DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 7 TEST -1 MARKS : 50

==========

2 MARKS : ANSWER ANY 10 Q 10X 2 = 20

1.The temperature T in Celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by T=x(10 – x). Prove that the rate of change of temperature at the midpoint of the rod is zero.

2.A particle moves so that the distance moved is according to the law s(t) $=\frac{t^3}{3}-t^2+3$. At what time the velocity and acceleration are zero.

3.Find the points on the curve y = x^3-3x^2+x-2at which the tangent is parallel to the line y = x .

4.Find the angle of intersection of the curve y = sin x with the positive x - axis.

5.Find the value in the interval $\left(\frac{1}{2}, 2\right)$satisfied by the Rolle's theorem for the function f(x) = x+$\frac{1}{2}$, xϵ [$\frac{1}{2}$, 2].

6.Suppose f (x) is a differentiable function for all x with $f'(x) \leq 29$ and f(2) = 17 .
What is the maximum value of f (7) ?

7.A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from −10°C to 100°C. Show that the rate of change of temperature at some time t is 5°C per second.

8.Expand $tan\,x$in ascending powers of x upto 5^{th} power for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

.**9.**Compute the limit$\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)$.

10. Evaluate : $\lim_{x\to 1^-}\left(\frac{log(1-x)}{cot(\pi x)}\right)$.

11. Evaluate : $\lim_{x\to 0^+} x log x$.

12. Evaluate : $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right)$, mϵ N

13. Prove that the function f(x) =x^3−2x −3 is strictly increasing in (2,∞).

3 MARKS : ANSWER ANY 10 Q 10 X 3 = 30

1. A particle is fired straight up from the ground to reach a height of s feet in t seconds where s(t) = 128t $-16t^2$
(i) Compute the maximum height of the particle reached.
(ii) What is the velocity when the particle hits the ground?

2. Find the equations of tangent and normal to the curve y = x^2+3x-2at the point (1,2).

3. If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, show that $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$.

4. Find the values in the interval (1,2) of the mean value theorem satisfied by the function f(x) = x −x^2 for 1 ≤ x ≤ 2

5. Write the Taylor series expansion of $\frac{1}{x}$ about x = 2 by finding the first three non-zero terms.

6. Evaluate :$\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

7. Evaluate the limit $\lim_{x\to 0}\left(\frac{sin\,x}{x^2}\right)$.

8. Evaluate : $\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$.

9. Using the l'Hôpital Rule, prove that $\lim_{x\to 0^+}(1+x)^{\frac{1}{x}}$ = e .

10. Evaluate :$\lim_{x\to 1} x^{\frac{1}{1-x}}$

11. Find the absolute maximum and absolute minimum values of the function f(x) =$2x^3+3x^2-12x$ on [−3,−2]

12. Prove that the function f(x)=x – sinx is increasing on the real line. Also discuss for the existence of local extrema.

13. Find the local extremum of the function f(x) = x^4+32x.

DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 7 TEST -2 MARKS : 50

==========

2 MARKS : ANSWER ANY 10 Q 10 X 2 = 20

1. Find the intervals of monotonicity and hence find the local extrema for the function f(x) = $x^{\frac{2}{3}}$.

2. Find the asymptotes of the curve f(x) = $\frac{2x^2-8}{x^2-16}$.

3. Sketch the graph of the function y = $\frac{3x}{x^2-1}$.

4. Explain why Rolle's theorem is not applicable to the functions in the respective intervals. f(x) = $\left|\frac{1}{x}\right|$, x ϵ [−1,1]

5. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x -axis for f(x) =x^2− x , x ϵ [0,1]

6. Write the Maclaurin series expansion of e^x

7. Write the Maclaurin series expansion of sin x

8. Write the Maclaurin series expansion of cos x

9. Write the Maclaurin series expansion of log(1−x) ; [-1,1]

10. Expand sin x in ascending powers x − $\frac{\pi}{4}$ up to three non-zero terms.

11. Evaluate the limits, if necessary use l'Hôpital Rule : $\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3}$

12. Find the slope of the tangent to the curves at the respective given points. y = $x^4 + 2x^2 - x$at x =1

3 MARKS : ANSWER ANY 10 QUESTIONS 10 X 3 = 30

13. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of s= $16t^2$in t seconds. (i) How long does the camera fall before it hits the ground?

(ii) What is the average velocity with which the camera falls during the last 2 seconds? (iii) What is the instantaneous velocity of the camera when it hits the ground?

14. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?

15. Find the point on the curve y = $x^2 - 5x + 4$at which the tangent is parallel to the line 3x+y = 7

16. Find the points on the curve y = $x^3 - 6x^2 + x + 3$where the normal is parallel to the line x + y = 1729

17. Suppose that for a function f(x) ,
$f'(x) \leq$ 1for all 1≤ x ≤ 4. Show that f(4) − f(1) ≤ 3

18. Write down the Taylor series expansion, of the function log x about x =1 upto three non-zero terms for x > 0.

19. Evaluate the limits,
if necessary use l'HôpitalRule : $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

20. Evaluate the limits,
if necessary use l'HôpitalRule : $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

21. Evaluate the limits,
if necessary use l'HôpitalRule : $\lim_{x\to 0} \left(\frac{1}{sinx} - \frac{1}{x}\right)$

22. Find the absolute extrema of the following functions on the given closed interval. f(x) = $x^2 - 12x + 10$[1,2]

23. Find the intervals of monotonicities and hence find the local extremum for the functions: f(x) = $2x^3 + 3x^2 - 12$

24. Find two positive numbers whose sum is 12 and their product is maximum.

25. Find two positive numbers whose product is 20 and their sum is minimum.

26. Find the smallest possible value of $x^2 + y^2$given that x + y = 10 .

DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 7 TEST -3 MARKS : 50

==========

5 MARKS : ANSWERS ANY 10 Q 10 X 5 = 50

1. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$.(1) Compute the maximum height of the particle reached.(2) What is the velocity when the particle hits the ground?

2. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds? (1) At what time the particle is at rest? (2) At what time the particle changes its direction? (3) Find the total distance travelled by the particle in the first 2 seconds.

3 .If we blow air into a balloon of spherical shape at a rate of $1000cm^3$ per second, at what rate the radius of the balloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

4. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

5. A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and travelling at 80 km/hr, while car B is 15 kilometres to the east of P and travelling at 100 km/hr. How fast is the distance between the two cars changing?

6. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.

(i) How long does the camera fall before it hits the ground? (ii) What is the average velocity with which the camera falls during the last 2 seconds?(iii) What is the instan- taneous velocity of the camera when it hits the ground?

7. A particle moves along a line according to the $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

(i) At what times the particle changes direction?

(ii) Find the total distance travelled by the particle in the first 4 seconds. (iii) Find the particle's acceleration each time the velocity is zero.

8. A conical water tank with vertex down of 12 metres height has a radius of 5metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

9. A ladder 17 meters long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.

(i) How fast is the top of the ladder moving down the wall? (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

10. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

11. Find the equation of the tangent and normal at any point to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t$, $t \in R$.

12. Find the acute angle between $y = x^2$ and $y = (x - 3)^2$.