ST. JOSEPH'S HR. SEC. SCHOOL - CHENNAI-14

SECOND MID TERM TEST - 2024

18/11/24

STD - 12 MATHEMATICS

C. SELVAM, P.G. ASST, (MATHS)

I

1. d $\pi/2$
2. c $\sqrt{3}/12$
3. c $1/e^2$
4. a $z-x$
5. b $2x+4$
6. b ∞
7. c $[\pi/4, \pi/2]$
8. b $\frac{dx}{(x+1)^2}$
9. a 0
10. c 1, 2

II

11. $f'(x) = 2x-2$, $x=2$, $f'(2) = 2 > 0$
Strictly increasing in $(2, \infty)$

12. $\lim_{x\to 0^+} \frac{\log x}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = 0$

13. $g_x = 6x+5 = 11$ | $g_y = 2y = -4$

14. $df = (2x+3)\,dx$.
$df = (2(3)+3)(0.02) = 0.18$.

15. $V = x^3$
$\frac{dv}{dx} = 3x^2 = 75$ units.

16. $f'(x) = 6x^2+6x-12$
$= x^2+x-2$
$f'(x) = 0 \Rightarrow (x+2)(x-1) = 0$ $\boxed{x=-2, 1}$

17. $f(x) = 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\cdots$

III

18. $\frac{dy}{dx} = 4x^3+2e^x = 2$
Eqn. tangent
$(y-2) = 2(x-0) \Rightarrow 2x-y-2=0$
eqn. normal
$y-2 = -\frac{1}{2}(x-0) \Rightarrow x+2y-4=0$

19. $f(a) = 1/a$ $f(b) = 1/b$
$f'(c) = -1/c^2$

$f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow -\frac{1}{c^2} = \frac{1/b-1/a}{b-a}$

$-\frac{1}{c^2} = \frac{1}{ab} \Rightarrow c^2 = ab \Rightarrow \boxed{c = \sqrt{ab}}$

20. $\log x = 0 + \frac{x-1}{1!}(1) + \frac{(x-1)^2}{2!}(-1) + \frac{(x-1)^3}{6}(2) + \frac{(x-1)^4}{24}(-6)$

$\log x = (x-1) - \frac{(x-1)}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$

21. $\lim_{x\to\infty} \frac{x^2[2-3/x^2]}{x^2[1-5/x+3/x^2]} = \lim_{x\to\infty} \frac{2-3/x^2}{1-5/x+3/x^2} =$

22. $f(x) = 24x^5 - 24x^3$
$f'(x) = 0 \Rightarrow x^5 - x^3 = 0 \Rightarrow x^3(x^2-1) = 0$
$x = 0, x = 1, x = -1$

In	$f'(x)$	Nature
$(-\infty, -1)$	−	St. dec
$(-1, 0)$	+	St. Incre
$(0, 1)$	−	St. dec
$(1, \infty)$	+	St. Incre

$f'(x) = 120x^4 - 72x^2$ | $f''(-1) = 48 > 0$.
$f''(0) = 0$
$f(-1) = -2 < 0$

Local mini $= -2$
Local maxi $= 0$

23. $\lim_{x\to\infty} \frac{x^2[2-8/x^2]}{x^2[1-16/x^2]} = 2$

$\lim_{x\to\infty} \frac{2-8/x^2}{1-16/x^2} = 2$

$y = 2$ is a horizontal asymptote.

24. $V = \pi r^2 h$ $r = 10$, $h = 20$, $dr = 0.1$ cm
$V(r) = 20\pi r^2$
$\Delta V = V(10.1) - V(10) = 20\pi(102.01 - 100)$
$\Delta V = 20\pi(2.01) = 40.2\pi$, $V = 20\pi r^2$
$\frac{dv}{dr} = 40\pi r \Rightarrow \boxed{dv = 40\pi}$
Ab. error $= 0.2\pi$ cm^3
Re. error $= 0.00497$
%. error $= 0.497\%$

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IV 25 a. $V = \frac{1}{3}\pi r^2 h$ $h = 2r$, $r = h/2$

$V = \frac{1}{3}\pi (h/2)^2 h = \frac{1}{12}\pi h^3$

$\frac{dv}{dt} = \frac{1}{12}\pi 3h^2 \frac{dh}{dt}$

$30 = \frac{1}{4}\pi (10)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{6}{5\pi}$ m/min

b. $f'(x) = 12x^2 + 6x - 6$

$f'(x) = 0 \Rightarrow (x - 1/2)(x+1) = 0$

In	$f'(x)$	nature
$(-\infty, -1)$	+	Str. Incr
$(-1, 1/2)$	−	Str. dec
$(1/2, \infty)$	+	Str. Incr

$f''(x) = 24x + 6$

$f''(x) = 0 \Rightarrow x = -1/4$

In	$f''(x)$	concavity
$(-\infty, -1/4)$	−	str. dec
$(-1/4, \infty)$	+	str. incre

$f''(1/2) = 18 > 0$

Local min $= -3/4 = f(1/2)$

$x = -1$

$f''(x) = -18 < 0$, Local max $= 6$

$x = -1/4$, $f(-1/4) = \frac{21}{8}$

pt. of inflection $(-1/4, 21/8)$

26. a. $y = 2/x$, $x^2 + 4y = 0 \Rightarrow x = -2$

$y = -1$

$x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -y/x$.

$m_1 = -1/2$

$x^2 + 4y = 0$

$2x + 4\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = m_2 = 1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = 3$

$\theta = \tan^{-1}(3)$

26 b. $g_x = e^y + 6xy$, $g_y = xe^y + 3x^2$

$g_{xy} = e^y + 6x$, $g_{yy} = xe^y$

$g_{xx} = 6y$ $g_{yx} = e^y + 6x$.

