SECOND MIDTERM EXAMINATION NOVEMBER 2024
XII STANDARD – MATHEMATICS

Time : 1.30 hrs **Max marks : 45**

I. Choose the correct answer. **10 x 1 = 10**

1. The area between $y^2 = 4x$ and its lattus rectum is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

2. The value of $\int_{-1}^{2}|x|dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

3. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(1) 2 (2) 2.5 (3) 3 (4) 3.5

4. The point on the curve $6y = x^3 + 2$ at which y- coordinate changes 8 times as fast as x-coordinate is

(1) $(4,11)$ (2) $(4,-11)$ (3) $(-4,11)$ (4) $(-4,-11)$

5. The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25 ?

(1) -8 (2) -4 (3) -2 (4) 0

6. The slope of the line normal to the curve $f(x) = 2cos4x$ at $x = \frac{\pi}{12}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

7. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

8. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(1) 0.4 cu.cm (2) 0.45 cu.cm (3) 2 cu.cm (4) 4.8 cu.cm

9. The change in the surface area $S = 6x^2$ of cube when the edge length varies from x_0 to $x_0 + dx$ is

(1) $12x_0 + dx$ (2) $12x_0dx$ (3) $6x_0dx$ (4) $6x_0 + dx$

10. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(1) $z - x$ (2) $y - z$ (3) $x - z$ (4) $y - x$

II. Answer any three of the following. (Question number 15 is compulsory) **3 x 2 = 6**

11. Evaluate : $\lim_{x\to 0}\frac{1-cosx}{x^2}$

12. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

13. Evaluate : $\int_0^3 (3x^2 - 4x + 5)dx$

14. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals $f(x) = tanx,\ x \in [0,\pi]$

15. If $u = \log(x^2 + y^2 + z^2)$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

III. Answer any three of the following. (Question number 20 is compulsory) $3 \times 3 = 9$

16. Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx$.

17. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

18. If $V(x,y) = e^x(x\cos y - y\sin y)$ then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

19. Write the Maclaurin series expansion for the function e^x

20. Find the equation of tangent and normal to the curve $y = x^2 + 3x - 2$at the point $(1,2)$

IV. Answer all the questions. $4 \times 5 = 20$

21.(a) A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero.

OR

(b) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

22. (a) Prove that the local minimum values for the function $f(x) = 4x^6 - 6x^4$ attain at -1 and 1

OR

(b). Evaluate : $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^2} dx$.

23.(a) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

OR

(b) If $v(x,y,z) = x^3 + y^3 + z^3 + 3xyz$, Show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

24. (a) If $u(x,y,z) = xy^2z^3, x = sint, y = cost, z = 1 + e^{2t}$, find $\frac{du}{dt}$

OR

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tanu$