - 9)
28.  $\lim_{x \rightarrow 0} \frac{2 \arcsin x}{3x}$ . (EX9.4 - 12)
29. Calculate  $\lim_{x \rightarrow 3} \frac{(x^2 - 6x + 5)}{x^3 - 8x + 7}$ . (EG 9.13)

**3 MARKS****SET 1**

- $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$ .(EX9.2 - 8)
- $\lim_{x \rightarrow 1} \frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$ .(EX9.2 - 10)
- $\lim_{x \rightarrow a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$ , ( $a > b$ ). (EX9.2 - 15)
- Show that (i)  $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{3n^2+7n+2} = \frac{1}{6}$ , (EX9.3 - 8)

**SET 2**

- Show that (ii)  $\lim_{n \rightarrow \infty} \frac{1^2+2^2+3^2+\dots+(3n)^2}{(1+2+\dots+5n)(2n+3)} = \frac{9}{25}$ , (EX9.3 - 8)
- Show that (iii)  $\lim_{n \rightarrow \infty} \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} = 1$ .(EX9.3 - 8)
- Evaluate  $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x-2}\right)^x$ . (EG 9.33)
- Do the limits of functions exist as  $x \rightarrow 0$ ? State reasons for your answer.  $\frac{\sin(x-[x])}{x-[x]}$ . (EG 9.35)

**SET 3**

- Evaluate  $\lim_{x \rightarrow 0} \frac{\sin^3\left(\frac{x}{2}\right)}{x^3}$ . (EX9.4 - 6)
- $\lim_{x \rightarrow 0} \frac{3^x-1}{\sqrt{x+1}-1}$ . (EX9.4 - 16)
- $\lim_{x \rightarrow 0} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$ . (EX9.4 - 22)
- $\lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$ . (EX9.4 - 23)
- If  $f$  and  $g$  are continuous functions with  $f(3) = 5$  and  $\lim_{x \rightarrow 3} [2f(x) - g(x)] = 4$ , find  $g(3)$ . (EX9.5 - 8)

**5 MARKS**

- (a)  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ , (b)  $\lim_{\theta \rightarrow 0} \frac{1-\cos \theta}{\theta} = 0$ . (RE9.1)
- Show that  $\lim_{x \rightarrow 0^+} x \left[ \left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \dots + \left\lfloor \frac{15}{x} \right\rfloor \right] = 120$ . (EG 9.31)
- For what value of  $a$  is this  $f(x) = \begin{cases} \frac{x^4-1}{x-1}, & \text{if } x \neq 1 \\ a, & \text{if } x = 1 \end{cases}$ , continuous at  $x = 1$ ? (EX9.5 - 6)
- A function  $f$  is defined as follows:  $f(x) = \begin{cases} 0, & \text{for } x < 0 \\ x, & \text{for } 0 \leq x < 1 \\ -x^2 + 4x - 2, & \text{for } 1 \leq x < 3 \\ 4 - x, & \text{for } x \geq 3 \end{cases}$ . Is the function continuous? (EX9.5 - 10)

**CHAPTER 10 – DIFFERENTIAL CALCULUS - DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION**

**2 MARKS**

**SET 1**

1. Find the slope of the tangent line to the graph of  $f(x) = 7x + 5$  at any point  $(x_0, f(x_0))$ .(EG 10.1)
2. Find the derivatives of the functions using first principle.  $f(x) = -x^2 + 2$ (EX10.1 - 1)
3. Examine the differentiability of functions in  $R$  by drawing the diagrams. (i)  $|\sin x|$  EX10.1 - 7)
4. Differentiate the following with respect to  $x$ : (viii) Find  $f'(3)$  and  $f'(5)$  if  $f(x) = |x - 4|$ .(EG 10.7)
5.  $g(t) = 4 \sec t + \tan t$ (EX10.2 - 6)

**SET 2**

6. Find the derivatives  $y = e^x \sin x$ (EX10.2 - 7)
7. Find the derivatives  $y = \frac{\sin x}{1 + \cos x}$ (EX10.2 - 9)
8.  $y = \frac{\sin x}{x^2}$ (EX10.2 - 12)
9.  $y = e^{-x} \cdot \log x$ (EX10.2 - 16)
10.  $y = \sin x^\circ$ (EX10.2 - 18)

**SET 3**

11.  $y = \log_{10} x$ (EX10.2 - 19)
12. Differentiate:  $y = (x^3 - 1)^{100}$ .(EG 10.10)
13.  $y = \operatorname{cosec} x \cdot \cot x$ (EX10.2 - 14)
14. Differentiate:  $y = e^{\sin x}$ .(EG 10.14)
15. Differentiate  $2^x$ .(EG 10.15)