27 a. $y = (1 + 1/x)^x$.

$\log y = \log (1 + 1/x)^x$.

$\log y = x \log (1 + 1/x)$

$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \log \frac{(1 + 1/x)}{1/x}$ ($\frac{0}{0}$ form)

$= \lim_{x\to\infty} \frac{1}{\frac{1 + 1/x}{-1/x^2}} (-1/x^2)$

$= \lim_{x\to\infty} \frac{1}{1 + 1/x}$

$\lim_{x\to\infty} (1 + 1/x)^x = e^0 = 1$

b. (i) $f(x)$ does not exist, Domain $R \setminus \{-1, 1\}$

(ii) x intercept $x = 0$, y intercept $y = 0$

(iii) $f'(x) = \frac{-3(x^2+1)}{(x^2-1)^2}$, $x^2 + 1 \neq 0$

$x^2 = 1$, $x = \pm 1$

critical value $x = -1$, $x = 1$

NO Local maximum, Local minimum

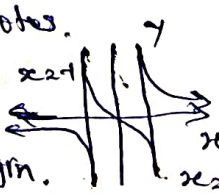
In	$f'(x)$	concavity
$(-\infty, -1)$	−	st. dec
$(-1, 1)$	−	st. dec
$(1, \infty)$	−	st. dec

Asymptotes.

$x = -1$ is a vertical asymptotes.

$y = 0$ is horizontal Asymptotes.

$y = \frac{3x}{x^2+1} \Rightarrow y = \frac{3x}{x^2-1}$



It is symmetrical about origin.

(iv) $f'(x) = \frac{-3x^2 - 3}{(x^2-1)^2}$, $f''(x) = \frac{x(6x^2 + 18)}{(x^2-1)^3} \leq 0$

$x^2 - 1 \geq 0$, $x \neq \pm 1$

In	$f''(x)$	concavity
$(-\infty, -1)$	−	concave down
$(-1, 0)$	+	up
$(0, 1)$	−	down
$(1, \infty)$	+	up.

$x = 0$ is point of inflection.

SECOND MID TERM TEST - 2024

Standard XII

Reg.No. | | | | | | |

MATHEMATICS

Time : 1.30 hrs *Marks : 50*

Part - I

I. **Choose the correct answer:** 10 x 1 = 10

1. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

   a) $\tan^{-1}\left(\frac{4}{3}\right)$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\tan^{-1}\left(\frac{3}{4}\right)$

2. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

   a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

3. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is

   a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$

4. If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is

   a) $z - x$ b) $y - z$ c) $x - z$ d) $y - x$

5. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

   a) $x^2 + y^2$ b) $2xu$ c) x^2u d) y^2u

6. The value of the $\lim_{x\to\infty} \frac{e^x}{x^m}$, $m \in N$ is

   a) 0 b) ∞ c) $-\infty$ d) 1

7. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

   a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ d) $\left[0, \frac{\pi}{4}\right]$

8. If $f(x) = \frac{x}{x+1}$, then its differential is given by

   a) $\frac{-1}{(x+1)^2}dx$ b) $\frac{1}{(x+1)^2}dx$ c) $\frac{1}{x+1}dx$ d) $\frac{-1}{x+1}dx$

9. The value of $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

   a) 0 b) 1 c) 2 d) ∞

10. The critical numbers of the function $f(x) = (x-2)^{2/3}(2x+1)$ is

    a) $-1, 2$ b) $1, -\frac{1}{2}$ c) $1, 2$ d) $-1, -\frac{1}{2}$

Part - II

II. **Answer any 5 questions. (Q.No.17 is compulsory)** 5 x 2 = 10

11. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$

12. Evaluate $\lim_{x\to 0^+} x \log x$ using L' Hopital rule.

13. Find the partial derivatives of the function $g(x, y) = 3x^2 + y^2 + 5x + 2$ at the point $(1, -2)$
14. Find df for $f(x) = x^2 + 3x$ & evaluate it for $x = 3$ and $dx = 0.02$
15. If the volume of the cube is $V = x^3$ for a cube of side 'x', then at $x = 5$, find the rate of change of volume with respect to 'x'.
16. Find the critical numbers of the function $f(x) = 2x^3 + 3x^2 - 12x$
17. Find the Maclaurin series of the function e^x

Part - III

III. Answer any 5 questions. (Q.No.24 is compulsory) **5 x 3 = 15**

18. Find the tangent and normal to the following curve at the given point on the curve $y = x^4 + 2e^x$ at $(0, 2)$
19. Show that the value in the conclusion of the mean value theorem for $f(x) = 1/x$ on a closed interval of positive numbers $[a,b]$ is $\sqrt{ab}$
20. Write down the Taylor series expansion, of the function log x about $x = 1$ upto three non-zero terms for $x > 0$
21. Evaluate using L' Hopital Rule: $\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3}$
22. Find the local extremum of the function $f(x) = 4x^6 - 6x^4$
23. Find the asymptotes of the curve $f(x) = \frac{2x^2-8}{x^2-16}$
24. A right circular cylinder has radius r = 10 cm and height h = 20 cm. Suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

Part - IV

IV. Answer all the questions. **3 x 5 = 15**

25. a) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are equal. How fast is the height of the pile increasing when the pile is 10 metre high?

(OR)

b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$, find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

26. a) Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$

(OR)

b) For the function $g(x, y) = xe^y + 3x^2y$, find g_{xy}, g_{xx}, g_{yy} and g_{yx}.

27. a) Evaluate using L'Hopital Rule: $\lim_{x\to 0} \left(1+\frac{1}{x}\right)^x$

(OR)

b) Sketch the graph of the function $y = \frac{3x}{x^2-1}$

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