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CLASS – XII – MATHEMATICS
(Chapter 9)

Time : 3 Hrs Marks : 90

PART – I

I. Answer ALL questions. 20x1 = 20

1) If $\int_0^x f(t)\,dt = x + \int_x^1 tf(t)\,dt$, then the value of $f(1)$

(1) $\frac{1}{2}$ (2) 2 (3) 1 (4) $\frac{3}{4}$

2) The value of $\int_0^1 \left(\sin^{-1} x\right)^2 dx$ is

(1) $\frac{\pi^2}{4}-1$ (2) $\frac{\pi^2}{4}+2$ (3) $\frac{\pi^2}{4}+1$ (4) $\frac{\pi^2}{4}-2$

3) For any value of $n \in \mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]\,dx$ is

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) 2

4) If $\int_0^a \frac{1}{4+x^2}\,dx = \frac{\pi}{8}$ then a is

(1) 4 (2) 1 (3) 3 (4) 2

5) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x\,dx$ is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 0 (4) $\frac{2}{3}$

6) The value of $\int_0^a \left(\sqrt{a^2-x^2}\right)^3 dx$ is

(1) $\frac{\pi a^3}{16}$ (2) $\frac{3\pi a^4}{16}$ (3) $\frac{3\pi a^2}{8}$ (4) $\frac{3\pi a^4}{8}$

7) If $f(x) = \int_1^x \frac{e^{\sin u}}{u}\,du, x>1$ and $\int_1^3 \frac{e^{\sin x^2}}{x}\,dx = \frac{1}{2}[f(a)-f(1)]$, then one of the possible value of a is

(1) 3 (2) 6 (3) 9 (5)

8) The value of $\int_0^{\infty} e^{-3x}x^2\,dx$ is

(1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

9) The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

10) The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) π

11) The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x}\right)dx$ is

(1) 4 (2) 3 (3) 2 (4) 0

12) If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

13) The value of $\int_{-1}^{2} |x|\,dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

14) The area between $y^2 = 4x$ and its latus rectum is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

15) The value of $\int_0^{\frac{\pi}{6}} \cos^3 3x\,dx$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

16) The value of $\int_0^1 x(1-x)^{99}\,dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

17) The value of $\int_0^{\pi} \sin^4 x\,dx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

18) The value of $\int_{-4}^{4} \left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right)+\tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right]dx$ is

(1) π (2) 2π (3) 3π (4) 4π

19) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

(1) πa^3 (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^3}{5}$ (4) $\frac{\pi a^3}{6}$

PART – II

II. Answer any seven questions. 7x2 = 14

(Question No.30 is Compulsory)

20) Find an approximate value of $\int_1^{1.5} x\,dx$ by applying the left-end rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$.

21) Evaluate : $\int_0^{1.5} \lfloor x^2 \rfloor \, dx$

22) Evaluate $\int_0^1 x^3 (1-x)^4 dx$.

23) Find the area of the region bounded by the line $6x+5y=30$, $x-$ axis and the lines $x=-1$ and $x=3$.

24) Evaluate $\int_0^1 x dx$, as the limit of a sum.

25) Evaluate $\int_0^\infty e^{-ax} x^n dx$, where $a>0$.

26) Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx$.

27) Find the value of the following:

$$\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x \, dx$$

28) Evaluate the following $\int_0^\infty x^5 e^{-3x} dx$

29) Evaluate $\int_b^\infty \frac{1}{a^2+x^2} dx$, $a>0, b \in \mathbb{R}$.

PART – III

III. Answer any seven questions. 7x3 = 21

(Question No.40 is Compulsory)

30) Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|} dx$.

31) Evaluate the integral as the limits of sums:

$$\int_0^1 (5x+4) dx$$

32) Prove that $\int_0^\infty e^{-x} x^n dx = n!$, where n is a positive integer.

33) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

34) Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx$.

35) Evaluate integral : $\int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

36) Evaluate $\int_0^1 x^5 (1-x^2)^5 dx$.

37) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x} dx$

38) Evaluate : $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1} x} \sin^{-1} x}{\sqrt{1-x^2}} dx$

39) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves $y=\log x$, $y=0$, $x=0$ and $y=2$.

PART – IV

IV. Answer any SEVEN questions. 7x5 = 35

40) Evaluate $\int_0^{2a} x^2 \sqrt{2ax-x^2} dx$.

41) Find the volume of the spherical cap of height 'h' cut of from a sphere of radius r.

42) Evaluate by using properties of integration :

$$\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} \, dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} \, dt$$

43) Find the area of the region bounded by $y=\cos x, y=\sin x$, the lines $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$.

44) The region enclosed by the circle $x^2+y^2=a^2$ is divided into two segments by the line $x=h$ Find the area of the smaller segment.

45) Show that $\int_0^1 \left(\tan^{-1} x + \tan^{-1}(1-x)\right) dx = \frac{\pi}{2} - \log_e 2$.

46) Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

47) Evaluate using properties of integration : $\int_0^\pi \frac{x \sin x}{1+\sin x} dx$

48) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin x + \cos x} dx$

49) Evaluate : $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

50) Evaluate using properties of integration :

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx$$