T

SECOND MID TERM TEST - 2024

Standard - XII

MATHS

Reg.No. | | | 2 | 4 | 1 | 8 |

Time: 1.30 hrs. *Marks:45*

PART - A

Choose the correct answer: **10×1=10**

1. The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?
   a) -8 b) -4 c) -2 d) 0
2. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1, 9]$ is
   a) 2 b) 2.5 c) 3 d) 3.5
3. What is the value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is
   a) 0 b) 1 c) 2 d) ∞
4. The maximum value of the function $x^2e^{-2x}, x > 0$ is
   a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$
5. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
   a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31
6. If $f(x, y) = e^{xy}$ then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
   a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$
7. If $w(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$, then $\frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial z}$ is
   a) xy + yz + zx b) x(y+z) c) y(z+x) d) 0
8. A binary operation on a set S is a function from
   a) $S \to S$ b) $(S \times S) \to S$ c) $S \to (S \times S)$ d) $(S \times S) \to (S \times S)$
9. In the set R of reala numbers '*' is defined as follows. Which one of the following is not a binary operation on R?
   a) a*b = min (a.b) b) a*b = (max (a, b) c) a*b = a d) $a*b = a^b$
10. Which one is the inverse of the statement $(p \vee q) \to (p \wedge q)$?
    a) $(p \wedge q) \to (p \vee q)$ b) $\neg(p \vee q) \to (p \wedge q)$
    c) $(\neg p \vee \neg q) \to (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \to (\neg p \vee \neg q)$

PART - B

Answer any three questions: **3×2=6**

11. A particle moves so that the distance moved is according to the law $s(t) = s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero.
12. Compute the limit $\lim_{x\to a}\left(\frac{x^n - a^n}{x - a}\right)$
13. Use the linear approximation to find approximate values of $(123)^{\frac{2}{3}}$
14. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

PART - C

3×3=9

Answer any three questions:

15. Find the partial derivatives of the following functions at the indicated point
$g(x, y) = 3x^2 + y^2 + 5x + 2, (1, -2)$

16. Suppose f(x) is a differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of f(7)?

17. Show that $F(x, y) = \frac{x^2 + 5xy - 10y^2}{3x + 7y}$ is a homogeneous function of degree 1.

18. Let $*$ be defined on R by $(a*b) = a + b + ab - 7$. Is $*$ binary on R? If so, find

$3 * \left(\frac{-7}{15}\right)$

PART - D

4×5=20

Answer all the questions:

19. a) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

(OR)

b) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is need along the river. What is the length of the minimum needed fencing material.

20. a) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of I.

(OR)

b) If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

21. a) Expand tanx in ascending powers of x upto 5^{th} power for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(OR)

b) Verify :
i) Closure property ii) commutative property iii) associative property iv) existence of identity and v) existence of inverse for the operation × 11 on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

22. a) Verify :
i) Closure property ii) commutative property iii) associative property iv) existence of identity and v) existence of inverse for following the operation on the given set $m*n = m + n - mn$; $m, n \in z$

(OR)

b) Show that $\neg(p \wedge q) \equiv \neg p \vee \neg q$
