RK ACADEMY

FULL PORTION 1

12th Standard

Maths

Reg.No. :

Exam Time : 03:00 Hrs Total Marks : 90

I. CHOOSE THE CORRECT ANSWER

20 x 1 = 20

1) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) adj A = $|A|A^{-1}$ (b) adj(AB) = (adj A)(adj B) (c) det A^{-1} = $(\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2) If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that λA^{-1} = A, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

3) The value of $\sum_{n=1}^{13}\left(i^n + i^{n-1}\right)$ is

(a) 1+ i (b) i (c) 1 (d) 0

4) If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

(a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

5) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

6) The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

7) The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line x = $\frac{-9}{2}$ is

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

8) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

9) The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{j})$ meets the plane $\vec{r}.(\hat{i} + \hat{j} - \hat{k})$ = 3 are

(a) (2, 1, 0) (b) (7, -1, -7) (c) (1, 2, -6) (d) (5, -1, 1)

10) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(a) y = 0 (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

11) The curve $y= ax^4 + bx^2$ with ab > 0

(a) has, no horizontal tangent (b) is concave up (c) is concave down (d) has no points of inflection

12) If f (x, y) = e^{xy} then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(a) xye^{xy} (b) $(1 + xy)e^{xy}$ (c) $(1 + y)e^{xy}$ (d) $(1 + x)e^{xy}$

13) The value of $\int_0^{\pi} \sin^4 x dx$ is

(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

14) The order of the differential equation of all circles with centre at (h, k) and radius 'a' is

(a) 2 (b) 3 (c) 4 (d) 1

15) The number of arbitrary constants in the general solutions of order n and n +1 are respectively

(a) n-1,n (b) n,n+1 (c) n+1,n+2 (d) n+1,n

16) A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d) 2

17) The operation * defined by $a*b=\frac{ab}{7}$ is not a binary operation on

(a) Q^+ (b) Z (c) R (d) C

18) If a = cos θ + i sin θ, then $\frac{1+a}{1-a}$ = __________

(a) cot $\frac{\theta}{2}$ (b) cot θ (c) i cot $\frac{\theta}{2}$ (d) i tan$\frac{\theta}{2}$

19) The equation of the normal to the curve y = sin x at (0, 0) is __________

(a) x = 0 (b) y = 0 (c) x + y = 0 (d) x - y = 0

20) The area bounded by the curve y^2 = 4x and the lines. x = 1, x = 4 and x - axis in the first quadrant is _________

(a) $\frac{11}{3}$ (b) $\frac{17}{3}$ (c) $\frac{28}{3}$ (d) $\frac{31}{3}$

II. ANSWER THE FOLLOWING (Q.NO. 30 IS COMPULSORY) 7 x 2 = 14

21) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$$

22) Find the modulus of the following complex number $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$

23) Find the principal value of $sin^{-1}\left(sin\left(\frac{5\pi}{6}\right)\right)$

24) Find centre and radius of the following circles.

x^2+ (y + 2)^2 = 0

25) Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane 5x-y+z = 8.

26) Find the slope of the tangent to the curves at the respective given points.

x = a cos^3 t, y = b sin^3 t at t = $\frac{\pi}{2}$

27) Determine the order and degree (if exists) of the following differential equations:

dy + (xy − cos x)dx = 0

28) Compute P(X = k) for the binomial distribution, B(n, p) where

$n=10, p=\frac{1}{5}, k=4$

29) Construct a cubic equation with roots 2, −2, and 4.

30) If w=e^{xy},x=at^2,y=2at, find $\frac{dw}{dt}$

III. ANSWER THE FOLLOWING (Q.NO. 40 IS COMPULSORY) 7 x 3 = 21

31) Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

32) Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

33) Find the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix x =$\sqrt{2}$.

34) Find the magnitude and direction cosines of the torque of a force represented by $\hat{3i}+\hat{4j}-\hat{5k}$ about the point with position vector $\hat{2i}-\hat{3j}+\hat{4k}$ acting through a point whose position vector is $\hat{4i}+\hat{2j}-\hat{3k}$.

35) Evaluate the following limit, if necessary use l 'Hôpital Rule

$\lim_{x\to 0}\left(\frac{1}{sinx}-\frac{1}{x}\right)$

36) Find a linear approximation for the following functions at the indicated points.

f(x) = x^3 - 5x + 12, x_0 = 2

37) Evaluate the following integrals using properties of integration:

$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{1}{1+\sqrt{tanx}}dx$

38) In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variables.

39) Construct the truth table for the following statements.

(¬p ⟶ r) ∧ (p ↔ q)

40) Find the general solution of the differential equation $\frac{dy}{dx}=\sqrt{9-y^2}\quad(-3<\mathbf{y}<3)$.

