ST.ANNE'S ACADEMY
PUNNAI NAGAR, NAGERCOIL – 4.
CLASS – XII – MATHEMATICS
(Chapters 7, 8 & 9)

Time : 3 Hrs Marks : 90

PART – I

I. Answer ALL questions. 20x1 = 20

1) If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

2) If $v(x,y)=\log(e^x+e^y)$, then $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}$ is equal to

(1) e^x+e^y (2) $\frac{1}{e^x+e^y}$ (3) 2 (4) 1

3) Linear approximation for $g(x)=\cos x$ at $x=\frac{\pi}{2}$ is

(1) $x+\frac{\pi}{2}$ (2) $-x+\frac{\pi}{2}$ (3) $x-\frac{\pi}{2}$ (4) $-x-\frac{\pi}{2}$

4) If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

5) If $g(x,y)=3x^2-5y+2y^2, x(t)=e^t$ and $y(t)=\cos t$, then $\frac{dg}{dt}$ is equal to

(1) $6e^{2t}+5\sin t-4\cos t\sin t$
(2) $6e^{2t}-5\sin t+4\cos t\sin t$
(3) $3e^{2t}+5\sin t+4\cos t\sin t$
(4) $3e^{2t}-5\sin t+4\cos t\sin t$

6) If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is equal to

(1) xye^{xy} (2) $(1+xy)e^{xy}$
(3) $(1+y)e^{xy}$ (4) $(1+x)e^{xy}$

7) The value of $\int_{-1}^{2}|x|\,dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

8) If $f(x)=\int_0^x t\cos t\,dt$, then $\frac{df}{dx}=$

(1) $\cos x-x\sin x$ (2) $\sin x+x\cos x$
(3) $x\cos x$ (4) $x\sin x$

9) The value of the $\lim_{x\to 0}\left(\cot x-\frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

10) The number given by the Rolle's theorem for the function $x^3-3x^2, x\in[0,3]$ is

(1) 1 (2) $\sqrt{2}$ (3) $^3/_2$ (4) 2

11) The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

12) The curve $y=ax^4+bx^2$ with $ab>0$

(1) has no horizontal tangent (2) is concave up
(3) is concave down (4) has no points of inflection

13) The point on the curve $6y=x^3+2$ at which y-coordinate changes 8 times as fast as x-coordinate is

(1) (4,11) (2) (4,–11) (3) (–4,11) (4) (–4,–11)

14) The tangent to the curve $y^2-xy+9=0$ is vertical when

(1) $y=0$ (2) $y=\pm\sqrt{3}$ (3) $y=\frac{1}{2}$ (4) $y=\pm 3$

15) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3x\,dx\,m^3$ (2) $0.03x\,m^3$ (3) $0.03x^2m^3$ (4) $0.03x^3m^3$

16) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\cos x\,dx$ is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 0 (4) $\frac{2}{3}$

17) If $\int_0^a\frac{1}{4+x^2}dx=\frac{\pi}{8}$ then a is

(1) 4 (2) 1 (3) 3 (4) 2

18) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31

19) The change in the surface area $S=6x^2$ of a cube when the edge length varies from x_0 to x_0+dx is

(1) $12x_0+dx$ (2) $12x_0dx$ (3) $6x_0dx$ (4) $6x_0+dx$

20) Angle between $y^2=x$ and $x^2=y$ at the origin is

(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

PART – II

II. Answer any seven questions. 7x2 = 14

Question No.30 is Compulsory

21) If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

22) Let $V(x,y,z) = xy + yz + zx, x, y, z \in \mathbb{R}$. Find the differential dV.

23) Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$

24) Evaluate $\int_{0}^{\frac{\pi}{2}} \left(\sin^2 x + \cos^4 x\right) dx$

25) Evaluate $\int_{0}^{1} x dx$, as the limit of a sum.

26) Find the values in the interval $(1,2)$ of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$.

27) The temperature T in celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by $T = x(10-x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.

28) If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.

29) Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2,\infty)$.

30) Find Δf for the function f for the indicated values of $x, \Delta x$

$f(x) = x^2 + 2x + 3$; $x = -0.5, \Delta x = dx = 0.1$

PART – III

III. Answer any seven questions. **7x3 = 21**

Question No.40 is Compulsory

31) If $\lim_{\theta \to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$.

32) Write the Maclaurin series expansion of the following functions: $\sin x$

33) Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$.

34) Find the value of the following:

$\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x \, dx$

35) Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{4 + 5\sin x} = \frac{1}{3}\log_e 2$

36) Evaluate the following

$\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x} dx$

37) Show that the percentage error in the n^{th} root of a number is approximately 1/n times the percentage error in the number

38) Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

39) If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$

40 Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2 - 6x + 7}{x + 5}$.

PART – IV

IV. Answer all questions. **7x5 = 35**

41) a) Let $f(x,y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x,y) \in \mathbb{R}^2$. Check whether $f_{xy} = f_{yx}$

Or

b) Find the area of the region bounded by $y = \tan x$, $y = \cot x$ and the lines $x = 0$, $x = \frac{\pi}{2}$, $y = 0$.

42) a) Find the volume of the spherical cap of height 'h' cut of from a sphere of radius r.

Or

b) A right circular cylinder has radius $r = 10$ cm. and height $h = 20$ cm. Suppose that the radius of the cylinder is increased from 10 cm to 10.1cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

43) a) If $w(x,y,z) = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $z = e^t \cos t$, find $\frac{dw}{dt}$.

Or

b) Consider $f(x,y) = \frac{xy}{x^2 + y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is not continuous at $(0,0)$ and continuous at all other points of $\mathbb{R}^2$.

44) a) Evaluate the following using properties of integration : $\int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$

Or

b) Evaluate by using properties of integration :

$\int_0^{\sin^2 x} \sin^{-1}\sqrt{t} \, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t} \, dt$

45) a) Evaluate $\int_0^{\infty} \frac{x^n}{n^x} dx$, where n is a positive integer ≥ 2.

Or

b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

46) a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is 4/9 times volume of the cone.

Or

b) Find the intervals of monotonicity and local extrema of the function $f(x) = x \log x + 3x$.

47) a) Find intervals of concavity and points of inflexion for the following function:
$f(x) = \sin x + \cos x,\ 0 < x < 2\pi$

Or

b) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

ST.ANNE'S ACADEMY
Punnai Nagar, Nagercoil - 4.
Ph:948 99 00 886