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CLASS – XII – MATHEMATICS
(Chapter 7)

Time : 3 Hrs Marks : 90

PART – I

I. Answer ALL questions. 20x1 = 20

1) The function $\sin^4 x + \cos^4 x$ is increasing in the interval

(1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

2) The volume of a sphere is increasing in volume at the rate of $3\pi\,cm^3/sec$. The rate of change of its radius when radius is ½ cm

(1) 3 cm/s (2) 2 cm/s (3) 1 cm/s (4) $\frac{1}{2}$ cm/s

3) The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(1) -8 (2) -4 (3) -2 (4) 0

4) The number given by the Mean value theorem for the function $\frac{1}{x}$, $x \in [1,9]$ is

(1) 2 (2) 2.5 (3) 3 (4) 3.5

5) The curve $y = ax^4 + bx^2$ with $ab > 0$

(1) has no horizontal tangent (2) is concave up
(3) is concave down (4) has no points of inflection

6) The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

7) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(1) 2 (2) 2.5 (3) 3 (4) 3.5

8) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical if

(1) $y = 0$ (2) $y = \pm\sqrt{3}$ (3) $y = \frac{1}{2}$ (4) $y = \pm 3$

9) The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at

(1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$

10) The maximum value of the product of two positive numbers, when their sum of the squares is 200, is

(1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

11) A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(1) $\frac{3}{25}$ rad/sec (2) $\frac{4}{25}$ rad/sec
(3) $\frac{1}{5}$ rad/sec (4) $\frac{1}{3}$ rad/sec

12) The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

13) The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$ is

(1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

14) The maximum value of the function $x^2 e^{-2x}, x > 0$ is

(1) $\frac{1}{e}$ (2) $\frac{1}{2e}$ (3) $\frac{1}{e^2}$ (4) $\frac{4}{e^4}$

15) The point of inflection of the curve $y = (x-1)^3$ is

(1) $(0,0)$ (2) $(0,1)$ (3) $(1,0)$ (4) $(1,1)$

16) The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is

(1) $(4,11)$ (2) $(4,-11)$ (3) $(-4,11)$ (4) $(-4,-11)$

17) Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

18) One of the closest points on the curve $x^2 - y^2 = 4$ to the point $(6,0)$ is

(1) $(2,0)$ (2) $\left(\sqrt{5},1\right)$ (3) $\left(3,\sqrt{5}\right)$ (4) $\left(\sqrt{13},-\sqrt{3}\right)$

19) The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

(1) $t = 0$ (2) $t = \frac{1}{3}$ (3) $t = 1$ (4) $t = 3$

20) The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

PART – II

II. Answer any seven questions. 7x2 = 14

(Question No.30 is Compulsory)

21) For the function $f(x)=x^2$, compute the average rate of changes in the interval $[1,1.5]$ and the instantaneous rate of changes at the point $x=1.5$

22) If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x)=\sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is 27 metres.

23) Find the angle of intersection of the curve $y=\sin x$ with the positive x-axis.

24) Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following function:

$$f(x)=\sqrt{x}-\frac{x}{3},\ x\in[0,9]$$

25) Evaluate the limit $\lim\limits_{x\to 0}\left(\dfrac{\sin mx}{x}\right)$.

26) Prove that the function $f(x)=x^2-2x-3$ is strictly increasing in $(2,\infty)$.

27) Find the local extremum of the function $f(x)=x^4+32x$.

28) Find the asymptotes of the following curve :

$$f(x)=\frac{x^2}{x^2-1}$$

29) Evaluate : $\lim\limits_{x\to\infty}\left(\dfrac{x^2+17x+29}{x^4}\right)$.

30) A particle moves so that the distance moved is $s(t)=\dfrac{t^3}{3}-t^2+3$. At what time the velocity and acceleration are zero.

PART – III

III. Answer any seven questions. 7x3 = 21

(Question No.40 is Compulsory)

31) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. water flows into the tank at a rate 10 cu. m/min, how fast is the depth of the water increases

32) Find the tangent and normal to the following curve at the given points on the curve.

$x=\cos t, y=2\sin^2 t$ at $t=\dfrac{\pi}{3}$

33) Show that the value in the conclusion of the mean value theorem for $f(x)=Ax^2+Bx+C$ on any interval $[a,b]$ is $\dfrac{a+b}{2}$.

34) Write the Maclaurin series expansion of the function:

$$\tan^{-1}(x)\ ;\ -1\le x\le 1$$

35) Using the l'Hôpital Rule, prove that

$$\lim_{x\to 0^+}(1+x)^{\frac{1}{x}}=e.$$

36) Find the absolute extrema of the function on the given closed interval.

$$f(x)=6x^{\frac{4}{3}}-3x^{\frac{1}{3}}\quad ;\quad [-1,1]$$

37) Find the local extrema for the function using second derivative test :

$$f(x)=x\log x$$

38) Prove that among all the rectangles of the given area square has the least perimeter.

39) Evaluate : $\lim\limits_{x\to 1}x^{\frac{1}{1-x}}$.

40) Using mean value theorem prove that for, $a>0, b>0, |e^{-a}-e^{-b}|<|a-b|$.

PART – IV

IV. Answer any SEVEN questions. 7x5 = 35

41) A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

42) Find intervals of concavity and points of inflexion for the following function:

$$f(x)=\sin x+\cos x,\ 0<x<2\pi$$

43) Find the intervals of monotonicities and hence find the local extremum for the function: $f(x)=\dfrac{e^x}{1-e^x}$

44) Evaluate : $\lim\limits_{x\to\infty}(1+2x)^{\frac{1}{2\log x}}$.

45) Expand $\tan x$ in ascending powers of x upto 5th power for $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$.

46) A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.

47) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

48) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

49) Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$.

50) Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from $(1,1)$.