**SET 4**

16.  $y = e^{\sqrt{x}}$ (EX10.3 - 5)
17.  $y = xe^{-x^2}$ (EX10.3 - 15)
18.  $y = 5^{\frac{-1}{x}}$ (EX10.3 - 20)
19.  $y = e^{x \cos x}$ (EX10.3 - 27)
20. Differentiate:  $y = x^{\sqrt{x}}$ .(EG 10.23)

**SET 5**

21. Find  $\frac{dy}{dx}$  if  $x^2 + y^2 = 1$ .(EG 10.17)
22. Find  $\frac{dy}{dx}$  if  $x = at^2, y = 2at, t \neq 0$ .(EG 10.26)
23. Find  $y', y''$  and  $y'''$  if  $y = x^3 - 6x^2 - 5x + 3$ .(EG 10.31)
24. Find  $y'''$  if  $y = \frac{1}{x}$ .(EG 10.32)
25. Find  $f''$  if  $f(x) = x \cos x$ .(EG 10.33)

**3 MARKS****SET 1**

1. Determine whether the function is differentiable at the indicated values. (iii)  $f(x) = |x| + |x - 1|$  at  $x = 0, 1$ (EX10.1 - 3)

- Show that the functions are not differentiable at the indicated value of  $x$ .  $f(x) = \begin{cases} -x + 2, & x \leq 2 \\ 2x - 4, & x > 2 \end{cases}$ ,  $x = 2$  (EX10.1 - 4)
- Draw the function  $f'(x)$  if  $f(x) = 2x^2 - 5x + 3$  (EX10.2 - 20)
- Differentiate  $(2x + 1)^5(x^3 - x + 1)^4$ . (EG 10.13)
- If  $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ , find  $y'$ . (EG 10.16)

**SET 2**

- $y = \sin^2(\cos kx)$  (EX10.3 - 23)
- $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$  (EX10.3 - 28)
- Find  $\frac{dy}{dx}$  if  $x^4 + x^2y^3 - y^5 = 2x + 1$ . (EG 10.19)
- Find  $\frac{dy}{dx}$  if  $\sin y = y \cos 2x$ . (EG 10.20)
- If  $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ , find  $y'$ . (EG 10.24)

**SET 3**

- Find  $\frac{dy}{dx}$  if  $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$ . (EG 10.27)
- Find the derivative of  $\tan^{-1}(1 + x^2)$  with respect to  $x^2 + x + 1$ . (EG 10.29)
- Differentiate  $\sin(ax^2 + bx + c)$  with respect to  $\cos(lx^2 + mx + n)$ . (EG 10.30)
- Find  $f''$  if  $x^4 + y^4 = 16$ . (EG 10.34)
- Find the second order derivative if  $x$  and  $y$  are given by  $x = a \cos t$ ,  $y = a \sin t$ . (EG 10.35)

**SET 1**

- Find  $\frac{d^2y}{dx^2}$  if  $x^2 + y^2 = 4$ . (EG 10.36)
- $x = a \cos^3 t$ ,  $y = a \sin^3 t$  (EX10.4 - 13)
- $x = \frac{1-t^2}{1+t^2}$ ,  $y = \frac{2t}{1+t^2}$ . (EX10.4 - 15)
- Find the derivative of  $\sin x^2$  with respect to  $x^2$ . (EX10.4 - 19)
- If  $y = \sin^{-1} x$  then find  $y''$ . (EX10.4 - 23)

**5 MARKS****SET 1**

- Differentiate:  $y = \frac{x^3 \sqrt{x^2+1}}{(3x+2)^5}$ . (EG 10.22)
- Find the derivative with  $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$  with respect to  $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ . (EX10.4 - 22)
- If  $y = e^{\tan^{-1} x}$  show that  $(1 + x^2)y'' + (2x - 1)y' = 0$ . (EX10.4 - 24)
- If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ , show that  $(1 - x^2)y_2 - 3xy_1 - y = 0$ . (EX10.4 - 25)
- If  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$  then prove that at  $\theta = \frac{\pi}{2}$ ,  $y'' = \frac{1}{a}$ . (EX10.4 - 26)
- If  $\sin y = x \sin(a + y)$ , then prove that  $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ ,  $a \neq n\pi$ . (EX10.4 - 27)
- If  $y = (\cos^{-1} x)^2$ , prove that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$ . Hence find  $y_2$  when  $x = 0$ . (EX10.4 - 28)

