



2 MARKS: ANSWERS ANY 10 Q

10 X 2 = 20

- Find the differential equation for the family of all straight lines passing through the origin.
- Form the differential equation by eliminating the arbitrary constants A and B from $y = A \cos x + B \sin x$.
- Find the differential equation of the family of circles passing through the points (a,0) and (-a,0).
- Show that $x^2 + y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.
- For the differential equations, determine its order, degree (if exists) $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$
- Find the differential equation of the family of (i) all non-vertical lines in a plane (ii) all non-horizontal lines in a plane.
- Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$
- Show that the expressions is a solution of the corresponding given differential equation. $y = 2x^2; xy' = 2y$
- The slope of the tangent to the curve at any point is the reciprocal of four times the ordinate at that point. The curve passes through (2,5). Find the equation of the curve.
- Solve the differential equations: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
- Solve the differential equations: $y dx + (1+x^2)\tan^{-1} x dy = 0$

3 MARKS: ANSWERS ANY 10 Q

10 X 3 = 30

- Show that $y = a \cos(\log x) + b \sin(\log x)$, $x > 0$ is a solution of the differential equation $x^2 y'' + x y' + y = 0$
- Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$
- Solve: $\frac{dy}{dx} = (3x + y + 4)^2$
- Solve: $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$
- Solve $\frac{dy}{dx} + 2y = e^{-x}$
- Solve: $ye^y dx = (y^3 + 2xe^y) dy$
- For the differential equations, determine its order, degree (if exists) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$
- Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.
- Show that the expressions is a solution of the corresponding given differential equation. $y = ae^x + be^{-x}$; $y'' - y = 0$
- Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^x \left(\frac{d^2y}{dx^2}\right) - 1 = 0$
- Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$.
- Solve the differential equations $\frac{dy}{dx} = e^{x+y} + x^3 e^y$
- Solve the Linear differential equations: $\cos x \frac{dy}{dx} + y \sin x = 1$
- Solve the Linear differential equations: $\frac{dy}{dx} + y = x \log x$



5 MARKS : ANSWERS ANY 10 Q 10 X 5 = 50

1. Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0.$$

2. Show that the differential equation representing the family of curves $y^2 = 2a\left(x + a^{\frac{2}{3}}\right)$ where a is a positive parameter, is

$$\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$$

3. Find the particular solution of $(1 + x^3)dy - x^2y dx = 0$ satisfying the condition $y(1) = 2$.

4. Solve: $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$.

5. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1,0)$.

6. Solve the following differential equations: (v) $(e^y+1)\cos x dx + e^y \sin x dy = 0$

7. Solve $(x^2 - 3y^2) dx + 2xy dy = 0$.

8. Solve $(y + \sqrt{x^2 + y^2}) dx - x dy = 0, y(1) = 0$.

9. Solve $(2x + 3y) dx + (y - x) dy = 0$.

10. Solve $(1 + 2e^{x/y}) dx + 2e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$

11. Solve the differential equations:

$$\left(1 + 3e^{\frac{y}{x}}\right) dy + 3e^{\frac{y}{x}} \left(1 - \frac{y}{x}\right) dy = 0, \text{ given that } y = 0 \text{ when } x = 1$$

12. Solve the differential equations:

$$(x^2 + y^2) dy = xy dx. \text{ It is given that } y(1) = 1 \text{ and } y(x_0) = e. \text{ Find the value of } x_0.$$

13. Solve $[y(1 - x \tan x) + x^2 \cos x] dx - x dy = 0$.

14. Solve $(1 + x^3) \frac{dy}{dx} + 6x^2y = 1 + x^2$.

15. Solve the Linear differential equation: $\frac{dy}{dx} + \frac{y}{x} = \sin x$

16. Solve the Linear differential equation $(y - e^{\sin^{-1}x}) \frac{dx}{dy} + \sqrt{1 - x^2} = 0$

17. Solve the Linear differential equation $(1 + x + xy^2) \frac{dy}{dx} + (y + y^3) = 0$

18. The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

19. In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F . Two hours later, the detective measured the body temperature again and found it to be 60°F . If the room temperature is 50°F , and assuming that the body temperature of the person before death was 98.6°F , at what time did the murder occur? [$\log(2.43) = 0.88789$; $\log(0.5) = -0.69315$]

- 20.** The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?
- 21.** Find the population of a city at any time t , given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.
- 22.** The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L \frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance and L , the coefficient of induction. Find the current i at time t when $E = 0$
- 23.** Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?
- 24.** Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C . Find (i) The temperature of water after 20 minutes (ii) The time when the temperature is 40°C
 $\left[\log_e \frac{11}{15} = -0.3101 ; \log_e 5 = 1.6094 \right]$
- 25.** At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was 180°F , and 10 minutes later it was 160°F . Assume that constant temperature of the kitchen was 70°F .
- (i) What was the temperature of the coffee at 10.15 A.M.?
(ii) The woman likes to drink coffee when its temperature is between 130°F and 140°F . between what times should she have drunk the coffee?
- 26.** A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C , and another 5 minutes later it has dropped to 65°C . Determine the temperature of the kitchen.