ST.ANNE'S ACADEMY

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CLASS – XII – MATHEMATICS

(Chapter 8)

Time : 3 Hrs Marks : 85

PART – I

I. Answer ALL questions. **15x1 = 15**

1) If $f(x)=\frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

2) If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

3) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(1) 0.4 cu.cm (2) 0.45 cu.cm
(3) 2 cu.cm (4) 4.8 cu.cm

4) If $v(x,y)=\log(e^x+e^y)$, then $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}$ is equal to

(1) e^x+e^y (2) $\frac{1}{e^x+e^y}$ (3) 2 (4) 1

5) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdx\,m^3$ (2) $0.03x\,m^3$
(3) $0.03x^2m^3$ (4) $0.03x^3m^3$

6) If $u(x,y)=x^2+3xy+y-2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(1) -4 (2) -3 (3) -7 (4) 13

7) If $w(x,y)=x^y, x>0$, then $\frac{\partial w}{\partial x}$ is equal to

(1) $x^y\log x$ (2) $y\log x$ (3) yx^{y-1} (4) $x\log$.

8) The change in the surface area $S=6x^2$ of a cube when the edge length varies from x_0 to x_0+dx is

(1) $12x_0+dx$ (2) $12x_0dx$ (3) $6x_0dx$ (4) $6x_0+dx$

9) The % error of fifth root of 31 is approximately how many times the percentage error in 31?

(1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31

10) If $w(x,y,z)=x^2(y-z)+y^2(z-x)+z^2(x-y)$ then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ is

(1) $xy+yz+zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

11) Linear approximation for $g(x)=\cos x$ at $x=\frac{\pi}{2}$ is

(1) $x+\frac{\pi}{2}$ (2) $-x+\frac{\pi}{2}$ (3) $x-\frac{\pi}{2}$ (4) $-x-\frac{\pi}{2}$

12) If $g(x,y)=3x^2-5y+2y^2, x(t)=e^t$ and $y(t)=\cos t$, then $\frac{dg}{dt}$ is equal to

(1) $6e^{2t}+5\sin t-4\cos t\sin t$
(2) $6e^{2t}-5\sin t+4\cos t\sin t$
(3) $3e^{2t}+5\sin t+4\cos t\sin t$
(4) $3e^{2t}-5\sin t+4\cos t\sin t$

13) If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is equal to

(1) xye^{xy} (2) $(1+xy)e^{xy}$ (3) $(1+y)e^{xy}$ (4) $(1+x)e^{xy}$

14) If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is =

(1) $z-x$ (2) $y-z$ (3) $x-z$ (4) $y-x$

15) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%

PART – II

II. Answer any seven questions. 7x2 = 14

(Question No.25 is Compulsory)

16) Find a linear approximation for the following functions at the indicated points.

$g(x)=\sqrt{x^2+9},\ x_0=-4$

17) Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

18) Find differential dy of $y=(3+\sin(2x))^{2/3}$

19) If $w(x,y,z)=x^2y+y^2z+z^2x, x,y,z\in\mathbb{R}$, find the differential dw.

20) Determine whether the following function is homogeneous or not. If it is so, find the degree.

$h(x,y)=\frac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$

21) Let $f(x,y)=\sin(xy^2)+e^{x^3+5y}$ for all $(x,y)\in\mathbb{R}^2$ Calculate $\frac{\partial f}{\partial x}$

22) If $w(x,y)=x^3-3xy+2y^2, x,y\in\mathbb{R}$, find the linear approximation for w at $(1,-1)$.

23) Evaluate $\lim_{(x,y)\to(0,0)}\cos\left(\frac{x^3+y^2}{x+y+2}\right)$. If the limit exists.

24) Determine whether the following function is homogeneous or not. If it is so, find the degree. $f(x,y)=x^2y+6x^3+7$

25) Find $\lim_{(x,y)\to(0,0)}\cos\left(\frac{e^x\sin y}{y}\right)$, if the limit exists.

PART – III

III. Answer any seven questions. 7x3 = 21

(Question No.35 is Compulsory)

26) If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

27) Assuming $\log_{10}e=0.4343$, find an approximate value of $\log_{10}1003$.

28) Let $g(x,y)=\frac{e^y\sin x}{x}$, for $x\neq 0$ and $g(0,0)=1$ Show that g is continuous at $(0,0)$.

29) If $U(x,y,z)=\log(x^3+y^3+z^3)$, find $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}$.

30) If $u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$.

31) Let $g(x,y)=\frac{x^2y}{x^4+y^2}$ for $(x,y)\neq(0,0)$ and $f(0,0)=0$. Show that $\lim_{(x,y)\to(0,0)}g(x,y)=\frac{k}{1+k^2}$ along every parabola $y=kx^2, k\in\mathbb{R}\setminus\{0\}$.

32) Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

33) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following: (i) change in the volume (ii) change in the area

34) Let $f,g:(a,b)\to\mathbb{R}$ be differentiable functions. Show that $d(fg)=fdg+gdf$.

35) Let $u(x,y)=e^{-2y}\cos(2x)$ for all $(x,y)\in\mathbb{R}^2$ Prove that u is a harmonic function in $\mathbb{R}^2$.

PART – IV

IV. Answer any **SEVEN** questions. 7x5 = 35

36) Let $w(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}}, (x,y,z)\neq(0,0,0)$ Show that $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$.

37) Let $U(x,y)=e^x\sin y$, where $x=st^2,\ y=s^2t,\ s,t\in\mathbb{R}$ Find $\frac{\partial U}{\partial s},\frac{\partial U}{\partial t}$ and evaluate them at $s=t=1$.

38) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

39) If $u(x,y,z)=xy^2z^3,\ x=\sin t,\ y=\cos t,\ z=1+e^{2t}$, find $\frac{du}{dt}$.

40) Verify $\frac{dW}{dt}=\frac{\partial W}{\partial x}\frac{dx}{dt}+\frac{\partial W}{\partial y}\frac{dy}{dt}$ for $W(x,y)=x^2-2y^2+2xy$ and $x(t)=\cos t, y(t)=\sin t, t\in[0,2\pi]$.

41) Consider $f(x,y)=\frac{xy}{x^2+y^2}$ if $(x,y)\neq(0,0)$ and $f(0,0)=0$. Show that f is not continuous at

42) If $w(x,y,z)=x^2+y^2+z^2,\ x=e^t,\ y=e^t\sin t$ and $z=e^t\cos t$, find $\frac{dw}{dt}$.

43) If $w(x,y,z)=\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}$.

44) For $f(x,y)=\tan^{-1}\left(\frac{x}{y}\right)$ find the f_x, f_y, and show that $f_{xy}=f_{yx}$.

45) $W(x,y,z)=xy+yz+zx,\ x=u-v,\ y=uv,\ z=u+v,\ u,v\in\mathbb{R}$. Find $\frac{\partial W}{\partial u},\frac{\partial W}{\partial v}$, and evaluate them at $\left(\frac{1}{2},1\right)$.