## CHAPTER 11 – INTEGRAL CALCULUS

## 2 MARKS

## SET 1

- Integrate the with respect to  $x$ : (iii)  $\frac{\sin x}{\cos^2 x}$  (EG 11.2)
- Integrate the functions with respect to  $x$ : (ii)  $\frac{1}{(2-3x)^4}$  (EX 11.2 - 1)
- Evaluate  $30 \sec(2 - 15x) \tan(2 - 15x)$  (EX 11.2 - 4)
- Evaluate the with respect to  $x$ : (ii)  $\int \sqrt{15 - 2x} dx$  (EG 11.4)
- Integrate  $\frac{1}{\sqrt{1-25x^2}}$  (EG 11.7)

## SET 2

- If  $f'(x) = 3x^2 - 4x + 5$  and  $f(1) = 3$ , then find  $f(x)$ . (EG 11.10)
- If  $f'(x) = 4x - 5$  and  $f(2) = 1$ , find  $f(x)$ . (EX 11.4 - 1)
- If  $f'(x) = 9x^2 - 6x$  and  $f(0) = -3$ , find  $f(x)$ . (EX 11.4 - 2)
- Integrate the following with respect to  $x$ : (i)  $\cos 5x \sin 3x$  (EG 11.16)
- Evaluate:  $\int \sqrt{1 + \cos 2x} dx$ . (EG 11.20)

## SET 3

- Evaluate: (ii)  $\int e^x \log^2 e^x dx$ . (EG 11.26)
- Evaluate:  $e^{x \log a} e^x$  (EX 11.5 - 13)
- Evaluate  $\frac{e^x - e^{-x}}{e^x + e^{-x}}$  (EX 11.6 - 3)
- Evaluate  $\frac{\sin \sqrt{x}}{\sqrt{x}}$  (EX 11.6 - 5)
- Evaluate  $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$  (EX 11.6 - 9)

## SET 4

- Evaluate  $\alpha \beta x^{\alpha-1} e^{-\beta x^\alpha}$  (EX 11.6 - 12)
- Evaluate the integrals (ii)  $\int e^{-5x} \sin 3x dx$  (EG 11.36)
- Integrate the with respect to  $x$ :  $e^x (\tan x + \log \sec x)$  (EX 11.9 - 1)
- Find the integrals of the : (ii)  $\frac{1}{25-4x^2}$  (EX 11.10 - 1)
- Evaluate the following  $x$ : (i)  $\int \sqrt{4 - x^2} dx$  (EG 11.41)

## 3 MARKS

## SET 1

- Evaluate the following integrals: (i)  $\frac{12}{(4x-5)^3} + \frac{6}{3x+2} + 16e^{4x+3}$  (EG 11.9)

2. A train started from Madurai Junction towards Coimbatore at 3pm (time  $t = 0$ ) with velocity  $v(t) = 20t + 50$  kilometer per hour, where  $t$  is measured in hours. Find the distance covered by the train at 5pm. (EG 11.11)
3. Evaluate:  $\int (\tan x + \cot x)^2 dx$  (EG 11.22)
4. Evaluate:  $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$ . (EG 11.28)
5. Evaluate: (ii)  $\int \frac{x+3}{(x+2)^2(x+1)} dx$  (EG 11.29)

**SET 2**

6.  $(3x + 4)\sqrt{3x + 7}$  (EX 11.5 - 14)
7.  $\frac{1}{(x-1)(x+2)^2}$  (EX 11.5 - 18)
8.  $\frac{x^3}{(x-1)(x-2)}$ . (EX 11.5 - 20)
9.  $\sin^5 x \cos^3 x$  (EX 11.6 - 15)
10.  $\frac{x \sin^{-1} x}{\sqrt{1-x^2}}$  (EX 11.7 - 3)

**SET 3**

11.  $e^x \left( \frac{2+\sin 2x}{1+\cos 2x} \right)$  (EX 11.9 - 4)
12.  $\frac{\log x}{(1+\log x)^2}$  (EX 11.9 - 6)
13. Evaluate the integrals (iii)  $\int \frac{1}{\sqrt{12+4x-x^2}} dx$  (EG 11.39)
14.  $\frac{1}{\sqrt{x^2+4x+2}}$  (EX 11.10 - 2)
15. Integrate the following functions with respect to  $x$ : (ii)  $\sqrt{x^2 - 2x - 3}$  EX 11.12 - 1)