IV. ANSWER THE FOLLOWING 7 x 5 = 35

41) a)

(a) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

(OR)

b)

Find all cube roots of $\sqrt{3}+i$

42) a)

Solve: $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$

(OR)

b)

Solve $2tan^{-1}x=cos^{-1}\frac{1-a^2}{1+a^2}-cos^{-1}\frac{1-b^2}{1+b^2}, a>0, b>0$

43) a)

A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x -axis is an ellipse. Find the eccentricity.

(OR)

b)

Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

44) a)

If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

(OR)

b)

Sketch the graphs of the following function.

$y=-\frac{1}{3}\left(x^3-3x+2\right)$

45) a)

If w(x,y, z) = log $\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$ find $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}$

(OR)

b)

Find the area of the region bounded by y = tan x, y = cot x and the lines x = 0, x = $\frac{\pi}{2}$, y = 0

46) a)

The equation of electromotive force for an electric circuit containing resistance and self inductance is E = Ri + L$\frac{di}{dt}$, Where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when E = 0.

(OR)

b)

If X is the random variable with probability density function f(x) given by,

$$f(x)=\begin{cases} x+1 & -1\le x<0 \\ -x+1 & 0\le x<1 \\ 0 & otherwise \end{cases}$$

then find

(i) the distribution function F(x)

(ii) P(-0.5 ≤X ≤ 0.5)

47) a)

Let A be Q\{1}. Define $*$ on A by x*y = x + y − xy. Is $*$ binary on A? If so, examine the commutative and associative properties satisfied by $*$ on A.

(OR)

b)

If V = log r and $r^2=x^2+y^2+z^2$, then prove that $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\frac{1}{r^2}$

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I. CHOOSE THE CORRECT ANSWER

20 x 1 = 20

1) (b) adj(AB) = (adj A)(adj B)

2) (c) 19

3) (a) 1+ i

4) (a) $-\frac{q}{r}$

5) (d) $tan^{-1}\left(\frac{1}{2}\right)$

6) (c) $\sqrt{10}$

7) (c) an ellipse

8) (a) 0

9) (d) (5, -1, 1)

10) (d) $y = \pm 3$

11) (d) has no points of inflection

12) (b) $(1 + xy)e^{xy}$

13) (b) $\frac{3\pi}{8}$

14) (b) 3

15) (b) n,n+1

16) (d) 2

17) (b) Z

18) (c) i cot $\frac{\theta}{2}$

19) (c) x + y = 0

20) (c) $\frac{28}{3}$

II. ANSWER THE FOLLOWING (Q.NO. 30 IS COMPULSORY)

7 x 2 = 14

21) $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Let A = $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

A is a matrix of order (2 × 4)

∴ ρ(A) ≤ min(2, 4) = 2

The highest order of minor of A is 2

It is $\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix}$ = -6 + 6 = 0

Also, $\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix}$ = -1 + 0 = -1 ≠ 0

∴ ρ(A) = 2

22) $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$

Let z = $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$

= $\frac{(2-i)(1-i)+(1-2i)(1+i)}{(1+i)(1-i)}$

= $\frac{2-2i-i+i^2+1+i-2i-2i^2}{1^2-i^2}$

= $\frac{2-i-1+1-i+2}{2} = \frac{4-4i}{2}$

= $\frac{2(2-2i)}{2}$ = 2 - 2i

∴ |z| = $\sqrt{2^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

23) We know that $\sin^{-1}$: [-1, 1] $\rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is given by

$\sin^{-1}x = y$ if and only if $x = \sin y$ for $-1 \leq x \leq$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$. Thus

$sin^{-1}\left(sin\left(\frac{5\pi}{6}\right)\right) = sin^{-1}\left(sin\left(\frac{\pi}{6}\right)\right) = sin^{-1}\left(sin\frac{\pi}{6}\right)$, since $\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

24) Equation of the circle is $x^2 + (y + 2)^2 = 0$

Compare with $(x-h)^2+(y-k)^2 = r^2$

$h = 0, k = -2, r^2 = 0$

Centre (h, k) = (0, -2)

radius is 0.

25) Here (x_1, y_1, z_1) = (3, -4, -3) and direction ratios of the given straight line are (a, b, c) = (-4, -7, 12).

Direction ratios of the normal to the given plane are (A, B, C) = (5, -1, 1).

We observe that, the given point (x_1, y_1, z_1) = (3, 4, -3) satisfies the given plane 5x-y+z = 8

Next, aA+bB+cC = (-4)(5)+(-7)(-1)+(12)(1) = -1 $\neq$ 0.

So, the normal to the plane is not perpendicular to the line.

Hence, the given line does not lie in the plane.

