DALMIA HIGHER SECONDARY SCHOOL
DALMIAPURAM – 621651
Std : 12 MATHEMATICS TIME: 1.50HRS
CHAPTER – 8 TEST -1 MARKS : 50

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2 MARKS : ANSWERS ANY 10 Q **10 X 2 =20**

1. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

2. Let $g(x) = x^2 + sinx$. Calculate the differential dg .

3. If the radius of a sphere, with radius 10 cm, has to decrease by 0 .1 cm, approximately how much will its volume decrease?

4. Let $f(x)=\sqrt[3]{x}$. Find the linear approximation at x = 27 .
Use the linear approximation to approximate $\sqrt[3]{27.2}$.

5. Use the linear approximation to find approximate values of $\sqrt[4]{15}$

6. Find a linear approximation for the functions at the indicated points. $h(x)=\frac{x}{x+1}$, $x_0 = 1$

7. A sphere is made of ice having radius 10 cm.
Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following: (i) change in the volume (ii) change in the surface area

8. Find differential dyfor each of the following functions :$y= \frac{(1-2x)^3}{3-4x}$

9. Find df for $f(x) = x^3 + 3x$and evaluate it for (i) x = 2 and dx = 0.1
(ii) x = 3 and dx = 0.02

10. Assume that the cross section of the artery of human is circular.
A drug is given to a patient to dilate hisarteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

11. Find the partial derivatives of the following functions at the indicated points. $g(x,y) = 3x^2 + y^2 + 5x + 2$, (1,–2)

12. If $U(x,y,z) = \frac{x^2+y^2}{xy} + 3z^2y$, find $\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}$.

13. If $U(x,y,z) =\log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}$.

14. If $w(x,y) =x^3 - 3xy + y^2$, x,y ∈ Υ , find the linear approximation for w at (1, −1) .

15. Let V(x,y,z) = xy+yz+zx, x,y,z∈ Υ , Find the differential dV.

16. If $u(x,y) =x^2y + 3xy^4$, $x = e^t$ and y= sint ,
find $\frac{du}{dt}$ and evaluate it at t = 0 .

3 MARKS : ANSWERS ANY 10 Q **10 X 3 = 30**

17. Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

18. Let $w(x,y)=xy+\frac{e^y}{y^2+1}$for all (x,y)∈$\mathbb{R}^2$. Calculate $\frac{\partial^2 W}{\partial y \partial x}$ and $\frac{\partial^2 W}{\partial x \partial y}$.

19. Let $u(x,y) = e^{-2y}\cos(2x)$for all (x,y)∈$\mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

20. Let $g(x,y) = 2y+x^2$, x= 2r − s , $y = r^2+2s$, r ,s ∈Υ . Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$.

21. Use the linear approximation to find approximate values of $(123)^{\frac{2}{3}}$

22. Use the linear approximation to find approximate values of $\sqrt[3]{26}$

23. The time T , taken for a complete oscillation of single pendulum with length l , is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$,
Where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l .

24. Show that the percentage error in the n th root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

25. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimetres of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

26. Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, $g(x,y) = \frac{3x^2-xy}{x^2+y^2+3}$.

27. Let U(x,y,z) =xyz , $x = e^{-t}$, $y= e^{-t}\cos t$,z = sint , t ∈ Υ . Find $\frac{dU}{dt}$.

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CHAPTER – 8 TEST -2 MARKS : 50

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5 MARKS : ANSWERS ANY 10 Q **10 X 5 = 50**

1. Let $f(x,y) = \frac{3x-5y+8}{x^2+y^2+1}$ for all $(x,y) \in \mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$

2. Consider $f(x,y) = \frac{xy}{x^2+y^2}$ $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is not continuous at (0,0) and continuous at all other points of $\mathbb{R}^2$.

3.Consider $g(x,y) = \frac{2x^2y}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $g(0,0) = 0$. Show that g is continuous on $\mathbb{R}^2$.

4. Let $g(x,y) = \frac{x^2y}{x^4+y^2}$ for $(x, y) \neq (0, 0)$ and $f(0,0) = 0$.

(i) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = 0$ along every line y = mx, $m \in \mathbb{R}$.(ii) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = \frac{k}{1+k^2}$ along every parabola $y = kx^2$,$k \in \mathbb{R}\setminus\{0\}$.

5. $f(x,y) = \sin(xy^2) + e^{x^3+5y}$ for all $(x, y) \in \mathbb{R}^2$.

Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

6. For each of the following functions find the f_x ,$f_y$and show that $f_{xy}= f_{yx}$.(i) $f(x,y) = \frac{3x}{y+\sin x}$ (ii) $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$

7. If $V(x, y) = e^x (x \cos y - y \sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

8. If $w(x, y) = xy+ \sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

9. If $z(x, y) = x \tan^{-1}(xy)$, $x = t^2$, $y = s e^t$, $s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at s = t = 1.

10. Let $U(x, y) = e^x \sin y$, where $x = st^2$, $y = s^2t$, $s, t \in \mathbb{R}$. Find $\frac{\partial U}{\partial s}$, $\frac{\partial U}{\partial t}$ and evaluate them at s = t = 1.

11. $W(x, y, z) = xy+ yz+ zx$, $x = u - v$, $y = uv$, $z = u + v$, $u,v \in \mathbb{R}$. Find $\frac{\partial W}{\partial u}$, $\frac{\partial W}{\partial v}$ and evaluate them at $\left(\frac{1}{2}, 1\right)$.

12. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

13. Prove that $f(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous; what is the degree? Verify Euler's Theorem for f .

14. Prove that $g(x, y) = x \log\left(\frac{y}{x}\right)$ is homogeneous; what is the degree? Verify Euler's Theorem for g.

15. If $v(x, y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

16. If $w(x, y, z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$.