DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 6 TEST -1 MARKS : 50

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2 MARKS : ANSWER ALL QUESTIONS **10X 2 = 20**

1. Show that the vectors $\hat{\imath}+2\hat{\jmath}-3\hat{k}$, $2\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $3\hat{\imath}+\hat{\jmath}-\hat{k}$ are coplanar.

2. If $2\hat{\imath}-\hat{\jmath}+3\hat{k}$, $3\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\hat{\imath}+m\hat{\jmath}+4\hat{k}$ are coplanar, find the value of m.

3. If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that
$[\vec{a}+\vec{c},\vec{a}+\vec{b},\vec{a}+\vec{b}+\vec{c}]=[\vec{a},\vec{b},\vec{c}]$.

4. A straight line passes through the point (1,2,−3) and parallel to $4\hat{\imath}+5\hat{\jmath}-7\hat{k}$. Find (i) vector equation in parametric form (ii) vector equation in non-parametric form (iii) Cartesian equations of the straight line.

5. Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{\imath}-4\hat{\jmath}+12\hat{k})=5$

6. Find the acute angle between the planes $\vec{r}\cdot(2\hat{\imath}+2\hat{\jmath}+2\hat{k})=11$ and $4x-2y+2z=15$.

7. Prove by vector method that if a line is drawn from the centre of circle to the midpoint of a chord, then the line is perpendicular to the chord.

8. If $\vec{a}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}-2\hat{k}$, $\vec{c}=3\hat{\imath}+2\hat{\jmath}+\hat{k}$,
find $\vec{a}\cdot(\vec{b}\times\vec{c})$.

9. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors
$-6\hat{\imath}+14\hat{\jmath}+10\hat{k}$, $14\hat{\imath}-10\hat{\jmath}-6\hat{k}$ and $2\hat{\imath}+4\hat{\jmath}-2\hat{k}$.

10. Determine whether the three vectors $2\hat{\imath}+3\hat{\jmath}+\hat{k}$, $\hat{\imath}-2\hat{\jmath}+2\hat{k}$ and $3\hat{\imath}+\hat{\jmath}+3\hat{k}$ are coplanar.

3 MARKS : ANSWER ANY 10 QUESTIONS **10 X 3 = 30**

11. With usual notations, in any triangle ABC, prove the by vector method. $a^2=b^2+c^2-2bc\cos AC$

12. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

13. Find the magnitude and the direction cosines of the torque about the point (2,0,−1) of a force $2\hat{\imath}+\hat{\jmath}-\hat{k}$ whose line of action passes through the origin.

14. Show that the four points (6,−7,0), (16,−19,−4), (0,3,−6), (2,−5,10) lie on a same plane.

15. Prove that $[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$

16. Find the angle made by the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

17. Find the acute angle between the straight lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$ and state whether they are parallel or perpendicular.

18. Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines $\vec{r}=(\hat{\imath}+3\hat{\jmath}-\hat{k})+t(2\hat{\imath}+3\hat{\jmath}+2\hat{k})$ and $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z+3}{4}$ and perpendicular to both straight lines.

19. Determine whether the pair of straight lines $\vec{r}=(2\hat{\imath}+6\hat{\jmath}+3\hat{k})+t(2\hat{\imath}+3\hat{\jmath}+4\hat{k})$
$\vec{r}=(2\hat{\jmath}-3\hat{k})+s(\hat{\imath}+2\hat{\jmath}+3\hat{k})$ are parallel. Find the shortest distance between them.

20. Find the shortest distance between the two given straight lines $\vec{r}=(2\hat{\imath}+3\hat{\jmath}+4\hat{k})+t(-2\hat{\imath}+\hat{\jmath}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$.

21. Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$.

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CHAPTER – 6 TEST - 2 MARKS : 50

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2 MARKS : ANSWERS ANY 8 Q 8 X 2 = 16

2 MARKS : ANSWER ALL QUESTIONS 10 X 2 = 20

1. Prove that $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}] = 0$.

2. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors, show that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$.

3. If $\vec{a} = \hat{\imath}+2\hat{\jmath}+3\hat{k}$, $\vec{b} = 2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\vec{c} = 3\hat{\imath}+2\hat{\jmath}+\hat{k}$, and $\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a}+m\vec{b}+n\vec{c}$.
find the values of l ,m ,n

4. The vertices of ΔABC are A(7, 2,1), B(6,0,3) , and C(4, 2, 4) C. Find ∠ABC.

5. Show that the points (2,3, 4),(−1, 4,5) and (8,1,2) are collinear .

6. If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of m .

7. Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines.

8. Find the angle between the line $\vec{r} = (2\hat{\imath}-\hat{\jmath}+\hat{k})+t(\hat{\imath}+2\hat{\jmath}-2\hat{k})$ and the plane $\vec{r}\cdot(6\hat{\imath}+3\hat{\jmath}+2\hat{k}) = 8$.