26) Given $x = a\cos^3 t$; $y = b\sin^3 t$

$\frac{dx}{dt} = -3a\, cos^2\, t\, sin\, t$

$\frac{dy}{dt} = 3b\, sin^2\, t\, cot\, t$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{3bsin^2tcost}{3acos^2tsint} = \frac{-b}{a}$ tan t

Slope of the tangent at $t = \frac{\pi}{2}$ is

$m = \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{2}}$

$\frac{-b}{a} tan\frac{\pi}{2} = \frac{-b}{a} \times \infty = \infty$

$\therefore m = \infty$

27) dy + (xy − cos x)dx = 0 is a first order differential equation with degree 1

since the equation can be rewritten as

$\frac{dy}{dx}$ + xy - cos x = 0

28) $\therefore q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$

$\mathrm{P}(\mathrm{X} = x) = n\mathrm{C}_x p^x q^{n-x}, x = 0, 1, 2, \ldots\ldots n$

$\mathrm{P}(\mathrm{X} = k) = \mathrm{P}(\mathrm{X} = 4)$

$= 10\mathrm{C}_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^{10-4} = 210\left(\frac{1}{5^4}\right)\left(\frac{4^6}{5^6}\right) = 210\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$

29) Here ∝ = 2, β = -2 and ϒ = 4

$x^3-(2-2+4)x^2+(-4-8+8x)x-(2)(-2)(4) = 0$

$\Rightarrow x^3- 4x^2- 4x+16 = 0$

30) $\frac{dw}{dt} = 6a^2t^2e^{2a^2}t^3$

III. ANSWER THE FOLLOWING (Q.NO. 40 IS COMPULSORY) 7 x 3 = 21

31) Let $z = \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$

Here, $\frac{19+9i}{5-3i} = \frac{(19+9i)(5+3i)}{(5-3i)(5+3i)}$

$= \frac{(95-27)+i(45+57)}{5^2+3^2} = \frac{68+102i}{34}$

= 2 + 3i ....(1)

and $\frac{8+i}{1+2i} = \frac{(8+i)(1-2i)}{(1+2i)(1-2i)}$

$= \frac{(8+2)+i(1-16)}{1^2+2^2} = \frac{10-15i}{5}$

= 2 - 3i .... (2)

Now $z = \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$

$\Rightarrow z = (2 + 3i)^{15} - (2 - 3i)^{15}$ (by (1) and (2))

Then by definition, $\bar{z} = \left(\overline{(2+3i)^{15} - (2-3i)^{15}}\right)$

$= \left(\overline{2+3i}\right)^{15} - \left(\overline{2-3i}\right)^{15}$ (using properties of conjugates)

$= (2 - 3i)^{15} - (2 + 3i)^{15} = -((2 + 3i)^{15} - (2-3i)^{15})$

$\Rightarrow \bar{z} = -z$

Therefore, $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

32) Since $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a root, $x-\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a factor. To remove the outermost square root, we take $x+\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as another factor and find their product.

$\left(x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right) = x^2 - \frac{\sqrt{2}}{\sqrt{3}}$

Still we didn't achieve our goal. So we include another factor $x^2+\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ and get the product.

$$\left(x^2-\frac{\sqrt{2}}{\sqrt{3}}\right)\left(x^2+\frac{\sqrt{2}}{\sqrt{3}}\right)=x^4-\frac{2}{3}$$

So, $3x^4 - 2 = 0$ is a required polynomial equation with the integer coefficients.

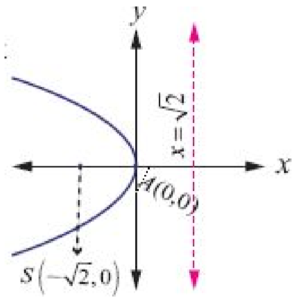
Now we identify the nature of roots of the given equation without solving the equation. The idea comes from the negativity, equality to 0, positivity of $\Delta = b^2 - 4ac$.

33) Parabola is open left and axis of symmetry as x-axis and vertex (0, 0)

Then the equation of the required parabola is

$(y - 0)^2 = -4\sqrt{2}(x - 0)$

$y^2 = -4\sqrt{2}\, x$



34) Given $\vec{F} = 3\hat{i}+4\hat{j}-5\hat{k}$

$\vec{r}$ = (Force acting through the point) - (force acting to the point)

$= (4\hat{i}+2\hat{j}-3\hat{k})-(2\hat{i}-\hat{j}+4\hat{k})$

$= 2\hat{i}+5\hat{j}-7\hat{k}$

Torque = $\vec{c} = \hat{r}\times\hat{F}$

$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 5 & -7\\ 3 & 4 & -5\end{vmatrix} = \hat{i}\begin{vmatrix}5 & -7\\ 4 & -5\end{vmatrix} - \hat{j}\begin{vmatrix}2 & -7\\ 3 & -5\end{vmatrix} + \hat{k}\begin{vmatrix}2 & 5\\ 3 & 4\end{vmatrix}$

$\hat{i} = \hat{i}(-25+28)-\hat{j}(-10+21)+\hat{j}(8-15)$

$= 3\hat{i}-11\hat{j}-7\hat{k}$

$\therefore$ MagnitudeoftheTorque = $\sqrt{3^2+(-11)^2+(-7)^2}$

$= \sqrt{9+121+49}=\sqrt{179}$

Hence, the direction cosines are $\left(\frac{3}{\sqrt{179}},\frac{-11}{\sqrt{179}},\frac{-7}{\sqrt{179}}\right)$.