**5 MARKS**

1. At a particular moment, a student needs to stop his speedy bike to avoid a collision with the barrier ahead at a distance 40 metres away from him. Immediately he slows (retardation) the bike under braking at a rate of  $8 \text{ metre/second}^2$ . If the bike is moving at a speed of  $24 \text{ m/s}$ , when the brakes are applied, would it stop before collision? (EG 11.14)
2. Evaluate the following integrals
  - (i)  $\int \frac{3x+5}{x^2+4x+7} dx$
  - (ii)  $\int \frac{x+1}{x^2-3x+1} dx$
  - (iii)  $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$
  - (iv)  $\int \frac{5x-7}{\sqrt{3x-x^2-2}} dx$  (EG 11.40)
3. Integrate the following with respect to  $x$ :
  - (1) (i)  $\frac{2x-3}{x^2+4x-12}$  (ii)  $\frac{5x-2}{2+2x+x^2}$  (iii)  $\frac{3x+1}{2x^2-2x+3}$  (EX 11.11 - 1)
  - (2) (i)  $\frac{2x+1}{\sqrt{9+4x-x^2}}$  (ii)  $\frac{x+2}{\sqrt{x^2-1}}$  (iii)  $\frac{2x+3}{\sqrt{x^2+4x+1}}$  (EX 11.11 - 2)

**CHAPTER 12 PROBABILITY THEORY****2 MARKS**



**SET 1**

1. An integer is chosen at random from the first ten positive integers. Find the probability that it is (i) an even number (ii) multiple of three.(EG 12.2)
2. Suppose a fair die is rolled. Find the probability of getting (i) an even number (ii) multiple of three.(EG 12.5)
3. An experiment has the four possible mutually exclusive and exhaustive outcomes  $A, B, C,$  and  $D$ . Check whether the assignments of probability are permissible.(i)  $P(A) = 0.15, P(B) = 0.30, P(C) = 0.43, P(D) = 0.12$ (EX 12.1 - 1)
4. If two coins are tossed simultaneously, then find the probability of getting (i) one head and one tail (ii) at most two tails(EX 12.1 - 2)
5. What is the chance that (i) non-leap year (ii) leap year should have fifty three Sundays?(EX 12.1 - 4)

**SET 2**

6. (i) The odds that the event  $A$  occurs is 5 to 7, find  $P(A)$ .(ii) Suppose  $P(B) = \frac{2}{5}$ . Express the odds that the event  $B$  occurs.(EX 12.1 - 10)
7. If  $\bar{A}$  is the complementary event of  $A$ , then  $P(\bar{A}) = 1 - P(A)$ .(TH12.4)
8. Find the probability of getting the number 7, when a usual die is rolled.(EG 12.12)
9. Nine coins are tossed once, find the probability to get at least two heads.(EG 12.13)
10. If  $A$  and  $B$  are any two events, then  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .(TH12.6)

**SET 3**

11. If  $A$  and  $B$  are any two events and  $\bar{B}$  is the complementary events of  $B$ , then  $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ .(TH12.5)
12. A die is thrown twice. Let  $A$  be the event, 'First die shows 5' and  $B$  be the event, 'second die shows 5'. Find  $(A \cup B)$ .(EX 12.2 - 3)
13. If  $P(A) = 0.6, P(B) = 0.5,$  and  $P(A \cap B) = 0.2$ , Find (i)  $P(A/B)$  (ii)  $P(\bar{A}/B)$  (iii)  $P(A/\bar{B})$ .(EG 12.16)
14. A die is rolled. If it shows an odd number, then find the probability of getting 5.(EG 12.17)
15. If  $A$  and  $B$  are two independent events such that  $P(A) = 0.4$  and  $P(A \cup B) = 0.9$ . Find  $P(B)$ .(EG 12.20)

**SET 4**

16. Can two events be mutually exclusive and independent simultaneously?(EX 12.3 - 1)
17. If  $A$  and  $B$  are two events such that  $P(A \cup B) = 0.7, P(A \cap B) = 0.2,$  and  $P(B) = 0.5$ , then show that  $A$  and  $B$  are independent.(EX 12.3 - 2)
18. If  $P(A) = 0.5, P(B) = 0.8$  and  $P(B/A) = 0.8$ , find  $P(A/B)$  and  $P(A \cup B)$ .(EX 12.3 - 4)