9. Find the angle between the planes
$\vec{r}\cdot(\hat{\imath}+\hat{\jmath}-2\hat{k}) = 3$ and 2x− 2y+z = 2

10. Find the length of the perpendicular from the point (1,−2,3) to the plane x−y+z = 5 .

3 MARKS : ANSWER ANY 10 QUESTIONS 10 X 3 = 30

11. Find the angle between the straight lines $\vec{r} = (2\hat{\imath}+3\hat{\jmath}+\hat{k})+t(\hat{\imath}-\hat{\jmath}+\hat{k})$ and the plane 2x −y +z = 5.

12. Find the distance between the parallel planes x + 2y −2z +1 = 0 and 2x +4y −4z + 5 = 0 .

13. Find the coordinates of the point where the straight line $\vec{r} = (2\hat{\imath}-\hat{\jmath}+2\hat{k}) + t(3\hat{\imath}+4\hat{\jmath}+2\hat{k})$ intersects the plane x − y + z − 5 = 0 .

14. Prove by vector method that an angle in a semi-circle is a right angle.

15. Prove by vector method that the area of the quadrilateral ABCD having diagonals AC and BD is $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$

16. Find the torque of the resultant of the three forces represented by $-3\hat{\imath}+6\hat{\jmath}-3\hat{k}$, $4\hat{\imath}-10\hat{\jmath}+12\hat{k}$ and $4\hat{\imath}+7\hat{\jmath}$ acting at the point with position vector $8\hat{\imath}-6\hat{\jmath}-4\hat{k}$ about the point with position vector $18\hat{\imath}+3\hat{\jmath}-9\hat{k}$.

17. For any vector $\vec{a}$ prove that
$\hat{\imath}\times(\vec{a}\times\hat{\imath})+\hat{\jmath}\times(\vec{a}\times\hat{\jmath})+\hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$.

18. Find the acute angle between the lines.
$\vec{r} = (4\hat{\imath}-\hat{\jmath})+t(\hat{\imath}+2\hat{\jmath}-2\hat{k})$, $\vec{r} = (\hat{\imath}-2\hat{\jmath}+4\hat{k})+s(-\hat{\imath}-2\hat{\jmath}+2\hat{k})$

19. Find the acute angle between the lines 2x = 3y = − z and 6x = −y = − 4z

20. If the straight lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-3}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, find the value of m .

21. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

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CHAPTER – 6 TEST -3 MARKS : 50

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5 MARKS : ANSWERS ANY 20 Q **20 X 5 =100**

1.By vector method, prove that cos(α+ β) = cosαcosβ − sinα sinβ

2.With usual notations, in any triangle ABC, prove by vector method that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

3.Prove by vector method that sin(α− β) = sinαcosβ – cosαsinβ .

4.(Apollonius's theorem)
If D is the midpoint of the side BC of a triangle ABC, then show by vector method that

$$|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2\left(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2\right)$$

5.Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

6. Using vector method, prove that cos(α− β) = cosαcosβ + sinα sinβ.

7. Prove by vector method that sin(α + β) = sinαcosβ + cosα sinβ.

8.If $\vec{a} = -2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$,$\vec{b} = 3\hat{\imath} - \hat{\jmath} + 3\hat{k}$,$\vec{c} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$. State whether they are equal.

9.If $\vec{a} = \hat{\imath} - \hat{\jmath}$,$\vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}$,$\vec{c} = 3\hat{\jmath} - \hat{k}$, and $\vec{c} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$, verify that

(i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

(ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

10. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$,$\vec{b} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$,$\vec{c} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, verify that

(i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$

(ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

11. Find the vector equation in parametric form and Cartesian equations of a straight passing through the points (-5,7,-4) and (13,-5, 2) . Find the point where the straight line crosses the xy-plane.

12. Find the points where the straight line passes through (6,7, 4) and (8, 4,9) cuts the xz and yz planes.

13. Find the point of intersection of the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and

$\frac{x-4}{5} = \frac{y-1}{2} = z$

14. Find the parametric form of vector equation of a straight line passing through the point of intersection of the straight lines

$\vec{r} = (\hat{\imath} + 3\hat{\jmath} - \hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, and perpendicular to both straight lines.

15. Determine whether the pair of straight lines

$\vec{r} = (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$,

$\vec{r} = (2\hat{\jmath} - 3\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ are parallel. Find the shortest distance between them.

16. Find the shortest distance between the two given straight lines

$\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + t(-2\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$.

17. Find the coordinate of the foot of the perpendicular drawn from the point (−1, 2, 3) to the straight line $\vec{r} = (\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$. Also, find the shortest distance from the given point to the straight line.

18. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (−1,2,0), (2,2 ,−1)and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

19. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

20. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane 2x+ 6y+6z = 9.

21. Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2,1), (1,−2,3) and parallel to the straight line passing through the points (2,1,−3) and (−1,5,−8).

22. Find the non-parametric form of vector equation of the plane passing through the point (1,−2, 4) and perpendicular to the plane x+2y−3z =11 and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.