35) $\lim\limits_{x\to 0}\left(\frac{1}{sinx}-\frac{1}{x}\right)=\lim\limits_{x\to 0}\left(\frac{x-sinx}{xsinx}\right)=\frac{0}{0}$ form

Which is indeterminate Applying L' Hopital rule we get,

$0+\frac{sinx}{-xsinx+cosx}+cosx=\frac{0}{2}=0\ \lim\limits_{x\to 1^+}\frac{1-cosx}{xcosx+sinx}=\frac{1-cos0}{0+sin0}=\frac{1-1}{0}=\frac{0}{0}$form

36) $f(x) = x^3 - 5x + 12,\ x_0 = 2$

$f(x_o) = 2^3 - 5(2) + 12$

$= 8 - 10 + 12 = 10$

$f'(x) = 3x^2 - 5$

$\Rightarrow f'(x_o) = 3(2^2) - 5 = 7$

$\therefore L(x) = f(x_o) + f'(x_o)(x - x_o)$

$= 10 + 7(x - 2)$

$= 10 + 7x - 14$

$L(x) = 7x - 4$

37) $I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{1}{1+\sqrt{tanx}}dx\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{1}{1+\frac{\sqrt{sin\quad x}}{\sqrt{cos\quad x}}}dx$

$=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{cos\quad x}}{\sqrt{cos\quad x}+\sqrt{sin\quad x}}dx\qquad ..(1)$

By the property, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

we get $I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{cos(\frac{\pi}{8}+\frac{3\pi}{8}-x)}}{\sqrt{cos(\frac{\pi}{8}+\frac{3\pi}{8}-x)}+\sqrt{sin(\frac{\pi}{8}+\frac{3\pi}{8}-x)}}dx$

$=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{cos(\frac{\pi}{2}-x)}}{\sqrt{cos(\frac{\pi}{2}-x)}+\sqrt{sin(\frac{\pi}{2}-x)}}dx$

$=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{sin\quad x}}{\sqrt{sin\quad x}+\sqrt{cos\quad x}}...(2)$

$(1)+(2)\rightarrow$

$2I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{cos\quad x}}{\sqrt{cos\quad x}+\sqrt{sin\quad x}}dx+\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}\frac{\sqrt{sin\quad x}}{\sqrt{sin\quad x}+\sqrt{sin\quad x}}$

$=\int_{\frac{\pi}{8}}^{3\frac{\pi}{8}}\frac{\sqrt{cos\quad x}+\sqrt{sin\quad x}}{\sqrt{cos\quad x}+\sqrt{sin\quad x}}dx=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}}dx=[x]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$

$2I=\frac{3\pi}{8}-\frac{\pi}{8}=\frac{2\pi}{8}=\frac{\pi}{4}$

$\therefore I=\frac{\pi}{8}$

38) n = 5, X B{n, p)

P(X = 1) = 0.4096

P(X = 2) 0.2048

$P(X = x) = {}^nC_x\, p^x q^{n-x}$, x = 0, 1, 2, .., n

ஃ$nC_1, p^1 q_4$ 0.4096

${}^5C_1,\ p^2q^4 = 0.4096$

${}^5C^2\ p^2q^3 = 0.2048$

$5pq^4 = 0.4096$ .....(1)

$10\ p^2\ q^3 = 0.2048$ ....(2)

Dividing (2) by (1) we get

$$\frac{5pq^4}{10p^2q^3} = 2$$

q = 4p

q = 4(1- q)

q = 4 - 4q

5q = 4

q = 4/5

$p = 1 - q = p = \frac{1}{5}$

$Mean = np = 5 \times \frac{1}{5} = 1$

$Variance = npq = \not{5} \times \frac{1}{\not{5}} \times \frac{4}{5} = \frac{4}{5}$

Distribution

(i) $P(X = x) = {}^5C_x\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{5-x}$, x = 0,1,2..n

39) Truth Table for $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

p	q	r	~ p	~ p → r	p ↔ q	(~p → r) ∧ (p ↔ q)
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	F	F

40) Given $\frac{dy}{dx} = \sqrt{9 - y^2}$

The given equation can be written as

$\frac{dy}{\sqrt{9-y^2}} = dx$ ....(1)

Now, Integrating equation (1), we get

$\int \frac{dy}{\sqrt{9-y^2}} = \int dx$

$\sin^{-1}\left(\frac{y}{3}\right) = x + c$ is the arbitrary constant.

Hence, $\sin^{-1}\left(\frac{y}{3}\right) = x + c$ the required differential equation.

IV. ANSWER THE FOLLOWING

7 x 5 = 35

41) a)

Given A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

AB = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

= $\begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-3 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$

= $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ = 4. I_3

BA = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

= $\begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix}$

= $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ = 4. I_3.

