**Class : 12**

Register Number

# SECOND MID TERM TEST - 2024

## MATHEMATICS

Time Allowed : 1.30 Hours] [Max. Marks : 50

### PART - I

**Answer all the Questions.** 10x1=10

1. The position of a particle moving along a horizontal line of any time $t$ is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

   (1) $t = 0$ (2) $t = \frac{1}{3}$ (3) $t = 1$ (4) $t = 3$

2. If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

   (1) $e^{x^2+y^2}$ (2) $2xu$ (3) $x^2u$ (4) $y^2u$

3. The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$ is

   (1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 0 (4) $\frac{2}{3}$

4. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

   (1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

5. The approximate change in the volume $V$ of a cube of side $x$ metres caused by increasing the side by 1% is

   (1) $0.3x dx \, m^3$ (2) $0.03x \, m^3$ (3) $0.03x^2 \, m^3$ (4) $0.03x^3 \, m^3$

6. The minimum value of the function $|3 - x| + 9$ is

   (1) 0 (2) 3 (3) 6 (4) 9

7. If $g(x, y) = 3x^2 - 5y + 2y^2, x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

   (1) $6e^{2t} + 5 \sin t - 4 \cos t \sin t$ (2) $6e^{2t} - 5 \sin t + 4 \cos t \sin t$
   (3) $3e^{2t} + 5 \sin t + 4 \cos t \sin t$ (4) $3e^{2t} - 5 \sin t + 4 \cos t \sin t$

8. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

   (1) $\frac{\pi a^3}{16}$ (2) $\frac{3\pi a^4}{16}$ (3) $\frac{3\pi a^2}{8}$ (4) $\frac{3\pi a^4}{8}$

9. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

   (1) $xye^{xy}$ (2) $(1 + xy)e^{xy}$ (3) $(1 + y)e^{xy}$ (4) $(1 + x)e^{xy}$

10. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

    (1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

### PART - II

**Answer any 4 questions. Question No : 16 is Compulsory.** 4x2=8

11. Find the slope of the tangent to the curves at the respective given points. $y = x^4 + 2x^2 - x$ at $x = 1$.

12. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals.
    $f(x) = \tan x, x \in [0, \pi]$

13. Show that $F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

14. Evaluate $\int_0^{\frac{\pi}{2}} cos^7 x\, dx$

15. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \; dx.$

16. Find $df$ for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

## PART - III

**Answer any 4 questions. Question No : 22 is Compulsory.** 4x3=12

17. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$.

18. The time $T$, taken for a complete oscillation of a single pendulum with length $l$, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where $g$ is a constant. Find the approximate percentage error in the calculated value of $T$ corresponding to an error of 2 percent in the value of $t$.

19. Write the Maclaurin series expansion of the following functions: $log(1-x); -1 \le x < 1$

20. If $u(x,y) = log(x^3 + y^3 + z^3)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

21. If $\lim_{\theta \to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$.

22. Evaluate: $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx.$

## PART - IV

**Answer all the questions** 4x5=20

23. A) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

(OR)

B) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where $c, r$ are constants, cut orthogonally.

24. A) For each of the following functions find the $f_x, f_y$, and show that $f_{xy} = f_{yx}$. $f(x,y) = tan^{-1}\left(\frac{x}{y}\right)$

(OR)

B) If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tan\, u.$

25. A) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

(OR)

B) Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, $x$-axis, $x = -3$ and $x = 3$.

26. A) Let $z(x,y) = x^3 - 3x^2y^3$, where $x = se^t, y = se^{-t}, s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

(OR)

B) Evaluate $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{tan\, x}} dx$