

**Standard 12****MATHEMATICS****Part - I**

Time: 1.30 Hours

Marks: 50

I. Choose the correct answer: **$10 \times 1 = 10$**

- 1) The slope of the normal to the curve $f(x) = 2 \cos 4x$ at $x = \frac{\pi}{12}$ is
 a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$
- 2) $\lim_{x \rightarrow 0} \left(\frac{\sin mx}{x} \right)$
 a) $\frac{1}{m}$ b) m c) 0 d) 1
- 3) The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is
 a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$
- 4) The point of inflection of the curve $y = (x-1)^3$ is
 a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)
- 5) If $u(x, y) = e^{x^2+y^2}$ then $\frac{\partial u}{\partial x}$ is
 a) $e^{x^2+y^2}$ b) $2xu$ c) x^2u d) y^2u
- 6) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in the calculation of the volume is
 a) 0.4 cu.cm b) 0.45 cu.cm c) 2 cu.cm d) 4.8 cu.cm
- 7) If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x+y}}$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$
 a) $\frac{1}{2}u$ b) $2u$ c) u d) $\frac{3}{2}u$
- 8) Subtraction is not a binary operation
 a) R b) Z c) N d) Q
- 9) If a compound statement involves 3 simple statements, then the number of rows in the truth table is
 a) 9 b) 8 c) 6 d) 3
- 10) $(p \wedge q) \vee (\neg p \wedge \neg q) \equiv$
 a) $p \leftrightarrow q$ b) $p \rightarrow q$ c) $p \leftarrow q$ d) $p \vee q$

Part - II**II. Answer any four questions only:** **$4 \times 2 = 8$**

- 11) Show that the curves $x^2 - y^2 = r^2$ and $xy = c_1 r$ where c_1, r are constants, cut orthogonally.
- 12) Find the absolute extreme for the function $f(x) = x^2 - 12x + 10$ on the interval [1, 2].
- 13) Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculation.
- 14) If $w(x, y) = x^3 - 3xy + 2y^2$, $x, y \in \mathbb{R}$ find the linear approximation for w at (1, -1).
- 15) On \mathbb{Z} , define * by $(m * n) = m^n + n^m$, $\forall m, n \in \mathbb{Z}$. Is a binary on \mathbb{Z} ?
- 16) How many rows are needed for following statement formulae?
 i) $p \vee \neg t \wedge (p \vee \neg s)$
 ii) $((p \wedge q) \vee (\neg r \vee \neg s) \wedge (\neg t \vee v))$

Part - III**III. Answer any four questions only:****4x3=12**17) Expand $\log(1+x)$ as a Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$ 18) Evaluate : $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

19) A circular plate expands uniformly under the influence of heat. If its radius increases from 10.5 cm to 10.75 cm then find an approximate change in the area and approximate percentage change in the area

20) If $V(x, y, z) = x^3 + y^3 + z^3 + 3xyz$, Prove that $\frac{\partial^2 u}{\partial y \partial z} = \frac{\partial^2 u}{\partial z \partial y}$ 21) Let * be defined on \mathbb{R} by $a * b = a + b + ab - 7$. Is * binary on \mathbb{R} ? If so, find

$$3 * \left(\frac{-7}{15} \right)$$

22) Verify whether $(p \wedge q) \wedge \neg(p \vee q)$ is tautology or contradiction or contingency**Part - IV****IV. Answer any four questions only:****4x5=20**

23) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min? How fast is the depth of the water increases when the water is 8 metres deep?

24) A rectangular page is to contain 24 cm² of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should the page so that the area of the paper used is minimum.25) If $w(x, y, z) = xy + yz + zx$, $x = u - v$, $y = uv$, $z = u + v$, $u, v \in \mathbb{R}$, Find $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$ 26) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x+y}} \right)$, Prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

27) Verify (i) closure property

ii) Commutative property iii) associative property iv) existence of identity

v) existence of inverse for following operation on the given set $m * n = m + n - mn$, $m, n \in \mathbb{Z}$ 28) Prove that $p \rightarrow (\neg p \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.