So, we get AB = BA = 4. I_3

⇒ $\left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = 1$

⇒ $B^{-1} = \frac{1}{4} = 1$

Writing the given set of equations in matrix form we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \left[\tfrac{1}{4}A\right] \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

∴ x = 2, y = 1, z = -1

Hence, the solution set is {2, 1, - 1}.

(OR)

b) We have to find $(\sqrt{3}+1)^{\frac{1}{3}}$. Let $z = (\sqrt{3}+i)^{\frac{1}{3}}$. Then $z^3 = \sqrt{3}+i = r\,(cos\theta + isin\theta)$

Then, $r = \sqrt{3+1} = 2$ and $\alpha = \theta = \frac{\pi}{6}$ ($\because \sqrt{3}+i$ lies in the first quadrant)

Therefore, $z^3 = \sqrt{3}+i = 2\left(cos\frac{\pi}{6} + isin\frac{\pi}{6}\right)$

$\Rightarrow z = \sqrt[3]{2}\left(cos\left(\frac{\pi+12k\pi}{18}\right) + isin\left(\frac{\pi+12k\pi}{18}\right)\right)$, k = 0, 1, 2.

Taking k = 0, 1, 2, we get

k = 0, $z = 2^{\frac{1}{3}}\left(cos\frac{\pi}{18} + sin\frac{\pi}{18}\right)$

k = 1, $z = z^{\frac{1}{3}}\left(cos\frac{\pi}{18} + sin\frac{\pi}{18}\right)$

k = 2, $z = 2^{\frac{1}{3}}\left(cos\frac{25\pi}{18} + sin\frac{25\pi}{18}\right) = 2^{\frac{1}{3}}\left(-cos\frac{7\pi}{18} - sin\frac{7\pi}{18}\right)$

42) a) Put $\sqrt{\frac{x}{a}} = y \Rightarrow 2y + \frac{3}{y} = \frac{b}{a} + \frac{6a}{b}$

$\Rightarrow \frac{2y^2+3}{y} = \frac{b^2+6a^2}{ab}$

$\Rightarrow ab(2y^2+3) = \left(b^2+6a^2\right)y$

$\Rightarrow 2aby^2 + 3ab - \left(b^2+6a^2\right) = 0$

$\Rightarrow 2aby^2 - y\left(b^2+6a^2\right) + 3ab = 0$

$\Rightarrow 2aby^2 - b^2y - 6a^2 + 3ab = 0$

$\Rightarrow by(2ay-b) - 3a(2ay-b) = 0$

$\Rightarrow (2ay-b)(by-3a) = 0$

$\Rightarrow 2ay = b,\ by = 3a$

$\Rightarrow y = \frac{b}{2a}, y = \frac{3a}{b}$

Case (i) When $y = \frac{b}{2a}$

$\Rightarrow \sqrt{\frac{x}{a}} = \frac{b}{2a} \Rightarrow \frac{x}{a} = \frac{b^2}{4a^2} \Rightarrow x = \frac{b^2}{4a}$

Case (ii) When $y = \frac{3a}{b}$

$\sqrt{\frac{x}{a}} = \frac{3a}{b} \Rightarrow \frac{x}{a} = \frac{9a^2}{b^2} \Rightarrow x = \frac{9a^3}{b^2}$

∴ The roots are $\frac{b^2}{4a}, \frac{9a^3}{b^2}$

(OR)

b) $2tan^{-1}x = cos^{-1}\frac{1-a^2}{1+a^2} - cos^{-1}\frac{1-b^2}{1+b^2}, a > 0, b > 0$

Let $a = tan\,\theta\ b = tan\phi$

$\therefore cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = cos^{-1}\left(\frac{1-tan^2\theta}{1+tan^2\theta}\right)$

= $cos^{-1}(cos2\theta) = 2\theta$ ...(1)

$\left[\because cos2\theta = \frac{1-tan^2\theta}{1+tan^2\theta}\right]$

Also $cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = cos^{-1}\left(\frac{1-tan^2\phi}{1+tan^2}\right)$

= $cos^{-1}(cos2\Phi)$

$\therefore 2tan^{-1}x = 2\theta - 2\phi = 2(\theta - \phi)$

[using (1) and (2)]

$\Rightarrow tan^{-1}x = \theta - \phi$

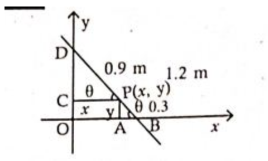
$\Rightarrow x = tan(\theta-\phi) = \frac{tan\theta - tan\phi}{1+tan\theta tan\phi}$

$\left[\because tan(A-B) = \frac{tanA - tanB}{1+tanAtanB}\right]$

$\Rightarrow x = \frac{a-b}{1+ab}$, a > 0

43) a) Let AB be the rod and P(x_1, y_1) be a point on the rod such that AP = 0.3 m.

