Legendre $(P_n(x))$	Hermite $(H_n(x))$	Laguerre $(L_n(x))$
Differential equation is $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$	Diff. eqn is $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0$	Diff. eqn is $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$
Polynomial equation $Y = P_n(x) = \sum_{r=0}^{N} \frac{(-1)^r (2n-2r)!}{2^n \cdot r!(n-2r)!(n-r)!} \cdot x^{n-2r}$	Polynomial eqn is $y = H_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \cdot n!}{r!(n-2r)!} \cdot (2x)^{n-2r}$ $y = H_n(-x) = \sum_{r=0}^{\infty} \frac{(-1)^r \cdot n!}{r!(n-2r)!} (-2x)^{n-2r}$	polynomial eqn $Y = L_n(x) = \sum_{r=0}^{n} \frac{(-1)^r \cdot n!}{(r!)^2 (n-r)!} \cdot x^r$
Generating function $\sum_{n=0}^{\infty} z^n P_n(x) = (1-2xz+z^2)^{-\frac{1}{2}}$	Generating fun $F(x,z) = e^{2zx-z^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} z^n$	Generating function $F(x,z) = \frac{e^{-xz/(1-z)}}{1-z} = \sum_{n=0}^{\infty} \frac{L_n(x)}{n!} z^n$
orthogonal property $\int_{-1}^{1} P_m(x) P_n(x)\,dx = \frac{2}{2n+1}\delta_{mn}$ $\delta_{mn} = 0 \quad m \neq n$ $\delta_{mn} = 1 \quad m = n$	orthogonal property $\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)\,dx = 0$ $\boxed{m \neq n}$ $\int_{-\infty}^{\infty} e^{-x^2} [H_n(x)]^2\,dx = \sqrt{\pi}\, 2^n n!\, \delta_{mn}$ $\boxed{m = n}$	orthogonal property $\int_{0}^{\infty} e^{-x} L_m(x) L_n(x)\,dx = 0 \quad m \neq n$ $\int_{0}^{\infty} e^{-x} [L_n(x)]^2\,dx = \frac{(-1)^{2n} \cdot n!}{(n!)^2} \cdot n!$ $m = n$

Legendre, $P_n(x)$	Hermite, $H_n(x)$	Laguerre, $L_n(x)$
Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n (x^2-1)^n$ $P_0(x) = 1$ $P_1(x) = x$ $P_2(x) = \frac{3x^2-1}{2}$ $P_3(x) = \frac{5x^3-3x}{2}$	Rodrigue's formula $H_n(x) = (-1)^n e^{x^2}\left(\frac{d}{dx}\right)^n e^{-x^2}$ $H_0(x) = 1$ $H_1(x) = 2x$ $H_2(x) = 4x^2-2$ $H_3(x) = 8x^3-12x$	Rodrigue's formula $L_n(x) = \frac{e^x}{n!}\left(\frac{d}{dx}\right)^n e^{-x} x^n$ $L_0(x) = 1$ $L_1(x) = 1-x$ $L_2(x) = \frac{1}{2!}(x^2-4x+2)$ $L_3(x) = \frac{1}{3!}(6-18x+9x^2-x^3)$
General solution $y = A P_n(x) + B Q_n(x)$	$y = Ay_1 + By_2$	$y = A L_n(x)$

Legendre $P_n(x)$	Hermite $H_n(x)$	Laguerre $L_n(x)$
Rodrique formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$ sub $n=0$ $P_0(x) = \frac{1}{2^0 \cdot 0!} \frac{d^0}{dx^0}(x^2-1)^0$ $= 1$ sub $n=1$ $P_1(x) = \frac{1}{2^1 \cdot 1!} \frac{d^1}{dx^1}(x^2-1)^1$ $= \frac{1}{2} \cdot \left(\frac{d}{dx}(x^2-1)\right)$ $= \frac{1}{2}(2x) = x$ sub $n=2$ $P_2(x) = \frac{1}{2^2 \cdot 2!} \frac{d^2}{dx^2}(x^2-1)^2$ $= \frac{1}{8} \frac{d^2}{dx^2}(x^4 - 2x^2 + 1)$ $= \frac{1}{8} \frac{d}{dx}(4x^3 - 4x)$ $= \frac{4}{8} \frac{d}{dx}(3x^2 - 1)$	Rodrique formula $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$ sub $n=0$ $H_0(x) = (-1)^0 e^{x^2} \frac{d^0}{dx^0} e^{-x^2}$ $= e^{x^2} e^{-x^2} = 1$ $n=1$ $H_1(x) = (-1)^1 e^{x^2} \frac{d}{dx} e^{-x^2}$ $= -e^{x^2}(-2x e^{-x^2})$ $= 2x$ $n=2$ $H_2(x) = (-1)^2 e^{x^2} \frac{d^2}{dx^2} e^{-x^2}$ $= e^{x^2} \frac{d}{dx}(-2x e^{-x^2})$ $= e^{x^2}[-2x(-2x e^{-x^2}) - 2e^{x^2}]$ $= -2 + 4x^2$ $= 4x^2 - 2$	Rodrique formula $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} x^n e^{-x}$ sub $n=0$ $L_0(x) = \frac{e^x}{0!}(x^0 e^{-x}) = 1$ sub $n=1$ $L_1(x) = \frac{e^x}{1!} \frac{d}{dx}(x e^{-x})$ $= e^x \frac{d}{dx}(x e^{-x})$ $= e^x (e^{-x} - x e^{-x})$ $= e^x e^{-x} - x e^x e^{-x}$ $= 1 - x$ $n=2$ $L_2(x) = \frac{e^x}{2!} \frac{d^2}{dx^2}(x^2 e^{-x})$ $= \frac{e^x}{2} \frac{d}{dx}(2x e^{-x} - x e^{-x} x^2)$ $= \frac{e^x}{2!}\left(2(e^{-x} - x e^{-x}) - (3x^2 e^{-x} + x^3(-x e^{-x})\right)$ $= \frac{1}{2!}(2 - 4x + x^2)$