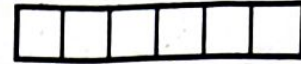


SMM

XII - Std

## SECOND MID TERM TEST - 2024

### MATHEMATICS



Time : 1.30 Hrs

Marks : 45

#### PART - I

Answer all the questions :-

10 X 1 = 10

1. The point on the curve  $6y = x^3 + 2$  at which  $y$  - coordinate changes 8 times as fast as  $x$  - coordinate is  
 a) (4, 11)                      b) (4, -11)                      c) (-4, 11)                      d) (-4, -11)
2. The value of the limit  $\lim_{x \rightarrow 0} \left( \cot x - \frac{1}{x} \right)$  is  
 a) 0                                  b) 1                                  c) 2                                  d)  $\infty$
3. The curve  $y = ax^4 + 6x^2$  with  $ab > 0$   
 a) has no horizontal tangent                      b) is concave up  
 c) is concave down                                  d) has no points of inflection
4. If  $f(x, y) = e^{xy}$ , then  $\frac{\partial^2 f}{\partial x \partial y}$  is equal to  
 a)  $xye^{xy}$                       b)  $(1 + xy)e^{xy}$                       c)  $(1 + y)e^{xy}$                       d)  $(1 + x)e^{xy}$
5. Linear approximation for  $g(x) = \cos x$  at  $x = \frac{\pi}{2}$  is  
 a)  $x + \frac{\pi}{2}$                       b)  $-x + \frac{\pi}{2}$                       c)  $x - \frac{\pi}{2}$                       d)  $-x - \frac{\pi}{2}$
6. Subtraction is not a binary operation in  
 a) R                                  b) Z                                  c) N                                  d) Q
7. If a compound statement involves 3 simple statements, then the number of rows in the truth table is  
 a) 9                                  b) 8                                  c) 6                                  d) 3
8. L - Hopital's Rule is not applicable for  
 a)  $\frac{0}{0}$                                   b)  $\infty - \infty$                                   c)  $\frac{\infty}{\infty}$                                   d)  $1^0$
9. If  $f$  and  $g$  are differentiable functions then  $d(fg)$  is  
 a)  $fdg + dfg$                       b)  $fdf - gdg$                       c)  $fdf + gdg$                       d)  $fdg - gdf$
10. Which of the following is not a binary operation of R?  
 a) +                                  b) -                                  c)  $\div$                                   d) X

#### PART - II

Answer any 3 questions. Q.No. 15 is Compulsory :-

3 x 2 = 6

11. If the volume of a cube of side length  $x$  is  $V = x^3$ . Find the rate of change of the volume with respect to  $x$  when  $x = 5$  units.
12. Prove that the function  $f(x) = x^2 - 2x - 3$  is strictly increasing in  $(2, \infty)$ .
13. Find  $df$  for  $f(x) = x^2 + 3x$  and evaluate it for  $x = 2$  and  $dx = 0.1$ .
14. Let  $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  by any two boolean matrices of the same type. Find  $A \cup B$  and  $A \cap B$ .

15. Prove that in an algebraic structure the identity element (if exists) must be unique

**PART - III**

Answer any 3 questions. Q.No. 20 is compulsory :-

3 X 3 = 9

16. Establish the equivalence property :  $p \rightarrow q \equiv \neg P \vee q$
17. Define an operation \* on Q as follows ;  $a * b = \left(\frac{a+b}{2}\right)$ ;  $a, b \in Q$ . Examine the closure commutative and associative properties satisfied by \* in Q.
18. Evaluate :  $\lim_{x \rightarrow 0^+} x \log x$
19. Show that  $f(x, y) = \frac{x^2 - y^2}{y^2 + 1}$  is continuous at every  $(x, y) \in R^2$ .
20. Use the linear approximation to find approximate values of  $(123)^{2/3}$

**PART - IV**

Answer all the questions :-

4 X 5 = 20

21. a) Establish the equivalence property connecting the bi - conditional with conditional :  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ . (OR)  
b) Prove that among all the rectangles of the given perimeter, the square has the maximum area.
22. a) If  $v(x, y) = e^x(x \cos y - y \sin y)$ , then prove that  $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ . (OR)  
b) Verify (i) closer property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation  $x_{11}$  on a subset  $A = \{1, 3, 4, 5, 9\}$  of the set of remainder  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .
23. a) Expand  $\log(1 + x)$  as a Maclaurin's series upto 4 non - zero terms for  $-1 < x \leq 1$ . (OR)  
b) Prove that  $f(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$  is homogeneous ; what is the degree? Verify Euler's theorem for f.
24. a) Find the angle between the curves  $y = x^2$  and  $x = y^2$  at their points of intersection  $(0, 0)$  and  $(1, 1)$  (OR)  
b) (i) Let  $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix}; x \in R - \{0\} \right\}$  and let \* be the matrix multiplication. Determine whether M is closed under \*. If so. Examine the commutative and associative properties satisfied by \* on M.  
(ii) If so, examine the existence of identity, existence of inverse properties for the operation \* on M.