Draw PD ⊥ x-axis and PC ⊥ y - axis.



Δ ADP ≅ Δ PCB

$\therefore \frac{PC}{DA} = \frac{PB}{AP} = \frac{BC}{PD}$

$\Rightarrow \frac{x_1}{DA} = \frac{0.9}{0.3} = \frac{BC}{y_1}$

$\Rightarrow DA = \frac{0.3x_1}{0.9} = \frac{x_1}{3}$

and BC = $\frac{0.9y_1}{0.3} = \frac{9}{3}y_1 = 3y_1$

Now OA = OD + DA

= $x_1 + \frac{x_1}{3} = \frac{4x_1}{3}$

OB = OC + BC = $y_1 + 3y_1 = 4y_1$

But $OA^2 + OB^2 = AB^2$

$\Rightarrow \left(\frac{4x_1}{3}\right)^2 + (4y_1)^2 = (1.2)^2$

$\Rightarrow \frac{x_1{}^2}{9} + \frac{y_1{}^2}{9} = \frac{1.44}{16} = 0.09 \cong 1$

$\therefore$ Locus of (x_1, y_1) is $\frac{x^2}{9} + \frac{y^2}{1} = 1$

Here $a^2 = 9$, $b^2 = 1$

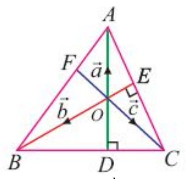
$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{9-1}{9}}$

= $\sqrt{\frac{8}{9}}$

e = $\frac{2\sqrt{2}}{3}$

(OR)

b) Consider a triangle ABC in which the two altitudes AD and BE intersect at O. Let CO be produced to meet AB at F. We take O as the origin and let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$



Since $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BC}$, we have $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$, and

hence we get $\overrightarrow{OA}$. $\overrightarrow{BC}$ = 0. That is, $\vec{a}.(\vec{c} - \vec{b}) = 0$, which means

$\vec{a}.\hat{c} - \hat{a}.\hat{b} = 0$....(1)

Similarly, since $\overrightarrow{BE}$ is perpendicular to $\overrightarrow{CA}$, we have $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{CA}$, and hence we get $\overrightarrow{OB}.\overrightarrow{CA}$ = 0.

That is, $\vec{b}.(\vec{a} - \vec{c}) = 0$

$\vec{a}.\hat{c} - \hat{b}.\hat{c} = 0$.......(2)

Adding equations (1) and (2), gives $\vec{a}.\hat{c} - \hat{b}.\hat{c} = 0$. That is, $\hat{c}(\hat{a} - \hat{b}) = 0$

That is $\overrightarrow{OC}$. $\overrightarrow{BA}$ = 0.

Therefore, $\overrightarrow{BA}$ is perpendicular to $\overrightarrow{OC}$.

Which implies that $\overrightarrow{CF}$ is perpendicular to $\overrightarrow{AB}$.

Hence, the perpendicular drawn from C to the side AB passes through O. Therefore, the altitudes are concurrent.

44) a) Let the two curves intersect at a point (x_0 , y_0) This leads to $(a-c)x_0{}^2 + (b-d)y_0{}^2 = 0$

Let us now find the slope of the curves at the point of intersection (x_0, y_0). The slopes of the curves are as follows :

For the curve $ax^2 + by^2 = 1$, $\frac{dy}{dx} = -\frac{ax}{by}$

For the curve $cx^2 + dy^2 = 1$, $\frac{dy}{dx} = -\frac{cx}{by}$

Now, two curves cut orthogonally, if the product of their slopes intersection (x_0, y_0) is −1. Hence, for the above two curves to cut orthogonally at (x_0, y_0) if

$(-\frac{ax_0}{by_0}) \times (-\frac{cx_0}{dy_0}) = -1$

That is, $acx_0{}^2 + bdy_0{}^2 = 0$,

together with $(a-c)x_0^2 + (b-d)y_0^2 = 0$

gives, $\frac{a-c}{ac} = \frac{b-d}{bd}$

That is, $\frac{1}{c} - \frac{1}{a} = \frac{1}{d} - \frac{1}{b}$.

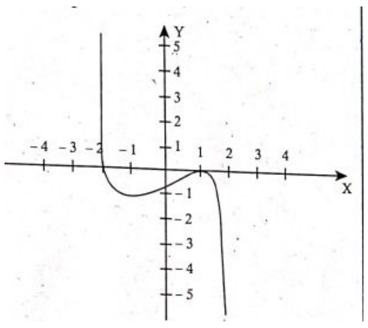
Hence, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

(OR)

b) Given$y = -\frac{1}{3}\left(x^3 - 3x + 2\right)$

Factorising the given function, we have

$y = f(x) = -\frac{1}{3}(x-1)\left(x^2 + x - 2\right)$



1. The domain and the range of the given function fix) are the entire real time.

2. Putting y = 0, we get x = 1. The other two roots are imaginary.

∴ X intercept is (1, 0)

Putting x = 0, we get $y = -\frac{2}{3}$

∴Y intercept is $\left(0, -\frac{2}{3}\right)$

3. $f'(x) = \frac{-1}{3}\left(3x^2 - 3\right) = -x^2 + 1$

f'(x) = 0

$\Rightarrow -x^2 = -1$

$\Rightarrow x = \pm 1$

∴The critical points are at x = 1, x = -1.