**3 MARKS****SET 1**

1. Three coins are tossed simultaneously, what is the probability of getting (i) exactly one head (ii) at least one head (iii) at most one head?(EG 12.3)
2. Suppose ten coins are tossed. Find the probability to get (i) exactly two heads (ii) at most two heads (iii) at least two heads(EG 12.4)
3. When a pair of fair dice is rolled, what are the probabilities of getting the sum (i) 7 (ii) 7 or 9 (iii) 7 or 12?(EG 12.6)

4. Three candidates  $X, Y$  and  $Z$  are going to play in a chess competition to win *FIDE* (World Chess Federation) cup this year.  $X$  is thrice as likely to win as  $Y$  and  $Y$  is twice as likely as to win  $Z$ . Find the respective probability of  $X, Y$  and  $Z$  to win the cup. (EG 12.7)
5. Three letters are written to three different persons and addresses on three envelopes are also written. Without looking at the addresses, what is the probability that (i) exactly one letter goes to the right envelopes (ii) none of the letters go into the right envelopes? (EG 12.8)

**SET 2**

6. Let the matrix  $M = \begin{bmatrix} x & y \\ z & 1 \end{bmatrix}$ . If  $x, y$  and  $z$  are chosen at random from the set  $\{1, 2, 3\}$ , and repetition is allowed (i.e.,  $x = y = z$ ), what is the probability that the given matrix  $M$  is a singular matrix? (EG 12.9)
7. Five mangoes and 4 apples are in a box. If two fruits are chosen at random, find the probability that (i) one is a mango and the other is an apple (ii) both are of the same variety. (EX 12.1 - 3)
8. Eight coins are tossed once, find the probability of getting (i) exactly two tails (ii) at least two tails (iii) at most two tails (EX 12.1 - 5)
9. Given that  $P(A) = 0.52, P(B) = 0.43$ , and  $P(A \cap B) = 0.24$ , find (i)  $P(A \cap \bar{B})$  (ii)  $P(A \cup B)$  (iii)  $P(\bar{A} \cap \bar{B})$  (iv)  $P(\bar{A} \cup \bar{B})$  (EG 12.14)
10. The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (i) she will get at least one of the two jobs (ii) she will get only one of the two jobs. (EG 12.15)

**SET 3**

11. If  $A$  and  $B$  are mutually exclusive events  $P(A) = \frac{3}{8}$  and  $P(B) = \frac{1}{8}$ , then find (i)  $P(\bar{A})$  (ii)  $P(A \cup B)$  (iii)  $P(\bar{A} \cap B)$  (iv)  $P(\bar{A} \cup \bar{B})$  (EX 12.2 - 1)
12. If  $A$  and  $B$  are two events associated with a random experiment for which  $P(A) = 0.35, P(A \text{ or } B) = 0.85$ , and  $P(A \text{ and } B) = 0.15$ . Find (i)  $P(\text{only } B)$  (ii)  $P(\bar{B})$  (iii)  $P(\text{only } A)$  (EX 12.2 - 2)
13. A town has 2 fire engines operating independently. The probability that a fire engine is available when needed is 0.96. (i) What is the probability that a fire engine is available when needed? (ii) What is the probability that neither is available when needed? (EX 12.2 - 5)
14. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that (i) it will get at least one of the two awards (ii) it will get only one of the awards. (EX 12.2 - 6)
15. If  $A$  and  $B$  are independent then (i)  $\bar{A}$  and  $\bar{B}$  are independent. (ii)  $A$  and  $\bar{B}$  are independent. (iii)  $\bar{A}$  and  $B$  are also independent. (TH 12.8)

**SET 4**

16. Two cards are drawn from a pack of 52 cards in succession. Find the probability that both are Jack when the first drawn card is (i) replaced (ii) not replaced (EG 12.18)
17. If  $A$  and  $B$  are two independent events such that  $P(A \cup B) = 0.6, P(A) = 0.2$ , find  $P(B)$ . (EX 12.3 - 3)
18. If for two events  $A$  and  $B, P(A) = \frac{3}{4}, P(B) = \frac{2}{5}$  and  $A \cup B = S$  (sample space), find the conditional probability  $P(A/B)$ . (EX 12.3 - 5)
19. Two thirds of students in a class are boys and rest girls. It is known that the probability of a girl getting a first grade is 0.85 and that of boys is 0.70. Find the probability that a student chosen at random will get first grade marks. (EX 12.3 - 9)
20. A year is selected at random. What is the probability that (i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays (EX 12.3 - 11)

