DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 2 TEST -1 MARKS : 50

==

2 MARKS : ANSWERS ANY 15 Q **15 X 2 = 30**

1.Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form

2.If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$ find $\frac{z_1}{z_2}$in the rectangular form

3.Find z^{-1}, if z = (2+3i)(1−i)

4.Show that (i) $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$is real

5.Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$

6.Show that $|z + 2 - i| < 2$ represents interior points of a circle. Find its centre and radius

.**7.**Represent the complex number (i) −1− i (ii) $1 + i\sqrt{3}$ in polar form.

8.Simplify$\sum_{n=1}^{12} i^n$

9.Simplify$i^{59} + \frac{1}{i^{59}}$

10.Simplify$i i^2 i^3 \ldots i^{2000}$

11. Simplify$\sum_{n=1}^{10} i^{n+50}$

12. Evaluate$(z + w)^2$if z =5−2i and w =−1+3i

13. Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$(i) real (ii) purely imaginary.

14. Show that (i) $(2 + i\sqrt{3})^n - (2 - i\sqrt{3})^n$is purely imaginary

15. Which one of the points10 − 8 i , 11 +6i is closest to1+ i .

16. Find the square roots of 4 +3i

17. Show that the following equations represent a circle, and, find its centre and radius. $|z - 2 - i|$=3

18. Write in polar form of the following complex numbers $2 + i2\sqrt{3}$

19. Find the rectangular form of the complex numbers $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

20. Find the value of $\left(\frac{1+\sin\frac{\pi}{10} + i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10} - i\cos\frac{\pi}{10}}\right)^{10}$

3 MARKS : ANSWERS ANY 5 Q **5 X 3 = 15**

21. Write $\frac{3+4i}{5-12i}$in the x +iy form, hence find its real and imaginary parts.

22. If$\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form$\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1-2i}\right)^{15}$ is purely imaginary.

23. Which one of the points i, −2+i , and 3 is farthest from the origin?

24. If $|z|=2$ show that $3 \le |z + 3 + 4i| \le 7$

25. Show that the points 1 ,$\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

26. Show that the equation$z^2 = \bar{z}$ has four solutions

27. Find the square root of 6 − 8i .

5 MARKS : ANSWERS ANY 1 Q **1 X 5 = 5**

28. If z = x + iy is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$

29. If$(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3)\ldots(x_n + iy_n)$= a+ib , show that

(i) $(x_1{}^2 + y_1{}^2)(x_2{}^2 + y_2{}^2)(x_3{}^2 + y_3{}^2)\ldots(x_n{}^2 + y_n{}^2) = a^2 + b^2$

(ii) $\sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\frac{b}{a} + 2k\pi$, k ∈Z.

DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 2 TEST -2 MARKS : 50

==

3 MARKS : ANSWERS ANY 10 Q **10 X 3 = 30**

1.Obtain the Cartesian form of the locus of z in each of the following cases. (i) $|z| = |z - i|$

2.If z = (cosθ+isin θ), show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i \sin n\theta$

3.Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

4.Find the cube roots of unity.

5.Find the fourth roots of unity.

6. Find the values of the real numbers x and y, if the complex numbers $(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal.

7. If $z_1=3, z_2 = 7i$ and $z_3 = 5+4i$, show that $(z_1+z_2)\, z_3 = z_1z_3 + z_2z_3$

8. The complex numbers u ,vand w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$If v = 3−4i and w =4+3i , find u in rectangular form.

9. Prove the following properties: Re(z)=$\frac{z+\bar{z}}{2}$ and Im(z) = $\frac{z-\bar{z}}{2i}$

10. Find the modulus of the following complex numbers $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$

11. If | z | = 3, show that $7 \le |z+6-8i| \le 13$.

12. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| =1$, $|z_2| =2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3|=1$ show that $|9z_1z_2 + 4z_1z_3 + z_2z_3|=6$

13. Obtain the Cartesian equation for the locus of z = x+ iy in the cases: $|z-4|^2 - |z-1|^2 =16$

14. If $\omega \ne 1$ is a cube root of unity, show that

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1.$$

15. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.

16. If $\omega \ne 1$ is a cube root of unity, show that $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots(1+\omega^{2^{11}}) = 1$

5 MARKS : ANSWERS ANY 4 Q **4 X 5 = 20**

17. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that z =i tan$\theta$.

18. If cosα +cosβ+cosγ = sinα +sinβ+ sinγ= 0 show that
(i) cos3α +cos3β+cos3γ = 3 cos $(\alpha+\beta+\gamma)$and
(ii) sin3α +sin3β+ sin3γ= 3 sin $(\alpha+\beta+\gamma)$

19. If z= x + iy and arg $\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$

. If 2cosα = x + $\frac{1}{x}$and 2cos β = y + $\frac{1}{y}$show that
(i)$\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha-\beta)$(ii) xy −$\frac{1}{xy} = 2i\sin(\alpha+\beta)$
(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$(iv) $x^my^n + \frac{1}{x^my^n} = 2\cos(m\alpha+n\beta)$

20. Obtain the Cartesian form of the locus of z =x+ iy in the cases: $[Re(iz)]^2 = 3$

21. Let z_1, z_2 and $z_3$be complex numbers such that $|z_1| =|z_2| = |z_3| =r > 0$ and $z_1 + z_2 + z_3 \ne 0$ then prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$

22. Show that (i) $(2+i\sqrt{3})^n - (2-i\sqrt{3})^n$is purely imaginary (ii) $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real

23. If z = x + iy is a complex number such that $\left|\frac{z-4i}{z+4i}\right|$ show that the locus of z is real axis.

24. If z = x + iy is a complex number such that Im$\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$

25. If $\omega \ne 1$is a cube root of unity, show that the roots of equation $(z-1)^3 + 8 = 0$ are −1 ,1−2ω , $1-2\omega^2$.

26..Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.

27 . If $\omega \ne 1$ is a cube root of unity, show that
(i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$
(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots(1+\omega^{2^{11}}) = 1$