4. $f''(x) = \frac{-1}{3}(6x) = -2x$

f''(1) = -2, f''(-1) = 2.

∴f (x) is local minimum at x = 1,

$f(1) = \frac{-1}{3}(1 - 3 + 2)$

$\Rightarrow f(1) = \frac{-1}{3}(0) = 0$

f (x) is local minimum at x = -1,

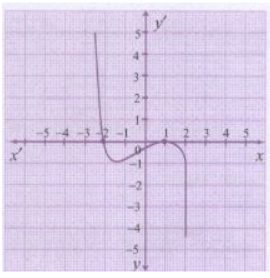
$\Rightarrow f(-1) = \frac{-1}{3}(-1 + 3 + 2) = \frac{-4}{3}$

∴The critical points are at x = 1, x = -1.

5. Since f''(x) = -2x<0∀x>0, the function is concave downward in the positive real line and f''(x) = -2x > 0∀<0,the function is concave upward in the negative real line.

6. f''(x) = 0 at x = 0 and f''(x) changes its sign when x = 0, the point of inflection is $(0, f(0)) = \left(0, \frac{-2}{3}\right)$

7. The curve has no asymptotes.



45) a)

Given w(x, y, z) = $\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$

Let (x, y, z) = $\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}$

⇒ w = log f ...(1)

⇒ e^w = f

f(λx, λy, λz) = $\frac{5\lambda^3x^3\lambda^4y^4+7\lambda^2y^2\lambda x\lambda^4z^4-75\lambda^3y^3\lambda^4z^4}{\lambda^2x^2+\lambda^2y^2}$

= $\frac{\lambda^7(5x^3y^4+7y^2xz^4-75y^3z^4)}{\lambda^2(x^2+y^2)} = \lambda^5 f(x, y, z)$

∴ f(x, y, z) is a homogeneous function of degree 5.

∴ By Euler's theorem,

$x.\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = 5.f$

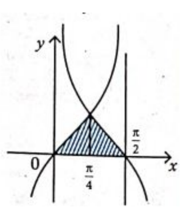
⇒ $x.\frac{\partial}{\partial x}(e^w) + y.\frac{\partial}{\partial y}(e^w) + z.\frac{\partial}{\partial z}(e^w) = 5.e^w$ [using (1)]

⇒ $x.e^w\frac{\partial w}{\partial x} + y.e^w\frac{\partial w}{\partial y} + z.e^w\frac{\partial w}{\partial z}(e^w) = 5e^w$

⇒ $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$ [Divided by e^w]

(OR)

b)



Given equation of the curves are y = tan x, y = cot x.

The intersection of y = tan x and y = cot x are

tan x = cot x ⇒ x = $\frac{\pi}{2}$

$\therefore$ Required area $= \int_0^{\frac{\pi}{4}} (tan\ x - cot\ x)dx$

$= [-log\ sin\ x + log\ sec\ x]_0^{\frac{\pi}{4}}$

$= log\left[\frac{sec\ x}{sin\ x}\right]_0^{\frac{\pi}{4}} == log\left[\frac{1}{sin\ x\ cos\ x}\right]_0^{\frac{\pi}{4}}$

$= -log(sin\quad x\ cos\ x)_0^{\frac{\pi}{4}}$

$= -log\left(sin\frac{\pi}{4}.cos\frac{\pi}{4}\right) + log(sin0\quad cos0)$

$= -log\left(\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}\right) + 0$

$= -log\left(\frac{1}{2}\right) = -(log1 - log2) = log2$

46) a)

Given E = Ri + L $\frac{di}{dt}$

$\frac{E}{L} = \frac{Ri}{L} + \frac{di}{dt}$

$\Rightarrow \frac{Ri}{L} + \frac{di}{dt} = \frac{E}{L}$

This is a linear differential equation

$Here \quad P = \frac{R}{L} and \quad Q = \frac{E}{L}$

$\therefore \int pdt = \int \frac{R}{L}dt = \frac{R}{L}t$

$\therefore I.F = e^{\int pdt} = e^{\frac{Rt}{L}}$

$\therefore$ Solution is $ie^{\int pdt} = \int Qe^{\int pdt}dt + C$

$\Rightarrow ie^{\frac{Rt}{L}} = \int \frac{E}{L}.e^{\frac{Rt}{L}}dt + C$

$\therefore ie^{\frac{Rt}{L}} = \frac{E}{L}\frac{e^{\frac{Rt}{L}}}{\frac{R}{L}}dt + C$