**SET 5**

21. Suppose the chances of hitting a target by a person  $X$  is 3 times in 4 shots, by  $Y$  is 4 times in 5 shots, and by  $Z$  is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits? (EX 12.3 - 12)
22. Urn-I contains 8 red and 4 blue balls and urn-II contains 5 red and 10 blue balls. One urn is chosen at random and two balls are drawn from it. Find the probability that both balls are red. (EG 12.24)

### 5 MARKS

#### SET 1

1. A coin is tossed twice. Events  $E$  and  $F$  are defined as follows  $E$  = Head on first toss,  $F$  = Head on second toss. Find (i)  $P(E \cup F)$   
(ii)  $P(E/F)$  (iii)  $P(\bar{E}/F)$  (iv) Are the events  $E$  and  $F$  independent? (EG 12.19)
2.  $X$  speaks truth in 70 percent of cases, and  $Y$  in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact? (EG 12.22)
3. A problem in Mathematics is given to three students whose chances of solving it are  $\frac{1}{3}, \frac{1}{4}$  and  $\frac{1}{5}$  (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it? (EX 12.3 - 6)
4. The probability that a car being filled with petrol will also need an oil change is 0.30, the probability that it needs a new oil filter is 0.40, and the probability that both the oil and filter need changing is 0.15. (i) If the oil had to be changed, what is the probability that a new oil filter is needed? (ii) If a new oil filter is needed, what is the probability that the oil has to be changed? (EX 12.3 - 7)
5. A factory has two machines  $I$  and  $II$ . Machine-I produces 40% of items of the output and Machine-II produces 60% of the items. Further 4% of items produced by Machine-I are defective and 5% produced by Machine-II are defective. If an item is drawn at random, find the probability that it is a defective item. (EG 12.25)

#### SET 2

6. A factory has two machines  $I$  and  $II$ . Machine  $I$  produces 40% of items of the output and Machine  $II$  produces 60% of the items. Further 4% of items produced by Machine  $I$  are defective and 5% produced by Machine  $II$  are defective. An item is drawn at random. If the drawn item is defective, find the probability that it was produced by Machine  $II$ . (See the previous example, compare the questions). (EG 12.26)
7. A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work? (EG 12.27)
8. The chances of  $X, Y$  and  $Z$  becoming managers of a certain company are 4: 2: 3. The probabilities that bonus scheme will be introduced if  $X, Y$  and  $Z$  become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that  $Z$  was appointed as the manager? (EG 12.28)
9. A consulting firm rents car from three agencies such that 50% from agency  $L$ , 30% from agency  $M$  and 20% from agency  $N$ . If 90% of the cars from  $L$ , 70% of cars from  $M$  and 60% of the cars from  $N$  are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency  $N$ ? (EG 12.29)

10. A factory has two Machines—*I* and *II*. Machine—*I* produces 60% of items and Machine—*II* produces 40% of the items of the total output. Further 2% of the items produced by Machine—*I* are defective whereas 4% produced by Machine—*II* are defective. If an item is drawn at random what is the probability that it is defective?(EX 12.4 - 1)

**SET 3**

11. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?(EX 12.4 - 2)
12. A firm manufactures PVC pipes in three plants viz, *X*, *Y* and *Z*. The daily production volumes from the three firms *X*, *Y* and *Z* are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant *X*, 4% from plant *Y* and 2% from plant *Z* are defective. A pipe is selected at random from a day's total production, (i) find the probability that the selected pipe is a defective one. (ii) if the selected pipe is a defective, then what is the probability that it was produced by plant *Y*? (EX 12.4 - 3)
13. The chances of *A*, *B* and *C* becoming manager of a certain company are 5: 3: 2. The probabilities that the office canteen will be improved if *A*, *B*, and *C* become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that *B* was appointed as the manager?(EX 12.4 - 4)
14. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.(EX 12.4 - 5)

[www.Padasalai.Net](http://www.Padasalai.Net)

[www.Padasalai.Net](http://www.Padasalai.Net)

[www.Padasalai.Net](http://www.Padasalai.Net)



**K. Magibalan**



**R. Pavithran**



**M. Hariharan**



**R. Kavinkumaran**



**M. Suthakar**



**R. Ragesh**



**N. Anichchem**



**M. Rithika**



**Harshini Ram**



**R. Kavinila**



**S. Srinithi**



**G. Aasika**



**S. Santhosh**



**M. Mosikeeran**



**T. Monoranjan**



**K.V. Sri Suriya dharshini**



**S. Gopikrishnan**



**C. Harish**



**S. Saptharishvaran**

*2023-24 centum holders of  
Mathematics in 10<sup>th</sup> standard*