SECOND MID-TERM TEST - 2024	Reg. No.						

XII - MATHEMATICS

Time Allowed : 1.30 Hrs. *Maximum Marks: 45*

I. Choose the correct answer. **10 x 1 = 10**

1. The point of inflection of the curve $y = (x-1)^3$ is
   a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)
2. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when
   a) $y = 0$ b) $y = \pm\sqrt{3}$ c) $y = \frac{1}{2}$ d) $y = \pm 3$
3. If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?
   a) 0 and 12 b) 5 and 17 c) 7 and 19 d) 16 and 24
4. A pair of dice numbered 1, 2,3, 4,5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is
   a) 1 b) 2 c) 3 d) 4
5. If $a * b = \sqrt{a^2 + b^2}$ on the real numbers then * is
   a) commutative but not associative b) associative but not commutative
   c) both commutative and associative d) neither commutative nor associative
6. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are
   a) 1 b) 2 c) 3 d) 4
7. The Maclaurin's series is obtained from the Tailors series by putting
   a) $x = a$ b)) $x = 0$ c) $a = 0$ d) $a = n$
8. $X : S \to R$ is said to discrete random variable if
   a) Range of X is countable b) Range of X is uncountable
   c) Range of X is N d) Range of X is R
9. The identity element under addition exists in
   a) N b) $C\backslash\{0\}$ c) $(0,\infty)$ d) $-3 \le x \le 3$
10. The fourth roorts of unity under multiplication satisfies the properties
    a) closure only b) closure and associative only
    c) closure, associative and identity
    d) closure, associative , identity and inverse

II. Answer any 3 questions. (Q.No.15 is compulsory) **3 x 2 = 6**

11. Let * be defined on R by $(a * b) = a + b + ab - 7$. Is * binary on R? If so, find $-2 * \left(\frac{5}{4}\right)$
12. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.
13. Show that $F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.
14. Evaluate : $\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$
15. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

III. Answer any 3 questions. (Q.No.20 is compulsory) **3 x 3 = 9**

16. If $u(x, y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

17. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three Boolean matrices of the same type. Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A \vee B) \wedge C$

18. The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$. Find the value of k.

19. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions: $f(x) = x^2 - x,\ x \in [0,1]$

20. Find the absolute extrema of the following functions on the given closed interval. $f(x) = x^2 - 12x + 10$; $[1,2]$

IV. Answer all the questions. **4 x 5 = 20**

21. a) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.
    (i) How fast is the top of the ladder moving down the wall?
    (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing? ***(OR)***

    b) Find the acute angle between $y = x^2$ and $y = (x - 3)^2$

22. a) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm. ***(OR)***

    b) If $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

23. a) If X is the random variable with distribution function F(x) given by,

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}(x^2 + x), & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$

    then find (i) the probability density function f (x) (ii) $P(0.3 \le X \le 0.6)$. ***(OR)***

    b) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

    Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$
    (iii) $P(X \le 4)$ (iv) $P(3 < X)$.

24. a) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5. ***(OR)***

    b) Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.