$i = \frac{E}{R}e^{\frac{Rt}{L}} + C$

$i = \frac{E}{R} + ce^{-\frac{Rt}{L}}$

When E = 0,

$i = 0 + ce^{-\frac{Rt}{L}}$

$\Rightarrow i = ce^{-\frac{Rt}{L}}$

(OR)

b)

$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ -x+1 & 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

(i) Distribution function

Case 1 : x < -1

F(x) = $\int_{-\infty}^{x} f(u)du$ = 0

Case 2 : -1 ≤ x < 0

$\int_{-\infty}^{x} f(u)du$

= $\int_{-\infty}^{x} f(x)dx = \left[\frac{x^2}{2} + x\right]_{-1}$

= $\left(\frac{u^2}{2} + u\right) = \frac{x^2}{2} + x - \left(\frac{1}{2} + 1\right)$

= $\frac{x^2}{2} + x + \frac{1}{2}$

Case 3 : 0 ≤ x < 1,

$F(X) = \int_0^x (-x+1)dx = \left[-\frac{x^2}{2} + x\right]_0^x$

= $\left(-\frac{x^2}{2} + x\right) - (0) = \frac{x^2}{2} + x$

When 1 ≤ x,

$F(x) = \int_1^x f(x)dx = \int_1^x 0dx$

$$= \therefore F(X) = \begin{cases} \frac{x^2}{2} + x + \frac{1}{2} & -1 \le x < 0 \\ -\frac{x^2}{2} + x & 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

(ii) p(0.5 ≤ X ≤ 0.5)

= $\int_{-0.5}^{0.5} f(x)dx = \int_{0.5}^{0} f(x)dx + \int_0^{0.5} f(x)dx$

= $\int_{-0.5}^{0} (x+1)dx + \int_0^{0.5} (-x+1)dx$

= $\left[\frac{x^2}{2} + x\right]_{-0.5}^{0} + \left[\frac{-x^2}{2} + x\right]_0^{0.5}$

= $0 - \left(\frac{0.5^2}{2} - 0.5\right) + \left(-\frac{(0.5)^2}{2} + 0.5\right) - 0$

= $-\left(\frac{.25}{2} - 0.5\right) + \left(\frac{-0.25}{2} + 0.5\right)$

= $\frac{.25}{2} + 0.5 - \frac{0.25}{2} + 0.5 = 0.25 + 1$

= 0.75

47) a)

given A = {Q\{1}}

A is defined on A by x*y = x+y-xy

Let x,y ≠ 1

∴ x*y = x + y - xy

Now to prove that x + y - xy ≠ 1

Let us assume that x + y - xy = 1

x+y-xy-1 = 0

(x-1)-y(x-1) = 0

(x-1)(1-y) = 0

x =1 or y = 1 which is a false [∵x, y ≠ 1]

∴ is a binary operation on A.

Commutative property:

Let x,y ∈A ⇒ x, y≠1

∴x*y = x+y-xy

and y*x = y+x-yx

⇒x+y = y*x∀x, y∈A

A has commutative property under *

Associative property:

Let x,y,z ∈A ⇒x,y,z≠1

Consider (x*y)*z = (x+y-xy)*z

= x +y~xy +z- (x +y-xy)z

= x +y-xy+ z-xz- yz + xyz

= x +y+z-xy-yz-zx +xyz ...(1)

= x +y+z - yz - x (y + z - yz)

= x +y +z-yz-zy-xz +xyz ..(2)

From (1) & (2), (x*y)*z = x*(y*z)

A has associative property under *.

(OR)

b) Given $r^2 = x^2 + y^2 + z^2$

$\log r^2 = \log (x^2 + y^2 + z^2)$

$\Rightarrow 2 \log r = \log (x^2 + y^2 + z^2)$

$\therefore 2V = \log (x^2 + y^2 + z^2)$ [∵ V = log r]

$\Rightarrow V = \frac{1}{2} \log (x^2 + y^2 + z^2)$

$$\frac{\partial V}{\partial x} = \frac{1}{2}\frac{2x}{x^2+y^2+z^2} = \frac{x}{x^2+y^2+z^2}$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{(x^2+y^2+z^2)(1)-x(2x)}{(x^2+y^2+z^2)^2}$$

$$= \frac{x^2+y^2-x^2}{(x^2+y^2+z^2)^2}$$

III^{ty} $\frac{\partial^2 V}{\partial y^2} = \frac{x^2+y^2-y^2}{(x^2+y^2+z^2)^2}$

$$\therefore \frac{\partial^2 V}{\partial z^2} = \frac{x^2+y^2-z^2}{(x^2+y^2+z^2)^2}$$

$$\therefore \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{y^2+z^2-x^2+z^2+x^2-y^2+x^2+y^2-z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^2} = \frac{1}{x^2+y^2+z^2}$$

$$= \frac{1}{r^2}$$

Hence proved.